Dark Energy and Neutrino Model

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References

• N.Weiner “Summer Institute 2004 lecture”
• R.D.Peccei, hep-ph/0411137
1. Introduction

Energy composition of the universe

- Neutrino, Photon 0. 5%
- Baryon 5%
- Dark Matter 25%
- Dark Energy 70%

Dark Energy
- Energy component with negative pressure. → Cosmic acceleration.
- Doesn’t make large scale structure unlike galaxies.

Candidates
- Cosmological Constant
- Quintessence
- k-essence
- Phantom energy
- Wet dark fluid
- Mass varying neutrino
Parameters & Equations

- Hubble rate:
  \[ H \equiv \frac{\dot{a}}{a} \]

- Energy density: \( \rho_j \) \( (j = \gamma, \nu, m, DE, \ldots) \)

- Critical density:
  \[ \rho_c \equiv \frac{3H^2}{8\pi G} \]

- Energy density in \( j \)th species over \( \rho_c \):
  \[ \Omega_j \equiv \frac{\rho_j}{\rho_c} \]

- Scale factor: \( a \)

- Redshift: \( z \)
  \[ 1 + z \equiv \frac{\lambda_{ob}}{\lambda_{em}} = \frac{a(t_0)}{a(t_1)} = \frac{1}{a(t_1)} \quad (a(t_0) = 1) \]

Examples
- \( z = 0 \) \( \Rightarrow \) The present.
- \( z = 1 \) \( \Rightarrow \) The universe is half the size of the present one.
• Pressure to energy-density ratio:
\[ \omega \equiv \frac{p}{\rho} \]

• Conservation of energy:
\[ \frac{d(\rho a^3)}{dt} + p \frac{da^3}{dt} = 0 \]
\[ \rightarrow \rho \propto a^{-3(1+\omega)} \]

Matter \hspace{1cm} \omega = 0 \hspace{1cm} \rightarrow \rho_m \propto a^{-3}
Radiation \hspace{1cm} \omega = 1/3 \hspace{1cm} \rightarrow \rho_r \propto a^{-4}
Cosmological Constant \hspace{1cm} \omega = -1 \hspace{1cm} \rightarrow \rho_\Lambda \propto a^0

• Observation: \(-1 < \omega < -0.7\)

• Einstein equation (Friedmann equation)
\[ H^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3} \rho \]
\[ \ddot{a} + H^2 + \frac{k}{a^2} - \Lambda = -8\pi G p \]
2. Mass varying neutrino model

—Motivation—

**Cosmological Constant**

- The constant added to the Einstein equation as vacuum energy to make a stable cosmic model.

- In quantum field theory, the value of the vacuum energy density is some **120** orders (55 orders in SUSY) of magnitude larger than the observed value.

- It is **unnatural** to exist a mechanism which does cancellation **120** orders and leaves a little quantity.

- Coincidence problem.

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Quintessence (time dependent)

• The constraint of potential is severe.
• $m_q \sim 10^{-33} \text{eV}$

Baryon

• We know too well. $\Leftrightarrow$ WMAP, BBN

Neutrino

• Neutrino density is quite uncertain.
  WMAP & terrestrial measurement of $m_\nu$:
  \[7 \times 10^{-4} < \Omega_\nu \leq 0.02\]
• Neutrino mass scale is close to DE scale:
  \[\sim (2 \times 10^{-3} \text{eV})^4\]
—Model Independent—

Assumption 1

Variable parameter:

\[ m_\nu (A) : \text{Function of scalar field } A \text{ ("acceleron").} \]

(R.Fardon, A.E.Nelson, N.Weiner)

\[ m_\nu (T) : \text{Function of temperature.} \]

(R.D.Peccei)

Assumption 2

The energy density in the dark sector has two component:

\[
\omega = \frac{\rho_{\text{dark}}}{\rho_{\text{dark}}}
\]

\[ \rho_{\text{dark}} = \rho_\nu + \rho_{\text{dark energy}} \]

\[ (\rho_{\text{dark energy}} = \rho_{\text{dark energy}}(m_\nu) = V(m_\nu)) \]

Assumption 3

\( \rho_{\text{dark}} \) is stationary with respect to variations in the neutrino mass:

\[
\frac{\partial \rho_{\text{dark}}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial \rho_{\text{dark energy}}}{\partial m_\nu} = 0
\]

Assumption 4

The background neutrinos at the present are non-relativistic:

\[ \rho_\nu^0 = m_\nu^0 n_\nu^0 \]
\[ \rho_{\text{dark}} = \rho_\nu + \rho_{\text{dark energy}}(m_\nu) \]
\[ = m_\nu n_\nu + \rho_{\text{dark energy}}(m_\nu) \]
The case of non-relativistic
—1 family model—

Neutrino energy density:

\[ \rho_\nu = m_\nu n_\nu \]

Relations of \( \rho, a, n, \omega \):

\[ \rho(=V) \propto a^{-3(1+\omega)} \propto n^{1+\omega} \]
\[ n \propto a^{-3} \]

Equation of state:

\[ \omega + 1 = -\frac{\partial \log \rho_{dark}}{3 \partial \log a} \]
\[ = -\frac{a}{3 \rho_{dark}} \left( m_\nu \frac{\partial n_\nu}{\partial a} + \frac{\partial m_\nu}{\partial a} n_\nu \right) \]
\[ + \rho_{dark \ energy} (m_\nu \frac{\partial m_\nu}{\partial a}) \]
\[ = \frac{m_\nu n_\nu}{\rho_{dark}} \]
\[ = \frac{m_\nu n_\nu}{m_\nu n_\nu + \rho_{dark \ energy}} \]
\[
\omega_0 + 1 = \frac{m_{\nu}^0 n_{\nu}^0}{m_{\nu}^0 n_{\nu}^0 + \rho_{\text{dark energy}}^0}
\]

[Estimate]

\[
\omega_0 \sim -0.9
\]

\[
\downarrow
\]

\[
m_{\nu}^0 n_{\nu}^0 = 0.1 \rho_{\text{dark}}^0
\]

\[
\rho_{\text{dark energy}}^0 = 0.9 \rho_{\text{dark}}^0
\]

\[
\rho_{\text{dark}}^0 \sim 2.73 \times 10^{-11} \ (eV)^4
\]

\[
n_{\nu}^0 \sim 8.83 \times 10^{-13} \ (eV)^3
\]

\[
\downarrow
\]

\[
m_{\nu}^0 = 3.09 \ (eV)
\]

This values for \( m_{\nu}^0 \) are not necessarily in contradiction with terrestrial limits on neutrino masses since the neutrino mass measured on earth may well differ from \( m_{\nu}^0 \), if there is an overdensity in the local group due to neutrino clustering.
The general case (Temperature dependent)

The neutrino density:

\[
\rho_\nu = T^4 F(\xi) \\
\xi = \frac{m_\nu(T)}{T} \\
F(\xi) = \frac{1}{\pi^2} \int_0^\infty dy y^2 \sqrt{y^2 + \xi^2} \frac{e^y + 1}{ey + 1}
\]

The stationary condition:

\[
\frac{\partial \rho_{dark}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial \rho_{dark \ energy}}{\partial m_\nu} = 0
\]

\[
= T^4 \frac{\partial F}{\partial \xi} \frac{\partial \xi}{\partial m_\nu} + \frac{\partial \rho_{dark \ energy}}{\partial m_\nu}
\]

\[
= T^3 \frac{\partial F}{\partial \xi} + \frac{\partial \rho_{dark \ energy}}{\partial m_\nu} = 0
\]

The equation of state:

\[
\omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{\rho_{dark \ energy}}{T^4 F(\xi)}]}
\]

\[
h(\xi) \equiv \frac{\xi \frac{\partial F(\xi)}{\partial \xi}}{F(\xi)}
\]
2.1. Power-law Potential

\[ V(m_\nu) = bm_\nu^a \]

There are two conditions for ordinary parameters.

(1) For the potential:

\[
m_\nu^0 n_\nu^0 + V(m_\nu^0) = 0.7 \rho_c
\]

\[
\Rightarrow m_\nu^0 n_\nu^0 + b(m_\nu^0)^a = 0.7 \rho_c
\]

(2) For the \( m_\nu \):

\[
\frac{\partial V(m_\nu)}{\partial m_\nu} = -T^3 \frac{\partial F}{\partial \xi} \quad \text{(Stationary Condition)}
\]

\[
downarrow
\]

\[
ab m_\nu^{a-1} = -\frac{T^3}{\pi^2} \int \frac{dyy^2(m_\nu/T)}{\sqrt{y^2 + (m_\nu/T)^2(e^y + 1)}}
\]

\[
\downarrow
\]

\[
ab(m_\nu^0)^{a-1} = -\frac{T_0^3}{\pi^2} \int \frac{dyy^2(m_\nu^0/T_0)}{\sqrt{y^2 + (m_\nu^0/T_0)^2(e^y + 1)}}
\]
The conditions for the power-law potential

\[ V(m_\nu) = bm_\nu^a \]

\[
\begin{cases}
  m_\nu^0 n_\nu^0 + b(m_\nu^0)^a = 0.7 \rho_c \\
  ab(m_\nu^0)^{a-1} = -\frac{T_0^3}{\pi^2} \int \frac{dy y^2(m_\nu^0/T_0)^2(e^y+1)}{\sqrt{y^2+(m_\nu^0/T_0)^2}}
\end{cases}
\]

The state of equation

\[
\omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{V(m_\nu)}{T^4F(\xi)}]}
\]

\[
= \frac{4 - h(\xi)}{3[1 - \frac{h(\xi)}{a}]}
\]

In the non-relativistic limit,

\[ \omega \to \omega_0, \quad h(\xi) \to 1, \quad m_\nu \to m_\nu^0; \]

\[ \omega_0 + 1 = \frac{a}{a - 1} \]
Example

\[ V(m_\nu) = -m_\nu^0 n_\nu^0 \left( \frac{\omega_0}{1 + \rho_0} \right) \left( \frac{m_\nu^0}{m_\nu} \right)^{1+\rho_0} \]

(R.D. Peccei)

\[ T^3 \frac{\partial F}{\partial \xi} + \frac{\partial V(m_\nu)}{\partial m_\nu} = 0 \text{ (Stationary Condition)} \]

\[ \downarrow \]

\[ m_\nu = m_\nu^0 \left( \frac{T}{T_0} \right)^3 g(\xi) \omega_0 \]

\[ g(\xi) = \frac{\int_0^\infty dy y^2 \xi}{\int_0^\infty \frac{dy y^2}{(ey+1)}} \]

\[ m = m_\nu / m_\nu^0 \]

\[ z = (T/T_0 - 1) \]

R.D. Peccei, hep-ph/0309800

\[ z \sim 20 \Rightarrow \sim O(10^8) \text{ (year)} \]
\[ \omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{V(m_\nu)}{T^4 F(\xi)}]} \quad (The \ state \ of \ equation) \]

\[ \downarrow \]

\[ \omega + 1 = \frac{4 - h(\xi)}{3[1 - \frac{\omega_0}{1+\omega_0} h(\xi) \}} \]

As the temperature goes down, quantum effects are important for the potential.
2.2. Log Potential

\[ V(m_\nu) = a \log\left(\frac{m_\nu}{b}\right) \]

(1) For the potential:

\[ m_\nu^0 n_\nu^0 + V(m_\nu^0) = 0.7 \rho_c \]

\[ \Rightarrow m_\nu^0 n_\nu^0 + a \log \frac{m_\nu^0}{b} = 0.7 \rho_c \]

(2) For the \( m_\nu \):

\[ \frac{\partial V(m_\nu)}{\partial m_\nu} = -T^3 \frac{\partial F}{\partial \xi} \quad (Stationary \ Condition) \]

\[ \downarrow \]

\[ \frac{a}{m_\nu} = -\frac{T^3}{\pi^2} \int \frac{dyy^2(m_\nu/T)}{\sqrt{y^2 + (m_\nu/T)^2(e^y + 1)}} \]

\[ \downarrow \]

\[ \frac{a}{m^0_\nu} = -\frac{T_0^3}{\pi^2} \int \frac{dyy^2(m^0_\nu/T_0)}{\sqrt{y^2 + (m^0_\nu/T_0)^2(e^y + 1)}} \]
The conditions for the log potential

\[ V(m_\nu) = a \log \left( \frac{m_\nu}{b} \right) \]

\[
\begin{align*}
& m_\nu^0 n_\nu^0 + a \log \frac{m_\nu^0}{b} = 0.7 \rho_c \\
& \frac{a}{m_\nu^0} = -\frac{T_0^3}{\pi^2} \int \frac{dy y^2 (m_\nu^0/T_0)}{\sqrt{y^2 + (m_\nu^0/T_0)^2 (ey+1)}}
\end{align*}
\]

\[ a \sim -2.80 \times 10^{-12} \]

\[ \log \left( \frac{m_\nu^0}{b} \right) \sim -9.01 \]
\[
\frac{a}{m_\nu} = -\frac{T^3}{\pi^2} \int \frac{dy y^2 (m_\nu/T)}{\sqrt{y^2 + (m_\nu/T)^2 (ey + 1)}}
\]

Different potentials have different temperature dependence of the neutrino mass.
The state of equation

\[
\omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{V(m_\nu)}{T^4 F(\xi)}]} = \frac{4 - h(\xi)}{3[1 - h(\xi) \log(\frac{m_\nu}{b})]}
\]

In the non-relativistic limit,

\[
\omega \to \omega_0 \, , \, h(\xi) \to 1 \, , \, m_\nu \to m^0_\nu;
\]

\[
\omega_0 + 1 = \frac{1}{1 - \log(\frac{m^0_\nu}{b})}
\]
3. SUSY model

(N.Weiner, "Summer Institute 2004 lecture")

\[ W = \lambda A N N \]

\[ A, N : \text{Chiral Superfield.} \]

Singlet under the SM gauge group.

\[ V = 4\lambda^2|\phi_a|^2|\phi_n|^2 + \lambda^2|\phi_n|^4 \]

The potential is decided automatically.

**Assumption**

\( \phi_a \) is a “acceleron”.

\[ \mathcal{L}_{mass} = m_D\nu_L\psi_n + m_D'\nu_L\psi_a + \lambda\phi_a\psi_n\psi_n + \lambda\phi_n\psi_a\psi_n \]

\[ M = \begin{pmatrix}
0 & m_D' & m_D \\
{m_D}' & 0 & \lambda\phi_n \\
m_D & \lambda\phi_n & \lambda\phi_a
\end{pmatrix} \]

\[ \lambda\phi_n \ll m_D', m_D \ll \lambda\phi_a \]

\[ \Rightarrow \pm m_D' + \frac{m_D^2}{2\lambda\phi_a}, \lambda\phi_a \]
\[ m_\nu \simeq \pm m'_D + \frac{m_D^2}{2\lambda \phi_a} \]

Two quasi-degenerate neutrinos exist.

\[ m_\nu \simeq m'_D + \frac{m_D^2}{2\lambda \phi_a} \]

\[ \Downarrow \]

\[ \phi_a \simeq \frac{m_D^2}{2\lambda (m_\nu - m'_D)} \]

\[ \Downarrow \]

\[ V = \frac{m_D^4 \phi_n^2}{(m_\nu - m'_D)^2} + \lambda^2 \phi_n^4 \]

(1) For the potential:

\[ m_\nu^0 n_\nu^0 + V(m_\nu^0) = 0.7 \rho_c \]

\[ \Rightarrow \quad \frac{m_D^4 \phi_n^2}{(m_\nu^0 - m'_D)^2} + \lambda^2 \phi_n^4 = 0.7 \rho_c \]
(2) For the \( m_\nu \):

\[
\frac{\partial V(m_\nu)}{\partial m_\nu} = -T^3 \frac{\partial F}{\partial \xi} \quad (\text{Stationary Condition})
\]

\[
-2 \frac{m_D^4 \phi_n^2}{(m_\nu - m_D')^3} = -\frac{T^3}{\pi^2} \int \frac{dyy^2(m_\nu/T)}{\sqrt{y^2 + (m_\nu/T)^2(e^y + 1)}}
\]

\[
2 \frac{m_D^4 \phi_n^2}{(m_\nu^0 - m_D')^3} = \frac{T_0^3}{\pi^2} \int \frac{dyy^2(m_\nu^0/T_0)}{\sqrt{y^2 + (m_\nu^0/T_0)^2(e^y + 1)}}
\]

The conditions for the potential

\[
V(m_\nu) = \frac{m_D^4 \phi_n^2}{(m_\nu - m_D')^2} + \lambda^2 \phi_n^4
\]

\[
\left\{ \begin{array}{l}
m_\nu m_\nu^0 + \frac{m_D^4 \phi_n^2}{(m_\nu^0 - m_D')^2} + \lambda^2 \phi_n^4 = 0.7 \rho_c \\
2 \frac{m_D^4 \phi_n^2}{(m_\nu^0 - m_D')^3} = \frac{T_0^3}{\pi^2} \int \frac{dyy^2(m_\nu^0/T_0)}{\sqrt{y^2 + (m_\nu^0/T_0)^2(e^y + 1)}}
\end{array} \right.
\]

\[
\lambda = 1 \\
\phi_n = 10^{-8} \quad (eV)
\]

\[\Rightarrow \quad m_D \sim 169 \quad (eV) \]

\[m_D' \sim -54.0 \quad (eV) \]

\[\lambda \phi_a \sim 251 \quad (eV)\]
\[
2 \frac{m_D^4 \phi_n^2}{(m_\nu - m_D')^3} = \frac{T^3}{\pi^2} \int \frac{dy y^2 (m_\nu / T)}{\sqrt{y^2 + (m_\nu / T)^2 (ey + 1)}}
\]

The state of equation

\[
\omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{V(m_\nu)}{T^4 F(\xi)}]}
\]

\[
= \frac{4 - h(\xi)}{3[1 + h(\xi) \frac{(m_\nu - m_D')^3}{2m_D^4 m_\nu} \left\{ \frac{m_D^4}{(m_\nu - m_D')^2 + \lambda^2 \phi_n^2} \right\}]}\]

In the non-relativistic limit,

\[
\omega \to \omega_0 , \quad h(\xi) \to 1 , \quad m_\nu \to m_\nu^0 ;
\]

\[
\omega_0 + 1 = \frac{1}{1 - \frac{(m_\nu^0 - m_D')^3}{2m_D^4 m_\nu^0} \left\{ \frac{m_D^4}{(m_\nu^0 - m_D')^2 + \lambda^2 \phi_n^2} \right\}}
\]
4. Future

- There are some models which are consistent with some observations in “Mass Varying Neutrinos” scenario.

![Graph with lines and labels: Power law Model, Log Model, SUSY Model]

- SUSY-breaking terms should be included in order to get the realistic model.