Dark Energy and Neutrino Model

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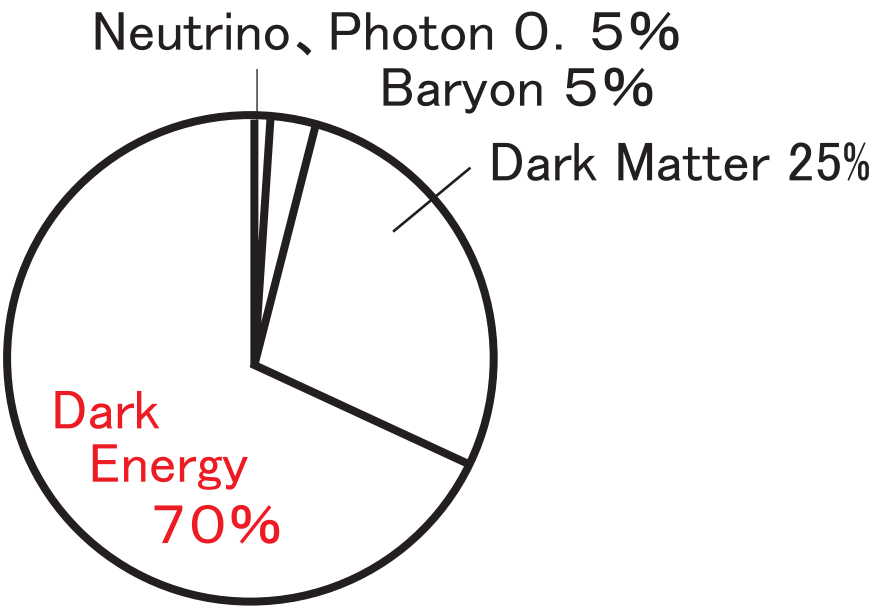
References

- R.Fardon,A.E.Nelson,N.Weiner,astro-ph/0309800
- N.Weiner "Summer Institute 2004 lecture"
- R.D.Peccei,hep-ph/0411137

1.Introduction

Energy composition of the universe



Dark Energy

- Energy component with negative pressure.
  →Cosmic acceleration.
- Doesn't make large scale structure unlike galaxies.

Candidates

- Cosmological Constant
- Quintessence
- k-essence
- Phantom energy
- Wet dark fluid
- Mass varying neutrino

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Parameters & Equations

- Hubble rate:

$$H \equiv \frac{\dot{a}}{a}$$

- Energy density : $\rho_j \quad (j = \gamma, \nu, m, DE, , ,)$
- Critical density:

$$\rho_c \equiv \frac{3H^2}{8\pi G}$$

- Energy density in j th species over ρ_c:

$$\Omega_j \equiv \frac{\rho_j}{\rho_c}$$

- Scale factor: a
- Redshift: z

$$1 + z \equiv \frac{\lambda_{ob}}{\lambda_{em}} = \frac{a(t_0)}{a(t_1)} = \frac{1}{a(t_1)} \quad (a(t_0) = 1)$$

<u>Examples</u>

$z = 0 \Rightarrow$ The present.

$z = 1 \Rightarrow$ The universe is half the size of the present one.

- Pressure to energy-density ratio:

$$\omega \equiv \frac{p}{\rho}$$

- Conservation of energy:

$$\frac{d(\rho a^3)}{dt} + p\frac{da^3}{dt} = 0$$
$$\rightarrow \rho \propto a^{-3(1+\omega)}$$

Matter	$\omega = 0$	$\rightarrow$	$\rho_m \propto a^{-3}$
Radiation	$\omega = 1/3$	$\rightarrow$	$\rho_r \propto a^{-4}$
Cosmological Constant	$\omega = -1$	$\rightarrow$	$\rho_\Lambda \propto a^0$

- Observation : $-1 < \omega < -0.7$

- Einstein equation(Friedmann equation)

$$H^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho$$
$$2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} - \Lambda = -8\pi G p$$

2.Mass varying neutrino model

—Motivation—

Cosmological Constant

- The constant added to the Einstein equation as vacuum energy to make a stable cosmic model.
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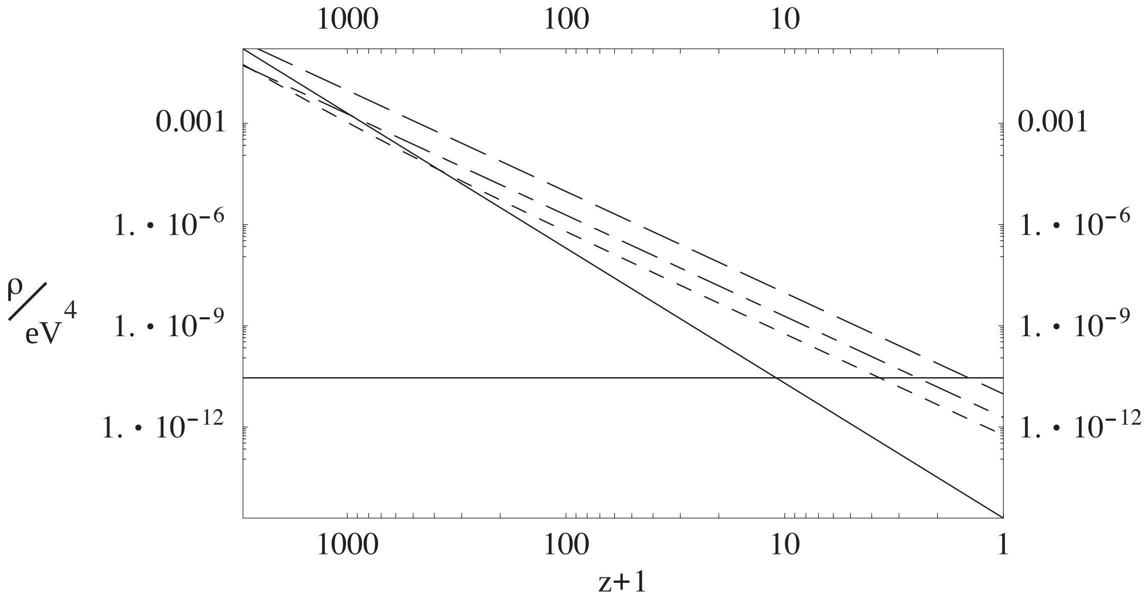
- In quantum field thory , the value of the vacuum energy density is some 120 orders (55 orders in SUSY) of magnitude larger than the observed value.
- It is unnatural to exist a mechanism which does cancellation 120 orders and leaves a little quantity.

- Coincidence problem.



R.Fardon,A.E.Nelson,N.Weiner,astro-ph/0309800

Quintessence (time dependent)

- The contraint of potential is severe.
- $m_q \sim 10^{-33} eV$

Baryon

- We know too well. $\Leftarrow$ WMAP,BBN

Neutrino

- Neutrino density is quite uncertain.
  WMAP & terrestrial measurement of m_ν:

$$7 \times 10^{-4} < \Omega_\nu \leq 0.02$$

- Neutrino mass scale is close to DE scale:

$$\sim (2 \times 10^{-3} eV)^4$$

—Model Independent—

Assumption 1

Variable parameter:

$m_\nu(A)$: Function of scalar field A("acceleron").

(R.Fardon,A.E.Nelson,N.Weiner)

$m_\nu(T)$: Function of temparature.

(R.D.Peccei)

Assumption 2

The energy density in the dark sector has two component:

$$\omega = \frac{p_{dark}}{\rho_{dark}}$$
$$\rho_{dark} = \rho_\nu + \rho_{dark\ energy}$$
$$(\rho_{dark\ energy} = \rho_{dark\ energy}(m_\nu) = V(m_\nu))$$

Assumption 3

ρ_{dark} is stationary with respect to variations in the neutrino mass:

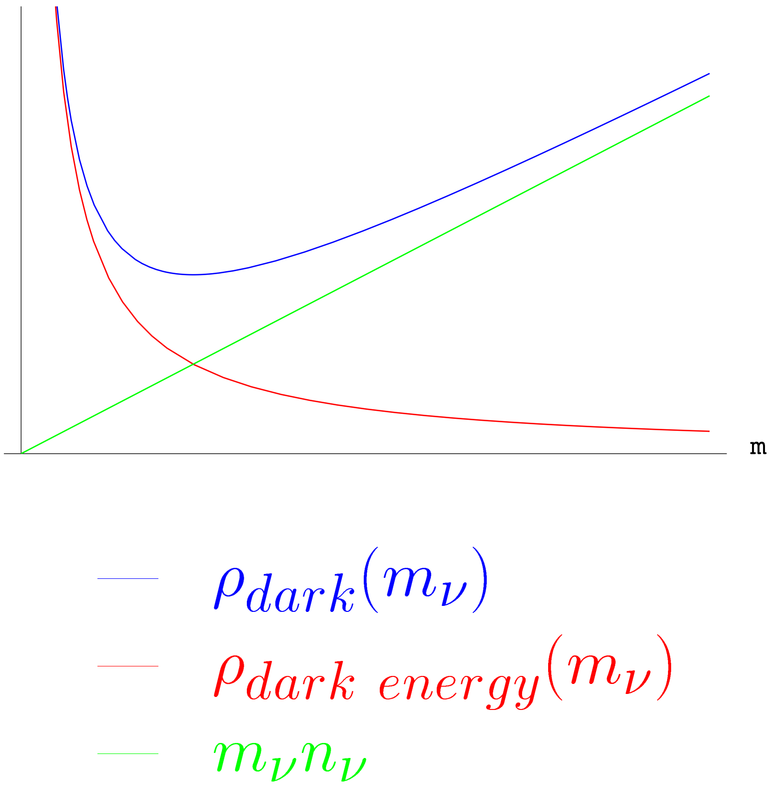
$$\frac{\partial \rho_{dark}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial \rho_{dark\ energy}}{\partial m_\nu} = 0$$

Assumption 4

The background neutirnos at the present are non-relativistic:

$$\rho_\nu^0 = m_\nu^0 n_\nu^0$$

$$\rho_{dark} = \rho_\nu + \rho_{dark\ energy}(m_\nu)$$
$$= m_\nu n_\nu + \rho_{dark\ energy}(m_\nu)$$



The case of non-relativistic

—1 family model—

Neutrino energy density:

$$\rho_\nu = m_\nu n_\nu$$

Relations of ρ , a , n , ω:

$$\rho(=V) \propto a^{-3(1+\omega)} \propto n^{1+\omega}$$
$$n \propto a^{-3}$$

Equation of state:

$$\begin{aligned}
\omega + 1 &= -\frac{\partial \log \rho_{dark}}{3\partial \log a} \\
&= -\frac{a}{3\rho_{dark}}\left(m_\nu \frac{\partial n_\nu}{\partial a} + \frac{\partial m_\nu}{\partial a} n_\nu \right. \\
&\qquad \left. + \rho_{dark\ energy}(m_\nu)\frac{\partial m_\nu}{\partial a}\right) \\
&= \frac{m_\nu n_\nu}{\rho_{dark}} \\
&= \frac{m_\nu n_\nu}{m_\nu n_\nu + \rho_{dark\ energy}}
\end{aligned}$$

$$\omega_0 + 1 = \frac{m_\nu^0 n_\nu^0}{m_\nu^0 n_\nu^0 + \rho^0_{dark\ energy}}$$

[Estimate]

$$\omega_0 \sim -0.9$$

$$\Downarrow$$

$$m_\nu^0 n_\nu^0 = 0.1 \rho^0_{dark}$$

$$\rho^0_{dark\ energy} = 0.9 \rho^0_{dark}$$

$$\rho^0_{dark} \sim 2.73 \times 10^{-11}\ (eV)^4$$

$$n_\nu^0 \sim 8.83 \times 10^{-13}\ (eV)^3$$

$$\Downarrow$$

$$m_\nu^0 = 3.09\ (eV)$$

This values for m_ν^0 are not neccessarily in contradiction with terrestrial limits on neutrino masses since the neutrino mass measured on earth may well differ from m_ν^0, if there is an overdensity in the local group due to neutrino clustering.

The general case (Temperature dependent)

The neutrino density:

$$\rho_\nu = T^4 F(\xi)$$
$$\xi = \frac{m_\nu(T)}{T}$$
$$F(\xi) = \frac{1}{\pi^2}\int_0^\infty \frac{dy y^2\sqrt{y^2+\xi^2}}{e^y+1}$$

The stationary condition:

$$\frac{\partial \rho_{dark}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial \rho_{dark\ energy}}{\partial m_\nu} = 0$$
$$= T^4\frac{\partial F}{\partial \xi}\frac{\partial \xi}{\partial m_\nu} + \frac{\partial \rho_{dark\ energy}}{\partial m_\nu}$$
$$= T^3\frac{\partial F}{\partial \xi} + \frac{\partial \rho_{dark\ energy}}{\partial m_\nu} = 0$$

The equation of state:

$$\omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{\rho_{dark\ energy}}{T^4 F(\xi)}]}$$
$$h(\xi) \equiv \frac{\xi\frac{\partial F(\xi)}{\partial \xi}}{F(\xi)}$$

2.1.Power-law Potential

$$V(m_\nu) = b m_\nu^a$$

There are two conditions for ordinary parameters.

(1) For the potential:

$$\begin{aligned} m_\nu^0 n_\nu^0 + V(m_\nu^0) &= 0.7\rho_c \\ \Rightarrow m_\nu^0 n_\nu^0 + b(m_\nu^0)^a &= 0.7\rho_c \end{aligned}$$

(2) For the m_ν:

$$\frac{\partial V(m_\nu)}{\partial m_\nu} = -T^3 \frac{\partial F}{\partial \xi} \quad (Stationary\ Condition)$$

$$\Downarrow$$

$$a b m_\nu^{a-1} = -\frac{T^3}{\pi^2} \int \frac{dy y^2 (m_\nu/T)}{\sqrt{y^2 + (m_\nu/T)^2}(e^y + 1)}$$

$$\Downarrow$$

$$a b (m_\nu^0)^{a-1} = -\frac{T_0^3}{\pi^2} \int \frac{dy y^2 (m_\nu^0/T_0)}{\sqrt{y^2 + (m_\nu^0/T_0)^2}(e^y + 1)}$$

The conditions for the power-law potential

$$V(m_\nu) = b m_\nu^a$$

$$\begin{cases} m_\nu^0 n_\nu^0 + b(m_\nu^0)^a = 0.7\rho_c \\ ab(m_\nu^0)^{a-1} = -\frac{T_0^3}{\pi^2}\int \frac{dy y^2 (m_\nu^0/T_0)}{\sqrt{y^2+(m_\nu^0/T_0)^2}(e^y+1)} \end{cases}$$

The state of equation

$$\omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{V(m_\nu)}{T^4 F(\xi)}]}$$

$$= \frac{4 - h(\xi)}{3[1 - \frac{h(\xi)}{a}]}$$

In the non-relativistic limit,

$$\omega \longrightarrow \omega_0 \ , \ h(\xi) \longrightarrow 1 \ , \ m_\nu \longrightarrow m_\nu^0;$$

$$\omega_0 + 1 = \frac{a}{a-1}$$

Example

$$V(m_\nu) = -m_\nu^0 n_\nu^0 [\frac{\omega_0}{1+\omega_0}](\frac{m_\nu}{m_\nu^0})^{(\frac{1+\omega_0}{\omega_0})}$$

(R.D.Peccei)

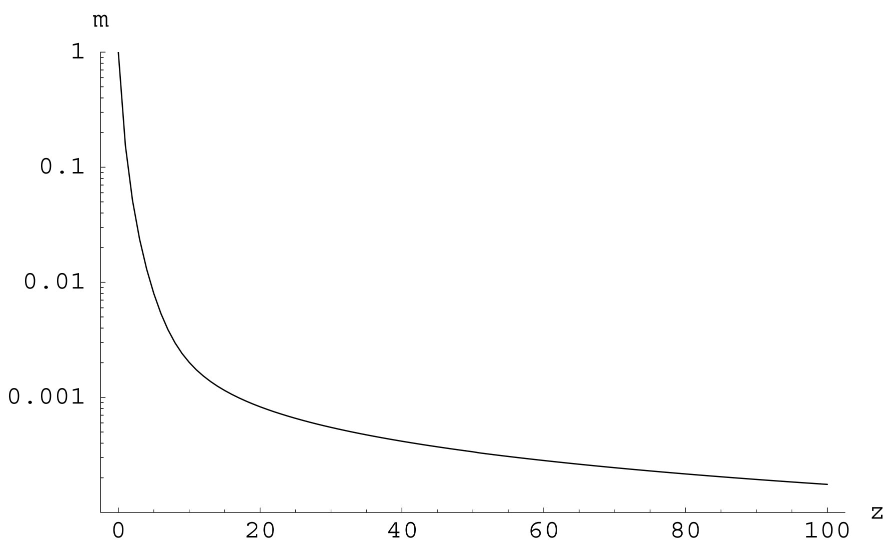
$$T^3\frac{\partial F}{\partial \xi} + \frac{\partial V(m_\nu)}{\partial m_\nu} = 0 \quad (Stationary\ Condition)$$

$$\Downarrow$$

$$m_\nu = m_\nu^0[(\frac{T}{T_0})^3 g(\xi)]^{\omega_0}$$

$$g(\xi) = \frac{\int_0^\infty \frac{dy y^2 \xi}{\sqrt{y^2+\xi^2}(e^y+1)}}{\int_0^\infty \frac{dy y^2}{(e^y+1)}}$$

$$m = m_\nu/m_\nu^0$$
$$z = (T/T_0 - 1)$$



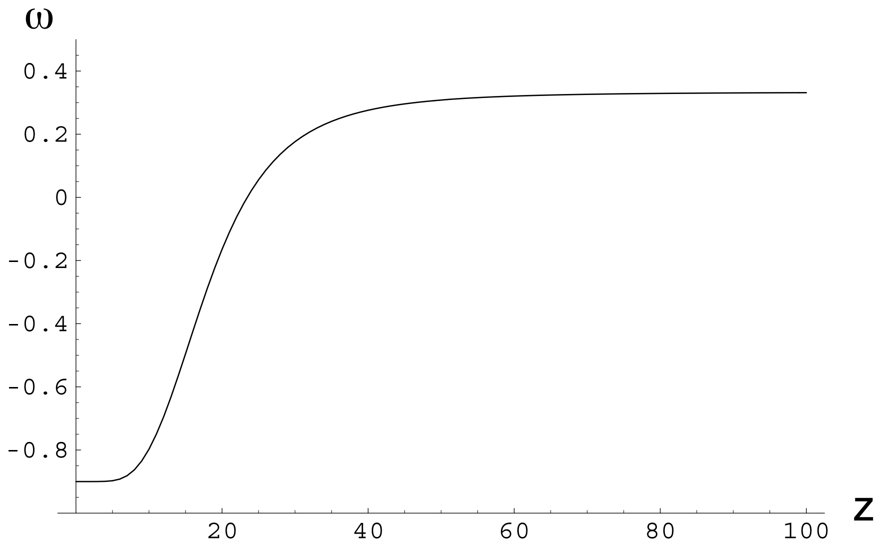
R.D.Peccei,hep-ph/0309800

$$z \sim 20 \Rightarrow \sim O(10^8) \quad (year)$$

$$\omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{V(m_\nu)}{T^4 F(\xi)}]} \quad (The\ state\ of\ equation)$$

$$\Downarrow$$

$$\omega + 1 = \frac{4 - h(\xi)}{3[1 - \frac{\omega_0}{1+\omega_0} h(\xi)]}$$



R.D.Peccei,hep-ph/0309800

As the temperature goes down, quantum effects are important for the potential.

2.2.Log Potential

$$V(m_\nu) = a\log(\frac{m_\nu}{b})$$

(1)For the potential:

$$m_\nu^0 n_\nu^0 + V(m_\nu^0) = 0.7\rho_c$$

$$\Rightarrow m_\nu^0 n_\nu^0 + a\log\frac{m_\nu^0}{b} = 0.7\rho_c$$

(2) For the m_ν:

$$\frac{\partial V(m_\nu)}{\partial m_\nu} = -T^3\frac{\partial F}{\partial \xi} \quad (Stationary\ Condition)$$

$$\Downarrow$$

$$\frac{a}{m_\nu} = -\frac{T^3}{\pi^2}\int \frac{dyy^2(m_\nu/T)}{\sqrt{y^2+(m_\nu/T)^2}(e^y+1)}$$

$$\Downarrow$$

$$\frac{a}{m_\nu^0} = -\frac{T_0^3}{\pi^2}\int \frac{dyy^2(m_\nu^0/T_0)}{\sqrt{y^2+(m_\nu^0/T_0)^2}(e^y+1)}$$

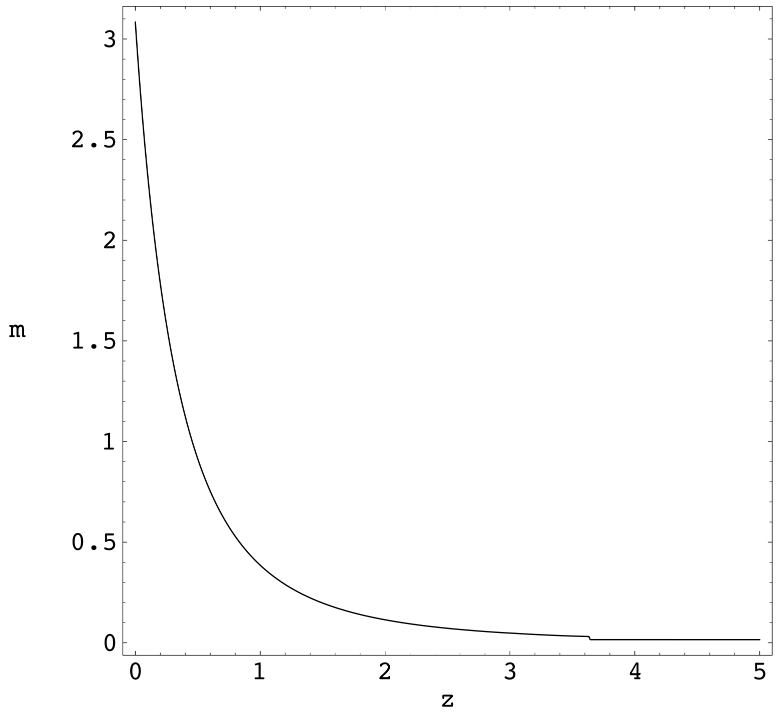
The conditions for the log potential

$$V(m_\nu) = a\log(\frac{m_\nu}{b})$$

$$\begin{cases} m_\nu^0 n_\nu^0 + a\log\frac{m_\nu^0}{b} = 0.7\rho_c \\ \frac{a}{m_\nu^0} = -\frac{T_0^3}{\pi^2}\int\frac{dy y^2 (m_\nu^0/T_0)}{\sqrt{y^2+(m_\nu^0/T_0)^2}(e^y+1)} \end{cases}$$

$$\Rightarrow \begin{array}{l} a \sim -2.80\times 10^{-12} \\ \log(\frac{m_\nu^0}{b}) \sim -9.01 \end{array}$$

$$\frac{a}{m_\nu} = -\frac{T^3}{\pi^2} \int \frac{dy y^2 (m_\nu / T)}{\sqrt{y^2 + (m_\nu/T)^2}(e^y + 1)}$$



Different potentials have different temperature dependence of the neutrino mass.

The state of equation

$$\omega + 1 = \frac{4 - h(\xi)}{3[1 + \frac{V(m_\nu)}{T^4 F(\xi)}]}$$
$$= \frac{4 - h(\xi)}{3[1 - h(\xi)\log(\frac{m_\nu}{b})]}$$

In the non-relativistic limit,

$$\omega \longrightarrow \omega_0 \ , \ h(\xi) \longrightarrow 1 \ , \ m_\nu \longrightarrow m_\nu^0;$$

$$\omega_0 + 1 = \frac{1}{1 - \log(\frac{m_\nu^0}{b})}$$

3. SUSY model

(N.Weiner, "Summer Institute 2004 lecture")

$$W = \lambda ANN$$

A, N : Chiral Superfield.
Singlet under the SM gauge group.

$$V = 4\lambda^2|\phi_a|^2|\phi_n|^2 + \lambda^2|\phi_n|^4$$

The potential is decided automatically.

Assumption

ϕ_a is a "acceleron".

$$\mathcal{L}_{mass} = m_D\nu_L\psi_n + m'_D\nu_L\psi_a + \lambda\phi_a\psi_n\psi_n + \lambda\phi_n\psi_a\psi_n$$

$$M = \begin{pmatrix} 0 & m'_D & m_D \\ m'_D & 0 & \lambda\phi_n \\ m_D & \lambda\phi_n & \lambda\phi_a \end{pmatrix}$$

$$\lambda\phi_n \ll m'_D, m_D \ll \lambda\phi_a$$

$$\Rightarrow \pm m'_D + \frac{m_D^2}{2\lambda\phi_a}, \;\; \lambda\phi_a$$

$$m_\nu \simeq \pm m'_D + \frac{m_D^2}{2\lambda\phi_a}$$

Two quasi-degenerate neutrinos exist.

$$\begin{aligned} m_\nu &\simeq m'_D + \frac{m_D^2}{2\lambda\phi_a} \\ &\Downarrow \\ \phi_a &\simeq \frac{m_D^2}{2\lambda(m_\nu - m'_D)} \\ &\Downarrow \\ V &= \frac{m_D^4 \phi_n^2}{(m_\nu - m'_D)^2} + \lambda^2 \phi_n^4 \end{aligned}$$

(1)For the potential:

$$m_\nu^0 n_\nu^0 + V(m_\nu^0) = 0.7\rho_c$$

$$\Rightarrow \quad \frac{m_D^4 \phi_n^2}{(m_\nu^0 - m'_D)^2} + \lambda^2 \phi_n^4 = 0.7\rho_c$$

(2) For the m_ν:

$$\frac{\partial V(m_\nu)}{\partial m_\nu} = -T^3 \frac{\partial F}{\partial \xi} \quad (Stationary\ Condition)$$

$$\Downarrow$$

$$-2\frac{m_D^4 \phi_n^2}{(m_\nu - m'_D)^3} = -\frac{T^3}{\pi^2} \int \frac{dy y^2 (m_\nu/T)}{\sqrt{y^2 + (m_\nu/T)^2}(e^y + 1)}$$

$$\Downarrow$$

$$2\frac{m_D^4 \phi_n^2}{(m_\nu^0 - m'_D)^3} = \frac{T_0^3}{\pi^2} \int \frac{dy y^2 (m_\nu^0/T_0)}{\sqrt{y^2 + (m_\nu^0/T_0)^2}(e^y + 1)}$$

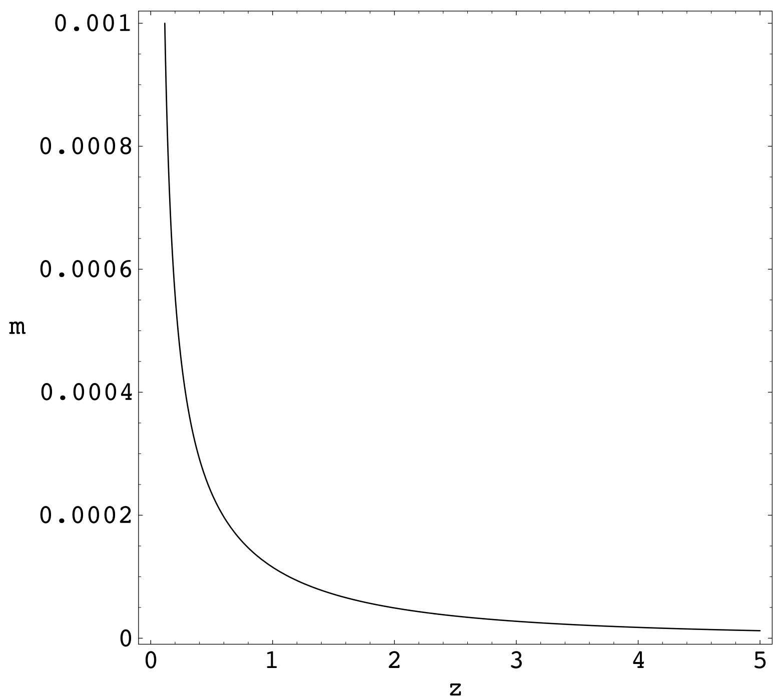
The conditions for the potential

$$V(m_\nu) = \frac{m_D^4 \phi_n^2}{(m_\nu - m'_D)^2} + \lambda^2 \phi_n^4$$

$$\begin{cases} m_\nu^0 n_\nu^0 + \frac{m_D^4 \phi_n^2}{(m_\nu^0 - m'_D)^2} + \lambda^2 \phi_n^4 = 0.7\rho_c \\ 2\frac{m_D^4 \phi_n^2}{(m_\nu^0 - m'_D)^3} = \frac{T_0^3}{\pi^2} \int \frac{dy y^2 (m_\nu^0/T_0)}{\sqrt{y^2 + (m_\nu^0/T_0)^2}(e^y + 1)} \end{cases}$$

$$\begin{aligned} & \lambda = 1 \\ & \phi_n = 10^{-8} \quad (eV) \\ \Rightarrow\ & m_D \sim 169 \quad (eV) \\ & m'_D \sim -54.0 \quad (eV) \\ & \lambda \phi_a \sim 251 \quad (eV) \end{aligned}$$

$$2\frac{m_D^4\phi_n^2}{(m_\nu - m'_D)^3} = \frac{T^3}{\pi^2}\int \frac{dy y^2(m_\nu/T)}{\sqrt{y^2+(m_\nu/T)^2}(e^y+1)}$$



The state of equation

$$\omega + 1 = \frac{4 - h(\xi)}{3[1+\frac{V(m_\nu)}{T^4 F(\xi)}]}$$

$$= \frac{4-h(\xi)}{3[1+h(\xi)\frac{(m_\nu - m'_D)^3}{2m_D^4 m_\nu}\{\frac{m_D^4}{(m_\nu - m'_D)^2}+\lambda^2\phi_n^2\}]}$$

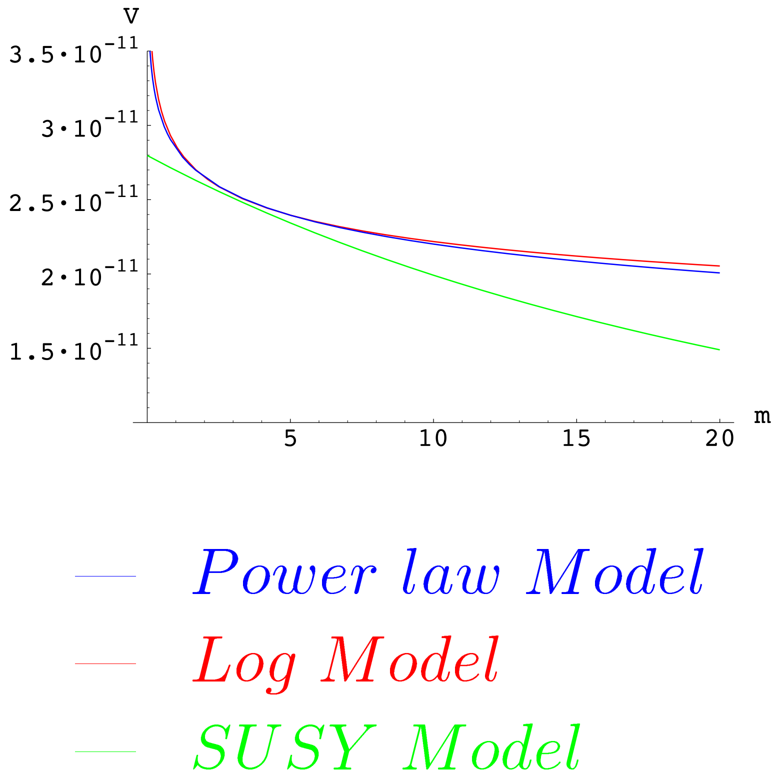
In the non-relativistic limit,

$$\omega \to \omega_0 \; , \; h(\xi) \to 1 \; , \; m_\nu \to m_\nu^0;$$

$$\omega_0 + 1 = \frac{1}{1 - \frac{(m_\nu^0 - m'_D)^3}{2m_D^4 m_\nu^0}\{\frac{m_D^4}{(m_\nu^0 - m'_D)^2}+\lambda^2\phi_n^2\}}$$

4. Future

- There are some models which are consistent with some observations in "Mass Varying Neutrnos" scenario.



- SUSY-breaking terms should be included in order to get the realistic model.