
Results on the Electroweak Phase Transition in the NMSSM with Explicit CPV

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Jan. 12, 2005

YITP Workshop, “CP violation and matter and anti-matter asymmetry”

Contents

- We study the PT in the NMSSM
- The **weak scale** vev. of the **singlet scalar** is considered
- We find **four different types** of phase transitions
- Three of which have **two-stage** nature
- One of the two-stage transitions admits strongly first order EWPT, even with **heavy squarks**
- We introduce tree-level CP violation

Introduction

Electroweak baryogenesis (EWBG)

- 📍 is related to physics at our reach
- 📍 require strongly first-order phase transition
→ light boson is required

EWBG in the Minimal SM

- 📍 light boson, $m_h > 50 \text{ GeV}$ conflict to present bound,
 $m_{h_{\text{SM}}} > 114 \text{ GeV}$
- 📍 CPV in the CKM matrix is too small to generate sufficient baryon number

Introduction

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- is related to physics at our reach
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EWBG in the MSSM

- with a light top squark
The bound on the lightest Higgs boson restricts the acceptable parameter space severely
- CPV in the soft SUSY X terms
first-order EWPT is **weakened**, when the stop-sector CP violation is large [K. Funakubo, S. T. and F. Toyoda, PTP101]

Introduction

Electroweak baryogenesis (EWBG)

- is related to physics at our reach
- require strongly first-order phase transition
→ light boson is required

EWBG in the NMSSM

- reduces to the MSSM in some limit
For such peculiar parameters, we expect the same behavior of the EWPT as that in the MSSM, which has been extensively studied
- We focus on the parameter space far from the MSSM

NMSSM

Next-to-Minimal Supersymmetric Standard Model

- Light Higgs
- CP Violation
- Parameter Set

[K. Funakubo and S. T. hep-ph/0409294]

The Model

$\langle \text{NMSSM} \rangle = \langle \text{MSSM} \rangle + (\text{one gauge singlet})$

$$W = f_d H_d Q D^c - f_u H_u Q U^c - \lambda N H_d H_u - \frac{\kappa}{3} N^3$$

- solves the μ -problem in the MSSM

μ -parameter is induced as $\mu = \lambda \langle N \rangle$

- no dimensional parameter in W

The scale by which NMSSM is characterized comes from SUSY-breaking terms

In the NMSSM, $m_{\text{SUSY}} \gg \langle H \rangle$ or $\langle N \rangle$ is unnatural

The Model

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$$W = f_d H_d Q D^c - f_u H_u Q U^c - \lambda N H_d H_u - \frac{\kappa}{3} N^3$$

SUSY-breaking soft terms:

$$\mathcal{L}_{\text{soft}} \ni -m_n^2 n^* n + \left[\lambda A_\lambda n \Phi_d \Phi_u + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.} \right]$$

• unremovable CP phases

• global Z_3 symmetry \rightarrow Domain wall problem

We assume it's already broken by higher dimensional operator

The Model - Higgs Sector

The tree-level Higgs potential:

$$\begin{aligned} V = & m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u + m_N^2 n^* n - (\lambda A_\lambda n \Phi_d \Phi_u + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.}) \\ & + \frac{g_2^2 + g_1^2}{8} (\Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u)^2 + \frac{g_2^2}{2} (\Phi_d^\dagger \Phi_u)(\Phi_u^\dagger \Phi_d) \\ & + |\lambda|^2 n^* n (\Phi_d^\dagger \Phi_d + \Phi_u^\dagger \Phi_u) + |\lambda \Phi_d \Phi_u + \kappa n^2|^2 \end{aligned}$$

Vacuum expectation values:

$$\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle \Phi_u \rangle = \frac{e^{i\theta}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle n \rangle = \frac{e^{i\phi}}{\sqrt{2}} v_n$$

Light Higgs

The structure of the mass matrix changes from that in the MSSM (3 scalar and 2 pseudoscalar if CP cons.)

$$\mathcal{M}^2 = \begin{pmatrix} \mathcal{M}_S^2|_{2 \times 2} & \\ & m_A^2 \end{pmatrix} \rightarrow \begin{pmatrix} \mathcal{M}_S^2|_{3 \times 3} & \mathcal{I} \\ \mathcal{I} & \mathcal{M}_P^2|_{2 \times 2} \end{pmatrix}$$

👉 “light-Higgs bosons” are allowed \rightarrow Strong 1st order PT

$$m_{h_1} < 114\text{GeV} \quad \text{with} \quad g_{ZZh_1} \sim 0.1$$

because of the small g_{ZZh_1}

Then the lightest Higgs scalar h_1 can escape from the bound

$$m_{h_{\text{SM}}} > 114\text{GeV}$$

Therefore, we can consider $h_2 \sim h_{\text{SM}}$

[Miller, et. al. NPB681]

Virtues of the NMSSM

• light Higgs \rightarrow Strong 1st order PT

Virtues of the NMSSM

- light Higgs \rightarrow Strong 1st order PT
- CP violation at the tree level, $\mathcal{I} \rightarrow$ Sufficient B number

$$\mathcal{I} = \text{Im}(\lambda\kappa^* e^{i(\theta-2\phi)})$$

from the vacuum condition, \mathcal{I} related to the other combinations

$$\mathcal{I} \sim \text{Im}(\lambda A_\lambda e^{i(\theta+\phi)}) \sim \text{Im}(\kappa A_\kappa e^{3i\phi})$$

- Only one combination of the phases is physical
- Our formalism is independent of parametrisation for the phases
- One can choose the phase combination not to affect the nEDM

Virtues of the NMSSM

- light Higgs \rightarrow Strong 1st order PT
- CP violation at the tree level, $\mathcal{I} \rightarrow$ Sufficient B number
- Pure NMSSM parameter set
vs. MSSM limit

$v_n \rightarrow \infty$ with λv_n and κv_n fixed \Rightarrow MSSM [Ellis, et. al. PRD39]

Moderate v_n value ~ 200 GeV, and soft masses $< \text{TeV}$

\rightarrow new features expected

- light Higgs bosons are realized
- mass bound on the charged Higgs bosons

[K. Funakubo and S. T. hep-ph/0409294]

Virtues of the NMSSM

- light Higgs \rightarrow Strong 1st order PT
- CP violation at the tree level, $\mathcal{I} \rightarrow$ Sufficient B number
- Pure NMSSM parameter set

These are Good features for Electroweak baryogenesis

EWPT

Electroweak Phase Transition in the NMSSM

- 🔴 Cubic Term
- 🔴 Types of the PT
- 🔴 Effects of the CPV

[K. Funakubo and S. T. hep-ph/0501052]

Effective Potential

The effective potential at finite temperature

$$V_{\text{eff}}(\mathbf{v}; T) = V_{\text{tree}}(\mathbf{v}) + \Delta V(\mathbf{v}; T)$$

The one-loop correction

$$\begin{aligned} \Delta V(\mathbf{v}; T) = & 3 \left[F_0(\bar{m}_Z^2) + \frac{T^4}{2\pi^2} I_B \left(\frac{\bar{m}_Z}{T} \right) + 2F_0(\bar{m}_W^2) + 2 \cdot \frac{T^4}{2\pi^2} I_B \left(\frac{\bar{m}_W}{T} \right) \right] \\ & - 2 \left[F_0(\bar{m}_{\psi_N}^2) + \frac{T^4}{2\pi^2} I_F \left(\frac{\bar{m}_{\psi_N}}{T} \right) \right] \\ & + N_C \sum_{q=t,b} \left\{ 2 \sum_{j=1,2} \left[F_0(\bar{m}_{\tilde{q}_j}^2) + \frac{T^4}{2\pi^2} I_B \left(\frac{\bar{m}_{\tilde{q}_j}}{T} \right) \right] - 4 \left[F_0(\bar{m}_q^2) + \frac{T^4}{2\pi^2} I_F \left(\frac{\bar{m}_q}{T} \right) \right] \right\} \end{aligned}$$

where \bar{m} denotes the field-dependent masses and

$$F_0(m^2) = \frac{1}{64\pi^2} (m^2)^2 \left(\log \frac{m^2}{M^2} - \frac{3}{2} \right), \quad I_{B(F)}(a) = \int_0^\infty dx x^2 \log \left(1 \mp e^{-\sqrt{x^2+a^2}} \right)$$

Effective Potential

The effective potential at finite temperature

$$V_{\text{eff}}(\mathbf{v}; T) = V_{\text{tree}}(\mathbf{v}) + \Delta V(\mathbf{v}; T)$$

The order parameters

$$\mathbf{v} = (v_1, v_2, v_3, v_4, v_5)$$

$$\equiv (v_d, v_u \cos \Delta\theta, v_u \sin \Delta\theta, v_n \cos \Delta\varphi, v_n \sin \Delta\varphi)$$

where $\Delta\theta = \theta - \theta_0$ and $\Delta\varphi = \varphi - \varphi_0$ (The $_0$ means $T = 0$ value)

EWPT is a first-order PT \Leftrightarrow the \mathbf{v} Jump vacuum to vacuum

We numerically minimize the $V_{\text{eff}}(\mathbf{v}; T)$ in 5 dim. space, and observe the phase transition

EWPT – naive argument

$$\text{order parameters : } \begin{cases} v_d = v \cos \beta(T) = y \cos \alpha(T) \cos \beta(T) \\ v_u = v \sin \beta(T) = y \cos \alpha(T) \sin \beta(T) \\ v_n = y \sin \alpha(T) \end{cases}$$

→ y^3 -term, even at the tree level [Pietroni, NPB402]

$$V = \frac{1}{2} \left((m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta) \cos^2 \alpha + m_N^2 \sin^2 \alpha \right) y^2 \\ - \left(R_\lambda \cos^2 \alpha \sin \alpha \cos \beta \sin \beta + \frac{1}{3} R_\kappa \sin^3 \alpha \right) y^3 + \dots$$

The y^3 -term makes the phase transition **along y -direction** strongly first order

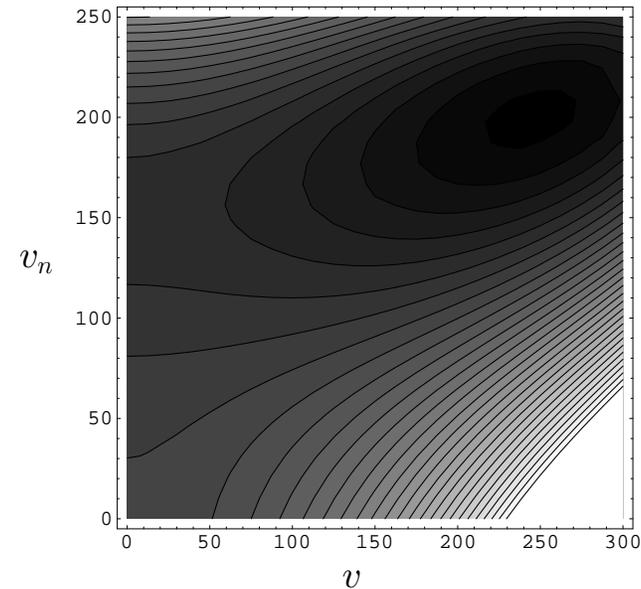
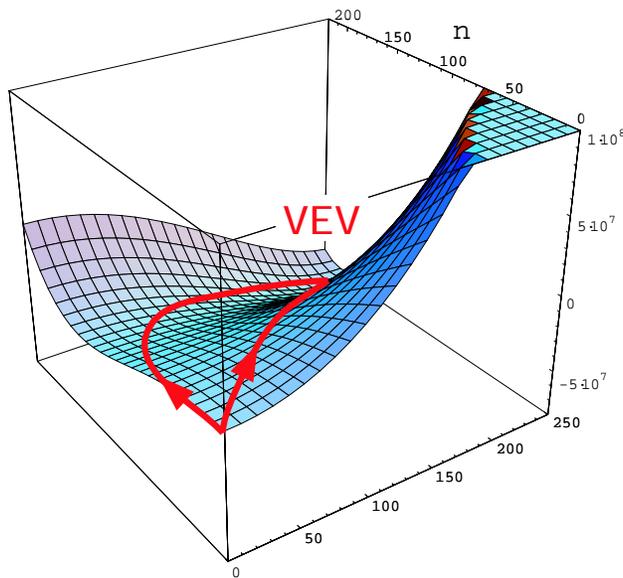
but

validity of the parametrization with a constant α is not obvious

EWPT – naive argument

$$\text{order parameters : } \begin{cases} v_d = v \cos \beta(T) = y \cos \alpha(T) \cos \beta(T) \\ v_u = v \sin \beta(T) = y \cos \alpha(T) \sin \beta(T) \\ v_n = y \sin \alpha(T) \end{cases}$$

→ y^3 -term, even at the tree level [Pietroni, NPB402]



The phase transitions in the NMSSM are classified into several types

Classification of the Phases

We found four phases

phase	order parameters	symmetries
EW	$v \neq 0, v_n \neq 0$	fully broken
I, I'	$v = 0, v_n \neq 0$	local $SU(2)_L \times U(1)_Y$
II	$v \neq 0, v_n = 0$	global $U(1)$
SYM	$v = v_n = 0$	$SU(2)_L \times U(1)_Y$, global $U(1)$

Symmetries

v : order parameter of the gauge symmetry

v_n : order parameter of a global $U(1)$ symmetry

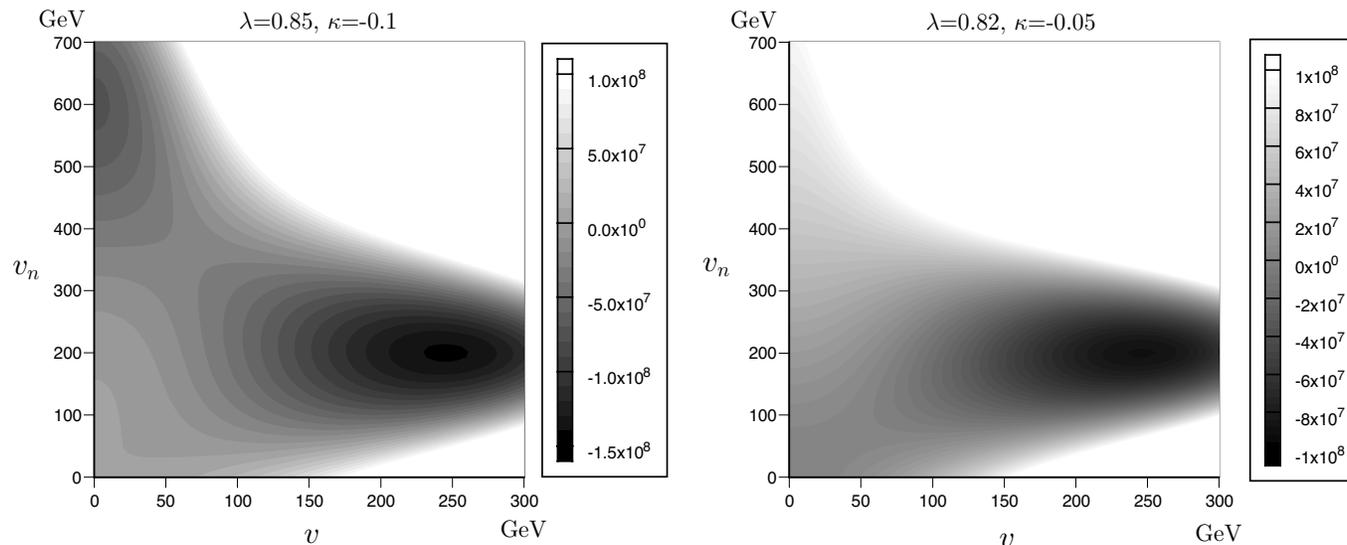
because in the subspace of $v_n = 0$ ($v_4 = v_5 = 0$), the effective potential is invariant under the global $U(1)$ transformation

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Examples which develop the phase-I' and II at high T



Classification of the Phases

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The existence of the phases-I, I' and II is a novel feature of the NMSSM

phase-I and I' can be understood: in subspace $v = 0$

$$\hat{V}_0(v_n) = V_0(0, 0, 0, v_n \cos \Delta\varphi, v_n \sin \Delta\varphi) = \frac{1}{2} m_N^2 v_n^2 - \frac{1}{3} \hat{R}_\kappa v_n^3 + \frac{|\kappa|^2}{4} v_n^4,$$

where $\hat{R}_\kappa \propto \kappa$

For a very small $|\kappa|$, the intermediate vacuum can develop far from the origin, phase-I'

Classification of the Phases

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Types of transitions

A: SYM \rightarrow I \Rightarrow EW B: SYM \rightarrow I' \Rightarrow EW

C: SYM \Rightarrow II \rightarrow EW D: SYM \Rightarrow EW

\triangle Type A: MSSM-like EWPT, which proceeds with almost constant $v_n(T)$

\odot Type B: Strongly first-order EWPT, **without light stop, It's NEW!**

\times Type C: **It's NEW**, but CP phase is ambiguous \rightarrow cannot produce net baryon number

\ominus Type D: Strongly first-order (need light Higgs), with light stop

EWPT – Type B

Parameters

- heavy stop
- small κ

$\tan \beta$	m_{H^\pm}	A_κ	v_n	λ	κ	$m_{\tilde{q}}$	$m_{\tilde{t}(\tilde{b})_R}$
5	600GeV	-100GeV	200GeV	0.85	-0.1	1000GeV	800GeV

Mass spectrum

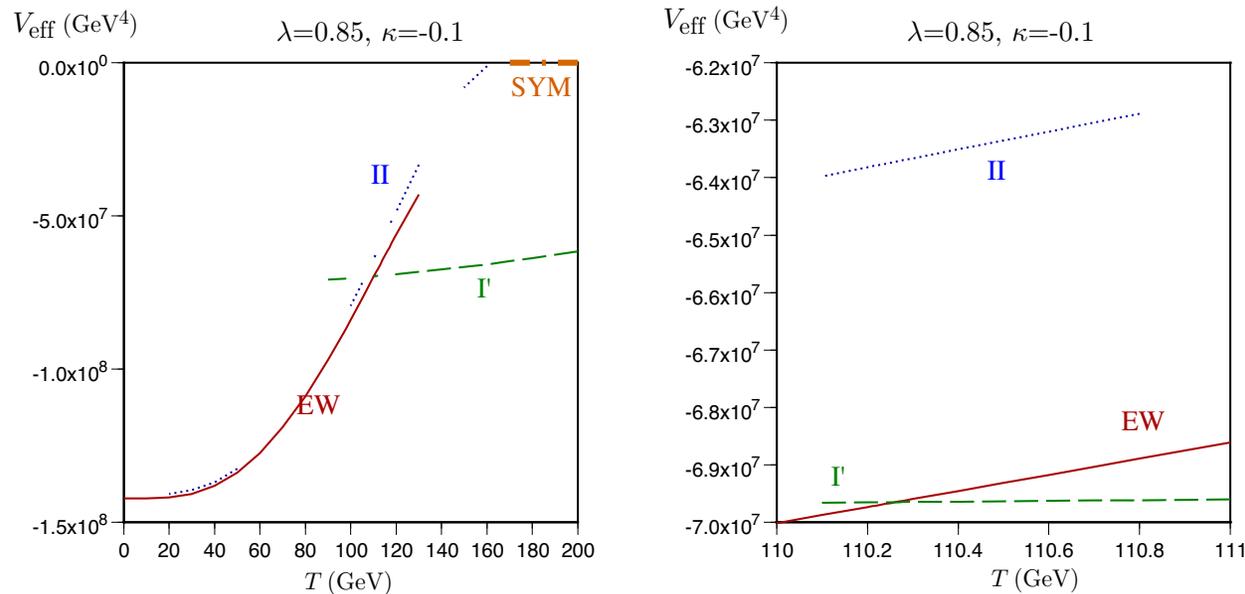
- SM-like Higgs is a third-lightest one
- h_2 and h_5 are pseudoscalar
- heaviest scalar mass and heaviest pseudoscalar mass are the same order
 $\text{Tr}\mathcal{M}_S \simeq \text{Tr}\mathcal{M}_P$ if m_{H^\pm} is large

	h_1	h_2	h_3	h_4	h_5
mass(GeV)	38.89	75.31	131.11	625.61	627.95
$g_{H_i ZZ}^2$	6.213×10^{-8}	0	0.999	6.816×10^{-5}	0

EWPT – Type B

The T dependence of the local minima of the V_{eff}

- distinct convergent points at the same T are displayed
- The values of the V_{eff} are subtracted by the value at the origin
- right-hand plot is the close view of the left-hand one near the T_C



- The T dependence of phase-I' is weak because at that phase, $v = 0$

$(v, v_n) = (208.13 \text{ GeV}, 248.85 \text{ GeV}) \rightarrow (0, 599.93 \text{ GeV})$ at $T_C = 110.26 \text{ GeV}$

\Rightarrow strongly first order PT; $v_C/T_C \sim 2$

CP-Violating Case

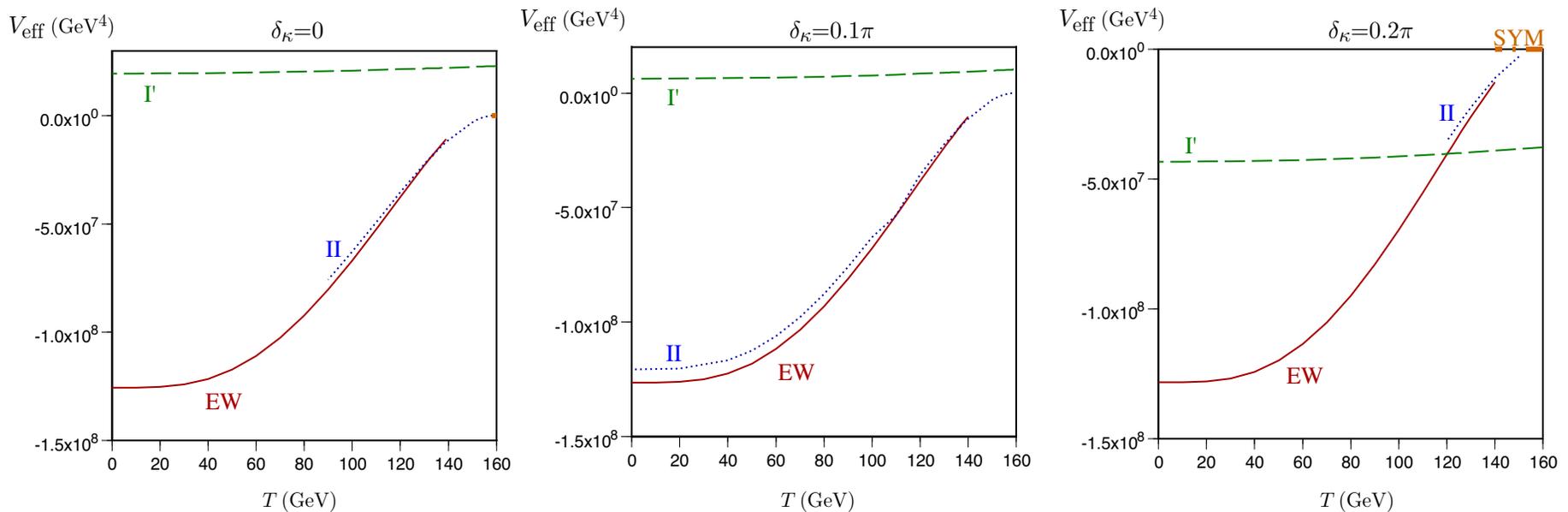
Source of the CPV

- There are infinite sets of CP violating parameters which yield the same value of \mathcal{I}
→ We constrain them so that the phase relevant to the nEDM vanish
- We take $\text{Arg}\kappa \equiv \delta_\kappa$ as an independent parameter

CP-Violating Case

Parameters

- almost same as Type-B example, but $(\lambda, \kappa) = (0.85, -0.1) \rightarrow (0.83, -0.07)$



- For $\delta_\kappa \gtrsim 0.3\pi$, the zero-temperature vacuum is in the phase-I'
- value of the phase-I' is decreases with δ_κ
- PT is type-C for $\delta_\kappa = 0, 0.1\pi \Rightarrow$ type-B for $\delta_\kappa = 0.2\pi$, strongly 1st order
- first-order EWPT is **not weakend** by tree-level CPV

Summary

In the NMSSM, we worked out to

- 👉 search allowed parameters (previous work)
- 👉 classify the phases into four types of the phase transitions

Our next interest is in

- 👉 For calculating the generated baryon number, sphaleron transition rate is needed
 - ← sphaleron solution and energy in the NMSSM
- 👉 drawing a phase diagram
 - ← more computer power
- 👉 application of the phases