FINITE GROUPS for FAMILY SYMMETRY

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PLAN

I. Motivation for (discrete) family symmetry: "A way to soften the SUSY flavor problem"

II. Dihedral D_N and Binary dihedral (dicyclic) Q_{2N} groups

III. SUSY models based on S_3 and Q_6

IV. Conclusion

I Motivation for (discrete) family symmetry

③ Why low-energy SUSY?

⇑

② What is the SUSY flavor problem?

⇑

① Softening the SUSY flavor problem

Why law-energy SUSY?

The natural scale of the SM is TOO LAW.

The SM is presumably mathematically inconsistent (trivial).

It is a cutoff theory.

$$m_H \lesssim 0.6 \text{ TeV}$$

$\Uparrow$

$$m_H < \frac{1}{2}\Lambda_{cutoff}$$

If the SM is extended to a more fundamental theory, it will contain a scale $\Lambda \gg M_Z$.

$\Downarrow$

$\Rightarrow \delta m_H^2 \sim \frac{\alpha}{\pi} \Lambda^2 + \cdots$

Unnatural ('t Hoof & Veltman) if

$$\delta m_H^2 \sim \frac{\alpha}{4\pi} \Lambda^2 > m_H^2$$

$$\Downarrow$$

$$\Lambda \lesssim \text{ few TeV}$$

* No "natural" extension of the SM with $\Lambda \gtrsim$ few TeV!!

* No "natural" Grand Unified Theories (GUTs)!!

SUSY non-renormalization theorem

⇓

No quardratic divergences

(only wave function renormalization)

⇓

$$\delta m_H^2 \sim \frac{\alpha}{4\pi} m_H^2 \ln \frac{\Lambda}{m_H} + \cdots$$

* Λ can be much larger than the Planck scale M_{PL}.

* SUSY allows a "natural" extension of the SM.

How do we like susy to be broken?

* Theoretical Possibilities
  - spontaneously
  - dynamically
  - explicitly
  - Higgs mechanism

  ⋮

* We donot know which one is realized in nature.

* We would like susy to be broken in such a way that no quadratic divergences appear.

Soft ~~susy~~

What is

the SUSY flavor problem?

* Renormalizability allows

105 = ciento cinco

new parameters for the minimal supersymmetric standard model (MSSM) without m_ν.

* All together

105 + 19 = ciento venti cuatro!

JOY for the experimentalists

PAIN for the theorists

The problem is not only this huge number of the independent parameters, but the existence of the strong experimental constraints.

[The theorists do not know why the new parameters should be highly <u>fine-tuned</u>.

Work in the super CKM basis

$$f_L \to U_L f_L \,,\quad \tilde{f}_L \to U_L \tilde{f}_L$$

not $\tilde{U}_L$!

Interactions are flavor diagonal, but $\tilde{f}_L$ is not a mass eigenstate.

$$p^2 \delta_{ij} - (\tilde{m}^2_{LL})_{ij}$$

$\downarrow$ CKM

$$p^2 \delta_{ij} - (U_L^\dagger \tilde{m}^2_{LL} U_L)_{ij}$$

$$= (p^2 - \tilde{m}_i^2)\delta_{ij} - (U_L^\dagger \tilde{m}^2_{LL} U_L)'_{ij}$$

$\equiv (\Delta_{LL})_{ij}$: non-diagonal part

$$\frac{1}{p^2 - \tilde{m}_i^2}\delta_{ij} + \frac{1}{p^2 - \tilde{m}_i^2}(\Delta_{LL})_{ij}\frac{1}{p^2 - \tilde{m}_j^2}$$

$+ \cdots$

$$(\delta_{ij})_{LL} = \frac{(\Delta_{LL})_{ij}}{\tilde{m}_i \tilde{m}_j}$$

RR

LR(= RL)

Hall, Kostelecky + Raby

Suppression of FCNC's and CP

$\Rightarrow$ constraints on δ's.

Universal or aligned

$$(U_L^\dagger \tilde{m}_{LL}^2 U_L)_{ij}$$

$$= m_i^2 \delta_{ij} + (U_L^\dagger \tilde{m}_{LL}^2 U_L)'_{ij}$$

$$\parallel$$

$$\Delta_{ij} \not\propto \delta_{ij}$$

$\Delta_{ij} \simeq 0$ if
- $\tilde{m}_{LL}^2 \propto \mathbb{1}$ (Universal)
- $\tilde{m}_{LL}^2 \propto m_F m_F^\dagger$ (alignment)

The real problem

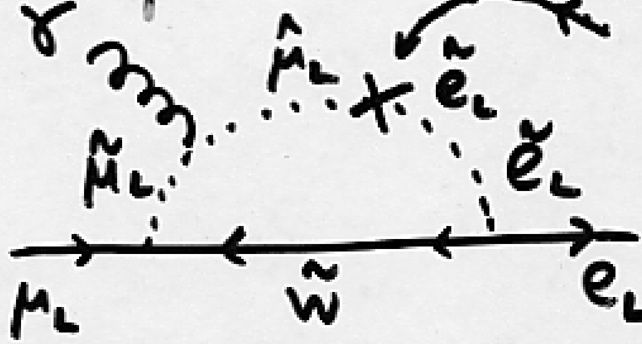
$105 = 43 + 63$ produce

$\uparrow$ ($\not{CP}$)

a lot of Flavor changing Neutral current (FCNC) and CP-violating processes.

* For instance:

FCNC $\mu \to e \gamma$



$(\Delta^e_{12})_{LL} \hat{\tilde{e}}^*_L \hat{\tilde{\mu}}_L$

$+ \cdots$

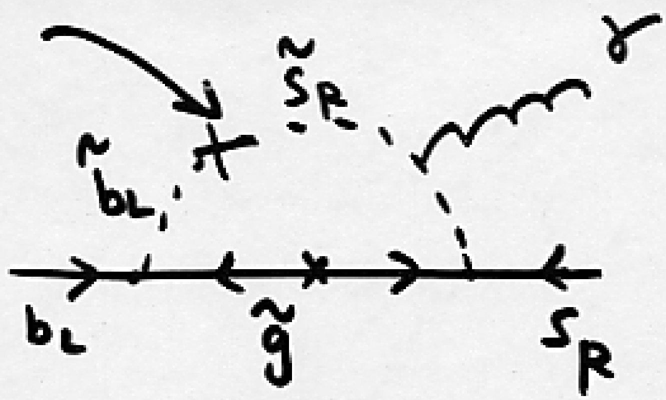
$B(\mu \to e\gamma) < 1.2 \times 10^{-11}$

$B(\tau \to e\gamma) < 2.7 \times 10^{-6}$

$B(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$

$\vdots$

$(\Delta^d_{32})_{LR}\, b_L s_R$



$b \to s\gamma$

$B(B \to \gamma X) = (3.3 \pm 0.4) \times 10^{-4}$ {Cleo, Belle}

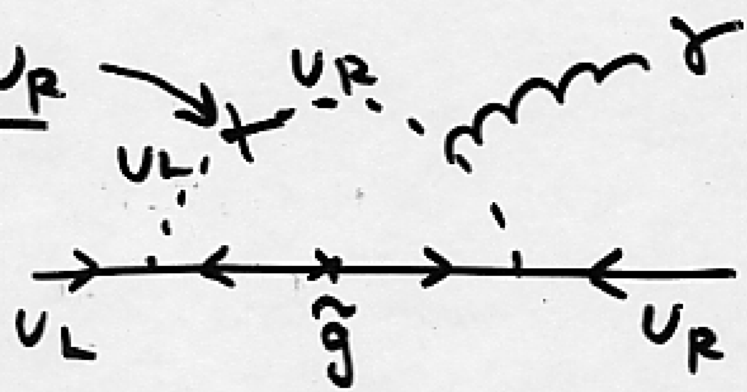
⋮

CP̸ Electric dipole moments (EDMs)

EDM of the neutron d_N

$(\Delta^u_{11})_{LR}\, u_L u_R$

$d_N \sim \mathrm{Im}$



$\frac{d_N}{e} \lesssim 10^{-26}\ \mathrm{cm}$

	Exp. bound		Exp. bound
$\lvert(\delta^{\ell}_{12})_{LL}\rvert$	$4\times10^{-5}\ \tilde{m}^2_{\tilde{\ell}}$	$\lvert(\delta^{\ell}_{12})_{RR}\rvert$	$9\times10^{-4}\ \tilde{m}^2_{\tilde{\ell}}$
$\lvert(\delta^{\ell}_{13})_{LL}\rvert$	$2\times10^{-2}\ \tilde{m}^2_{\tilde{\ell}}$	$\lvert(\delta^{\ell}_{13})_{RR}\rvert$	$3\times10^{-1}\ \tilde{m}^2_{\tilde{\ell}}$
$\lvert(\delta^{\ell}_{23})_{LL}\rvert$	$2\times10^{-2}\ \tilde{m}^2_{\tilde{\ell}}$	$\lvert(\delta^{\ell}_{23})_{RR}\rvert$	$3\times10^{-1}\ \tilde{m}^2_{\tilde{\ell}}$
$\lvert(\delta^{\ell}_{23})_{LL}(\delta^{\ell}_{13})_{LL}\rvert$	$1\times10^{-4}\ \tilde{m}^2_{\tilde{\ell}}$	$\lvert(\delta^{\ell}_{23})_{RR}(\delta^{\ell}_{13})_{RR}\rvert$	$9\times10^{-4}\ \tilde{m}^2_{\tilde{\ell}}$
$\lvert(\delta^{\ell}_{23})_{LL}(\delta^{\ell}_{13})_{RR}\rvert$	$2\times10^{-5}\ \tilde{m}^2_{\tilde{\ell}}$	$\lvert(\delta^{\ell}_{23})_{RR}(\delta^{\ell}_{13})_{LL}\rvert$	$2\times10^{-5}\ \tilde{m}^2_{\tilde{\ell}}$
$\lvert(\delta^{\ell}_{12})_{LR}\rvert$	$8.4\times10^{-7}\ \tilde{m}^2_{\tilde{\ell}}$	$\lvert(\delta^{\ell}_{13})_{LR}\rvert$	$1.7\times10^{-2}\ \tilde{m}^2_{\tilde{\ell}}$
$\lvert(\delta^{\ell}_{23})_{LR}\rvert$	$1.0\times10^{-2}\ \tilde{m}^2_{\tilde{\ell}}$	-	-
$\sqrt{\lvert\mathrm{Re}(\delta^{d}_{12})^2_{LL,RR}\rvert}$	$4.0\times10^{-2}\ \tilde{m}_{\tilde{q}}$	$\sqrt{\lvert\mathrm{Re}(\delta^{d}_{12})_{LL}(\delta^{d}_{12})_{RR}\rvert}$	$2.8\times10^{-3}\ \tilde{m}_{\tilde{q}}$
$\sqrt{\lvert\mathrm{Re}(\delta^{d}_{12})^2_{LR}\rvert}$	$4.4\times10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sqrt{\lvert\mathrm{Re}(\delta^{d}_{13})^2_{LL,RR}\rvert}$	$9.8\times10^{-2}\ \tilde{m}_{\tilde{q}}$
$\sqrt{\lvert\mathrm{Re}(\delta^{d}_{13})_{LL}(\delta^{d}_{13})_{RR}\rvert}$	$1.8\times10^{-2}\ \tilde{m}_{\tilde{q}}$	$\sqrt{\lvert\mathrm{Re}(\delta^{d}_{13})^2_{LR}\rvert}$	$3.3\times10^{-3}\ \tilde{m}_{\tilde{q}}$
$\sqrt{\lvert\mathrm{Re}(\delta^{u}_{12})^2_{LL,RR}\rvert}$	$1.0\times10^{-1}\ \tilde{m}_{\tilde{q}}$	$\sqrt{\lvert\mathrm{Re}(\delta^{u}_{12})_{LL}(\delta^{u}_{12})_{RR}\rvert}$	$1.7\times10^{-2}\ \tilde{m}_{\tilde{q}}$
$\sqrt{\lvert\mathrm{Re}(\delta^{u}_{12})^2_{LR}\rvert}$	$3.1\times10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sqrt{\lvert\mathrm{Im}(\delta^{d}_{12})^2_{LL,RR}\rvert}$	$3.2\times10^{-3}\ \tilde{m}_{\tilde{q}}$
$\sqrt{\lvert\mathrm{Im}(\delta^{d}_{12})_{LL}(\delta^{d}_{12})_{RR}\rvert}$	$2.2\times10^{-4}\ \tilde{m}_{\tilde{q}}$	$\sqrt{\lvert\mathrm{Im}(\delta^{d}_{12})^2_{LR}\rvert}$	$3.5\times10^{-4}\ \tilde{m}_{\tilde{q}}$
$\lvert(\delta^{d}_{23})_{LL,RR}\rvert$	$8.2\ \tilde{m}^2_{\tilde{q}}$	$\lvert(\delta^{d}_{23})_{LR}\rvert$	$1.6\times10^{-2}\ \tilde{m}^2_{\tilde{q}}$
$\lvert(\delta^{u}_{32})_{LL}\rvert$	$0.20\ \tilde{m}^4_{\tilde{q}}$	$\lvert(\delta^{u}_{32})_{LR}\rvert$	$5.5\times10^{-2}\ \tilde{m}^2_{\tilde{q}}$
$\lvert\mathrm{Im}(\delta^{d}_{12})_{LL,RR}\rvert$	$4.8\times10^{-1}\ \tilde{m}^2_{\tilde{q}}$	$\lvert\mathrm{Im}(\delta^{d}_{12})_{LR}\rvert$	$2.0\times10^{-5}\ \tilde{m}^2_{\tilde{q}}$
$\lvert\mathrm{Im}(\delta^{d}_{11})_{LR}\rvert$	$6.7\times10^{-8}\ \tilde{m}^2_{\tilde{q}}$	$\lvert\mathrm{Im}(\delta^{u}_{11})_{LR}\rvert$	$6.7\times10^{-8}\ \tilde{m}^2_{\tilde{q}}$
$\lvert\mathrm{Im}(\delta^{\ell}_{11})_{LR}\rvert$	$3.7\times10^{-8}\ \tilde{m}^2_{\tilde{\ell}}$	$\lvert(\delta^{u}_{12})_{LL}(\delta^{u}_{21})_{RR}\rvert$	$9.4\times10^{-4}\ \tilde{m}^3_{\tilde{q}}$
$\lvert(\delta^{u}_{13})_{LL}(\delta^{u}_{31})_{RR}\rvert$	$3.4\times10^{-6}\ \tilde{m}^3_{\tilde{q}}$	$\lvert(\delta^{d}_{12})_{LL}(\delta^{d}_{21})_{RR}\rvert$	$7.2\times10^{-4}\ \tilde{m}^3_{\tilde{q}}$
$\lvert(\delta^{d}_{13})_{LL}(\delta^{d}_{31})_{RR}\rvert$	$2.0\times10^{-5}\ \tilde{m}^3_{\tilde{q}}$	$\lvert(\delta^{d}_{23})_{LL}(\delta^{d}_{32})_{RR}\rvert$	$1.9\times10^{-4}\ \tilde{m}^3_{\tilde{q}}$

$\tilde{m}_{\tilde{e}}$ = average slepton mass / 100 GeV

$\tilde{m}_{\tilde{q}}$ = " squark mass / 500 GeV

Strong exp. constraints coming from Δm_K, ϵ_K, ϵ'/ϵ, ...

$\Downarrow$

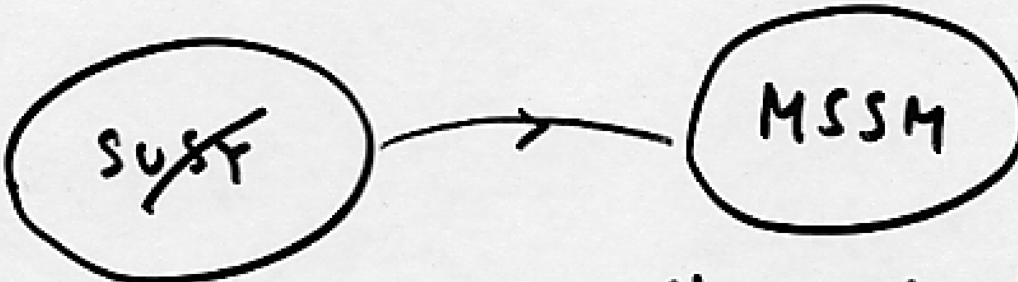
soft ~~SUSY~~ parameters are extremely flavorindependent.

WHY ??

possible solutions to the SUSY flavor problem

1. The hidden sector scenario

$\not{\text{SUSY}}$ in a sector, that is separated from the MSSM by interactions or in space-time.



- gravity mediation
- gauge mediation
- anomaly "
- ⋮ ⋮

2. RG Appraches

3. A certain set of superpartners are extremely heavy $\gtrsim O(\text{TeV})$.

Or, to use a family symmetry.

- Hall + Murayama, '95
- Babu, Kobayashi + Kubo, '03
- Hamaguchi, Kakizaki + Yamaguchi '03
- Maekawa, '03
- Kobayashi, Kubo + Terao, '03
- Ross, V-Sevilla + Vives, '04
- Maekawa + Yamashita, '04
- Choi, Kajiyama, Kubo + Lee, '04

⋮

II Group theory of Dihedral D_N and Dicyclic Q_{2N} Groups

The classification of the finite groups has been completed 1981 (Gorenstein).

$g \equiv$ order of a finit group
$= \#$ of the elements

* No non-abelian finite group exists for odd g

* For smaller g (≤ 31), only three type of non-abelian finite groups exist.

For $g \leq 31$

① Permution groups

* all permutations:

S_N, $N = 3, 4, 5, \cdots$

* even permutations:

A_N, $N = 4, 5, \cdots$

② Dihedral groups

* dihedral groups

D_N, $N = 3, 4, 6, \cdots \subset SO(3)$

* Dicyclic (binary dihedral) groups

Q_{2N}, $N = 2, 3, \cdots \subset SU(3)$

③ $Z_N \tilde{\times} Z_M$, $Z_N \times D_M$, $Z_N \times Q_{2M}$

$Z_N \times A_M$

Note

$D_3 \simeq S_3$

g	
6	$D_3 \equiv S_3$
8	$D_4, Q = Q_4$
10	D_5
12	$D_6, Q_6, T \simeq A_4$
14	D_7
16	$D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$
18	$D_9, Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T,$
	$Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

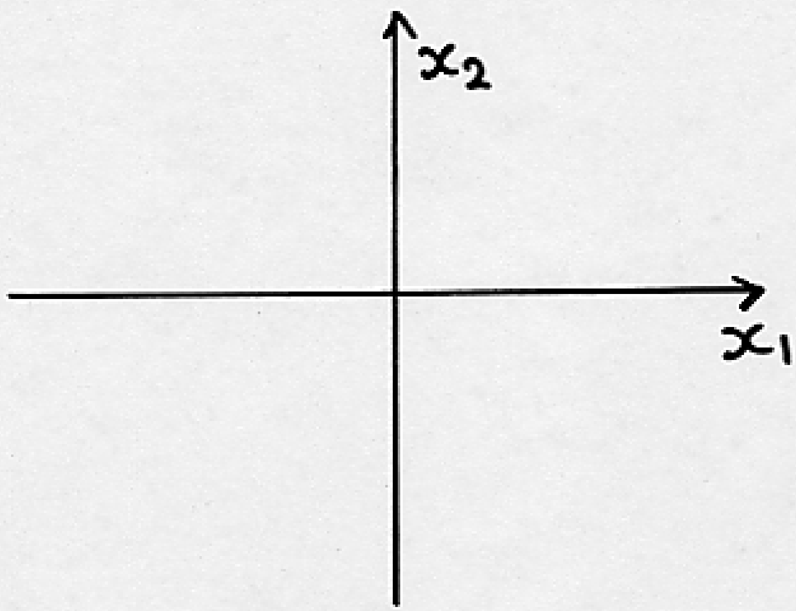
There remain thirteen others formed by twisted products of abelian factors. Only certain such twistings are permissable, namely (completing all $g \leq 31$)

Frampton + Kephart.
PRD 64 (2001) 086007.

Dihedral Symmetry

=

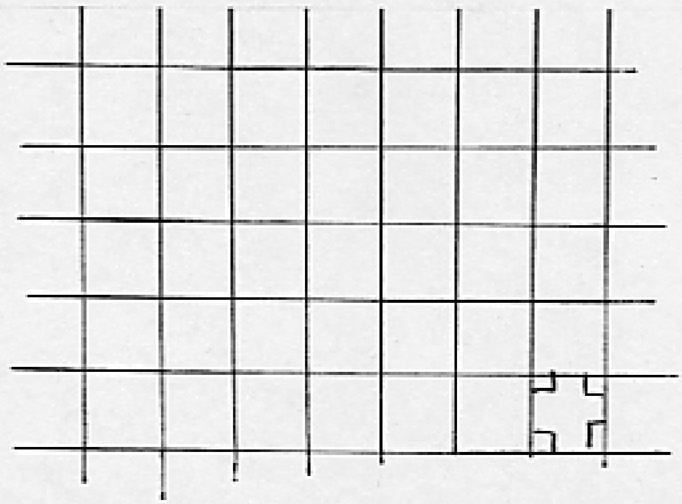
Symmetry of regular polygon
(正多角形)



$SO(2) \times P = O(2)$ symmetry

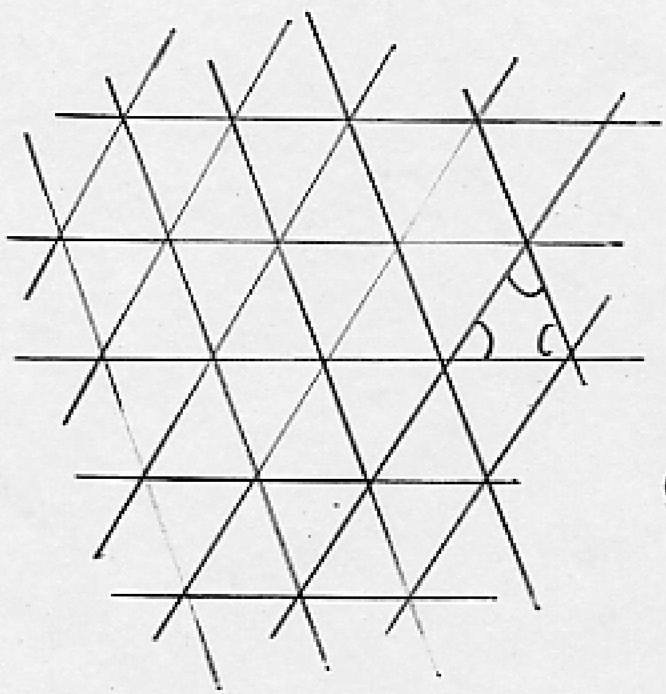
P : parity

Discretize the 2-d space!



$SO(2) \to Z_4$

$Z_4 \times P = D_4$



$SO(2) \to Z_6$

$Z_6 \times P = D_6$

60°

g	
6	$D_3 \equiv S_3$
8	$D_4, Q = Q_4$
10	D_5
12	D_6, Q_6, T
14	D_7
16	$D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$
18	$D_9, Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T,$
	$Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

Frampton + Kephart, PRD 64 (2001) 086007

g	
16	$Z_2 \tilde{\times} Z_8$ (two, excluding D_8), $Z_4 \tilde{\times} Z_4, Z_2 \tilde{\times} (Z_2 \times Z_4)$ (two)
18	$Z_2 \tilde{\times} (Z_3 \times Z_3)$
20	$Z_4 \tilde{\times} Z_5$
21	$Z_3 \tilde{\times} Z_7$
24	$Z_3 \tilde{\times} Q, Z_3 \tilde{\times} Z_8, Z_3 \tilde{\times} D_4$
27	$Z_9 \tilde{\times} Z_3, Z_3 \tilde{\times} (Z_3 \times Z_3)$

Recent Papers on Discrete Family Symmetry

S_3

1. J. Kubo, A. Mondragón, M. Mondragón and E. Rodríguez-Jáuregui, Prog. Theor. Phys. **109**, 795 (2003).
2. J. Kubo, Phys. Lett. **B578**, 156 (2004).
3. T. Kobayashi, J. Kubo and H. Terao, Phys. Lett. **B568**, 83 (2003).
4. Ki-Y. Choi, Y. Kajiyama, J. Kubo and H.M. Lee, Phys. Rev. **D70**, 055004 (2004).
5. S-L. Chen and E. Ma, Mod. Phys. Lett. **A19**, 1267 (2004).

   J. Kubo, H. Okada and F. Sakamaki, Phys. Rev. **D70**, 036007 (2004).

D_4

1. W. Grimus and L. Lavoura, Phys. Lett. **B572**, 76 (2003).
2. W. Grimus and L. Lavoura, Phys. Lett. **B579**, 113 (2004); JHEP **0405**, 016 (2004).
3. W. Grimus, A.S. Joshipura, S. Kaneko, L. Lavoura and M. Tanimoto, JHEP **0407**, 078 (2004).

A_4

1. E. Ma and G. Rajasekaran, Phys. Rev. **D64,** 113012 (2001); E. Ma, Mod. Phys. Lett. **A17,** 627 (2002); 2361 (2002) .
2. K.S. Babu, E. Ma and J.W.F. Valle, Phys. Lett. **B552,** 207 (2003).
3. K.S. Babu, T. Kobayashi and J. Kubo, Phys. Rev. **D67,** 075018 (2003).
4. M. Hirsch, J.C. Romao, S. Skadhauge, J.W.F. Valle and A. Villanova del Moral, Phys. Rev. **D69,** 093006 (2004).

Q_4

1. M. Frigerio, S. Kaneko, E. Ma and M. Tanimoto, hep-ph/0409187.

Q_6

1. K.S. Babu and J. Kubo, hep-ph/0411226.

Related Papers

1. Y. Koide, Phys. Rev. **D60,** 077301 (1999).

2. Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi and T. Fukuyama, Phys. Rev. **D66,** 093006 (2002).

3. T. Ohlsson and G. Seidl, Phys. Lett. **B537,** 95 (2002); Nucl. Phys. **B643,** 247 (2002).

4. T. Kitabayashi and M. Yasue, Phys .Rev. **D67,** 015006 (2003).

5. E. Ma and G. Rajasekaran, Phys. Rev. **D68,** 071302 (2003); E. Ma, Phys. Lett. **B583,** 157 (2004); Mod. Phys. Lett. **A19,** 577 (2004).

6. W. Grimus, A.S. Joshipura, L. Lavoura and M. Tanimoto, Eur. Phys. J. **C36** 227 (2004).

$\underline{\underline{D_N}}$ transformations:

$$\begin{pmatrix} \cos\phi_N & \sin\phi_N \\ -\sin\phi_N & \cos\phi_N \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\underset{Z_N}{\shortparallel} \qquad\qquad \underset{P}{\shortparallel}$$

$$\phi_N = \frac{2\pi}{N}$$

The "covering group" of D_N is $\underline{\underline{Q_{2N}}}$:

$$\begin{pmatrix} \cos\phi_{2N} & \sin\phi_{2N} \\ -\sin\phi_{2N} & \cos\phi_{2N} \end{pmatrix} \times \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

D_N and Q_{2N} have only one- and two-dimensional representations (like helicity, $\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \dots$)

Why dihedral?

① discrete or continues?

Nambu-Goldstones when spontaneously broken.

No known low-energy model with renormalizable interactions.

② abelian or non-abelian?

Abelian groups are not bad, but non-alelian groups are better, because one can really relate different entries of Yukawa's.

③ Two-dimensional irrps exist for D_N and Q_{2N}, but the dimension of the smallest, non-singlet irrps is

3 for S_4, A_4

4 for S_5, A_5 .....

III SUSY models based on S_3 and Q_6

S_3 model

- non-SUSY : Kubo, 2× Mondragon + R-Jauregui, '03
  Kubo, '04

- SUSY : Kobayashi, Kubo + Terao, '03
  Choi, Kajiyama, Kubo + Lee '04

$S_3 \times Z_2$ in the leptonic sector

⇓

6+2=8 parameters to describe 3+3+3+3=12 parameters, $m_{e,\mu,\tau}$, $m_{\nu_{1,2,3}}$ and V_{MNS}.

Predictions

(i) Inverted spectrum of m_ν

$$m_{\nu_1}, m_{\nu_2} > m_{\nu_3}$$

(ii) only $\varphi_3 = \rho_3 - \rho_e$ enters into M_ν and CP asymmetries for leptogenesis

(iii) Dirac Phase $\delta \neq 0$

(iv) Two Majorana Phases α, β are function of φ_3

(v) $m_{\nu_2} = f(\theta_{12}, \Delta^2 m_{32}, \Delta^2 m_{21}, \varphi_3)$

(vi) $s_{23} = \sin\theta_{23} = \frac{1}{\sqrt{2}} + O\left(\frac{m_e^2}{m_\mu^2}\right) \simeq 0.707$

$s_{23}^{exp} = 0.60 - 0.81 \ (2\sigma)$ (best fit: $1/\sqrt{2}$)

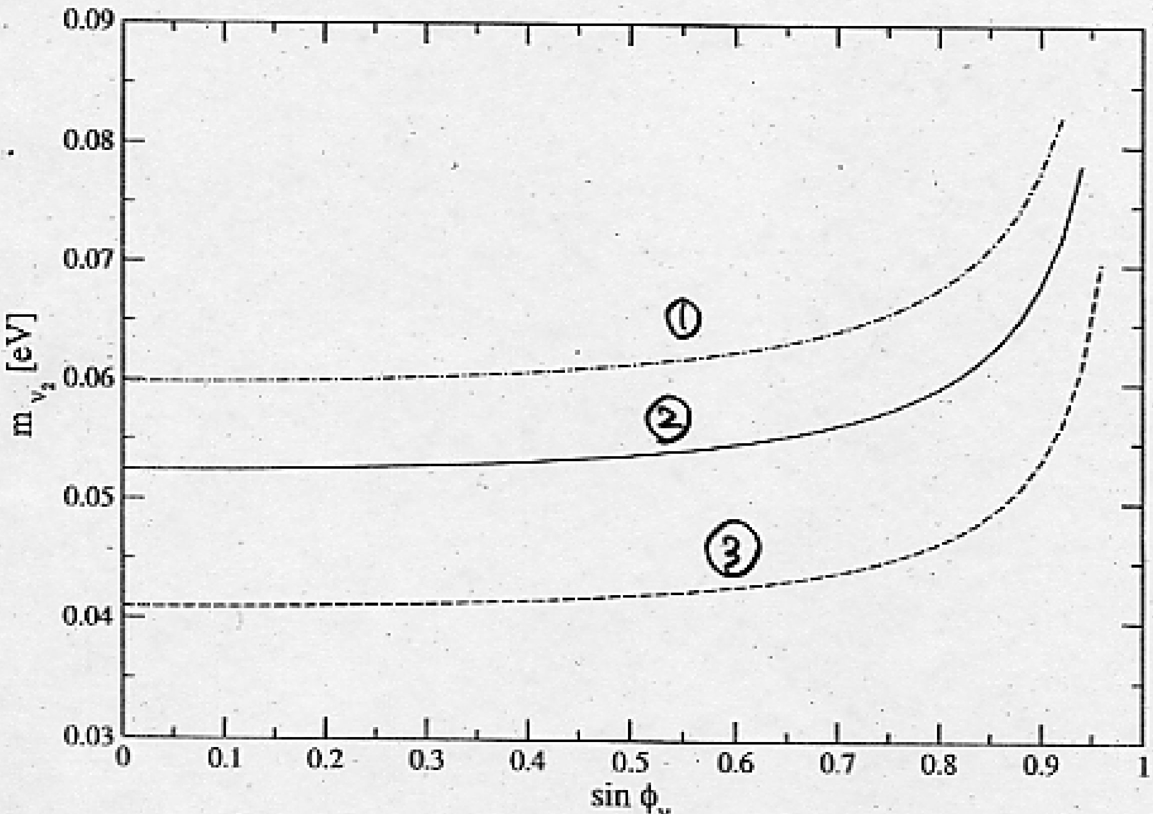
(vii) $s_{13} = \sin\theta_{13} \simeq \frac{m_e}{\sqrt{2}\, m_\mu} \simeq 3.4 \times 10^{-3}$

$|s_{13}^{exp}| \leq 0.19 \ (2\sigma)$

(best fit: 0.07)

$m_{\nu_2} - \sin\phi_\nu$

$$\begin{cases} m_{\nu_3}\sin\phi_\nu = m_{\nu_1}\sin\phi_1 = m_{\nu_2}\sin\phi_2 \\ 2\varphi_3 = \phi_1 + \phi_2 \end{cases}$$



Input:

$\sin^2\theta_{12} = 0.3$

$\Delta m^2_{21} = 6.9\times10^{-5}\ eV^2$

$\Delta m^2_{23} = (3.0,\ 2.3,\ 1.4)\times10^{-3}\ eV^2$

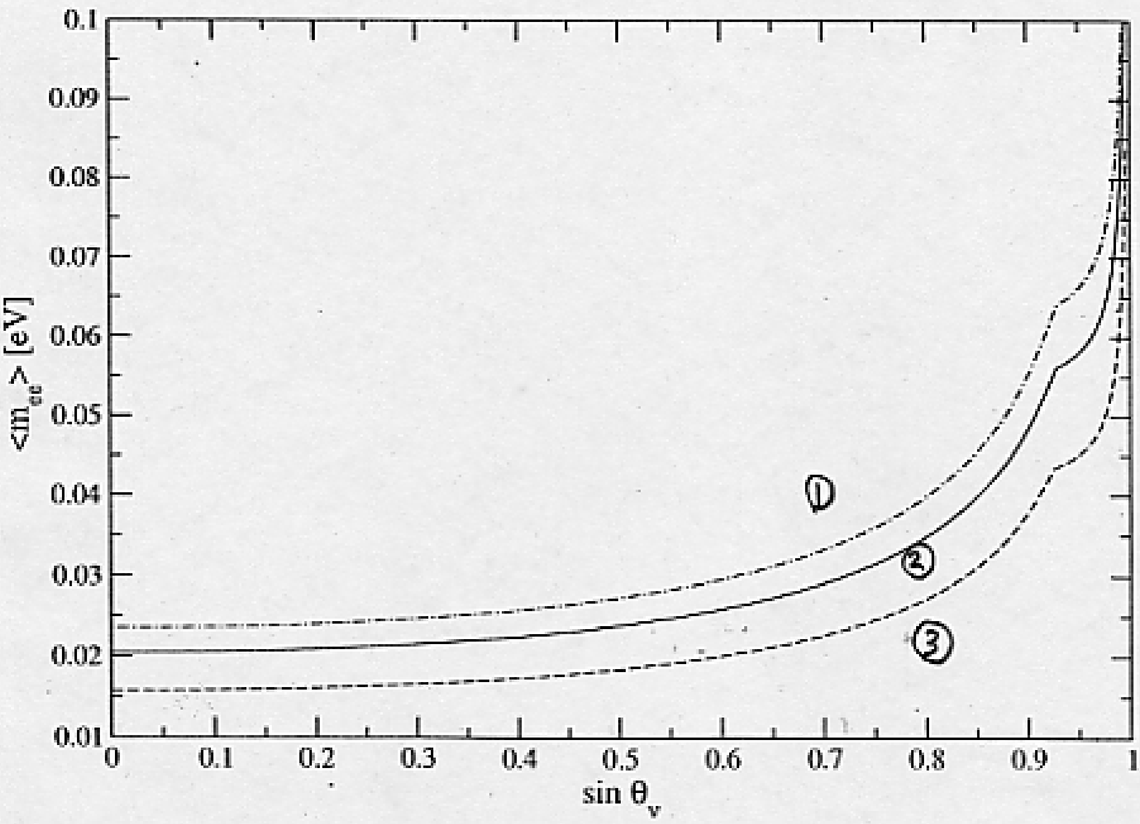
① ② ③

⟨Mee⟩ – Sin Φ_ν

Input: $\sin^2\theta_{12} = 0.3$, $\Delta m^2_{21} = 6.9\times10^{-5}$ eV^2

$\Delta m^2_{23} = (3.0, 2.3, 1.4)\times10^{-3}$ eV^2

① ② ③



The structure of the soft susy breaking (SSB) parameters is fixed by S_3.

- $$\tilde{m}^2_{LL,RR} = m_0^2 \begin{pmatrix} a_{L,R} & & \mathbb{1} \\ & a_{L,R} & \\ \mathbb{1} & & b_{L,R} \end{pmatrix}$$

- $$(\tilde{m}^2_{LR})_{ij} = (m_f)_{ij} \cdot A_{ij}$$

have the same structure as m_f.

$$(\delta_{ij})_{LL,RR} = U^\dagger_{L,R}\, \tilde{m}^2_{LL,RR}\, U_{L,R} / \tilde{m}^2$$

$$(\delta_{ij})_{LR} = U_L^\dagger\, \tilde{m}^2_{LR}\, U_R / \tilde{m}^2$$

Leptonic sector (LL and RR):

$$\begin{aligned}
\mu \to e\gamma \to \quad (\delta^{\ell}_{12})_{LL} &\simeq (\delta^{\ell}_{21})_{LL} \simeq 4.8 \times 10^{-3}\ \Delta a^{\ell}_{L}, \\
(\delta^{\ell}_{13})_{LL} &\simeq (\delta^{\ell}_{31})_{LL} \simeq -1.7 \times 10^{-5}\ \Delta a^{\ell}_{L}, \\
(\delta^{\ell}_{23})_{LL} &\simeq (\delta^{\ell}_{32})_{LL} \simeq 8.4 \times 10^{-8}\ \Delta a^{\ell}_{L}, \\
(\delta^{\ell}_{12})_{RR} &\simeq (\delta^{\ell}_{21})_{RR} \simeq 8.4 \times 10^{-8}\ \Delta a^{\ell}_{R}, \\
(\delta^{\ell}_{13})_{RR} &\simeq (\delta^{\ell}_{31})_{RR} \simeq 5.9 \times 10^{-2}\ \Delta a^{\ell}_{R}, \\
(\delta^{\ell}_{23})_{RR} &\simeq (\delta^{\ell}_{32})_{RR} \simeq -1.4 \times 10^{-6}\ \Delta a^{\ell}_{R}.
\end{aligned} \tag{19}$$

Leptonic sector (LR):

$$\begin{aligned}
Im(\delta^{\ell}_{ij})_{LR} &= 0, \\
\mu \to e\gamma \to \quad (\delta^{\ell}_{12})_{LR} &\simeq 5.1 \times 10^{-6}\ \left(\tilde{A}^{\ell}_{2} - \tilde{A}^{\ell}_{4}\right)\left(\frac{100\ \text{GeV}}{m_{\tilde{\ell}}}\right), \\
(\delta^{\ell}_{21})_{LR} &\simeq 2.5 \times 10^{-8}\ \left(\tilde{A}^{\ell}_{2} - \tilde{A}^{\ell}_{4}\right)\left(\frac{100\ \text{GeV}}{m_{\tilde{\ell}}}\right), \\
(\delta^{\ell}_{13})_{LR} &\simeq 3.1 \times 10^{-7}\ \left(\tilde{A}^{\ell}_{4} - \tilde{A}^{\ell}_{5}\right)\left(\frac{100\ \text{GeV}}{m_{\tilde{\ell}}}\right), \\
(\delta^{\ell}_{31})_{LR} &\simeq 1.1 \times 10^{-3}\ \left(\tilde{A}^{\ell}_{2} - \tilde{A}^{\ell}_{5}\right)\left(\frac{100\ \text{GeV}}{m_{\tilde{\ell}}}\right), \\
(\delta^{\ell}_{23})_{LR} &\simeq -1.5 \times 10^{-9}\ \left(\tilde{A}^{\ell}_{4} - \tilde{A}^{\ell}_{5}\right)\left(\frac{100\ \text{GeV}}{m_{\tilde{\ell}}}\right), \\
(\delta^{\ell}_{32})_{LR} &\simeq -2.5 \times 10^{-8}\ \left(\tilde{A}^{\ell}_{2} - \tilde{A}^{\ell}_{5}\right)\left(\frac{100\ \text{GeV}}{m_{\tilde{\ell}}}\right).
\end{aligned} \tag{20}$$

$$\boxed{(\delta^{\ell}_{12})_{LL} \simeq 4.8 \times 10^{-3}\, \Delta a_L}$$

$$\underbrace{4.8 \times 10^{-3}} = \frac{m_e}{m_\mu} = \sqrt{2}\,|V_{e3}|$$

$$\mu \to e\gamma \to \quad < 10^{-5} \left(\frac{100\ \mathrm{GeV}}{\tilde{m}_e} \right)^2$$

Two different Physics,
suppression of FCNCs
in the SSB sector and
the smallness of $|V_{e3}|$,
are related by the
family symmetry.

suppose $\tilde{m}^2_{LL}$ is universal at M_{GUT}.

At M_{ν_R}

$$\frac{\tilde{m}^2_{\ell,LL}}{\tilde{m}^2_{\ell}} \sim \begin{pmatrix} 1 & & ① \\ & 1 & \\ ① & & 1+\Delta a_L \end{pmatrix}$$

$$\Delta a_L \simeq \frac{1}{16\pi^2} \frac{m_\nu M_{\nu_R}}{v^2} \ln \frac{M_{GUT}}{M_{\nu_R}}$$

$$\simeq \begin{cases} 10^{-4} \\ 10^{-17} \\ 10^{-11} \end{cases} \text{for } M_{\nu_R} \simeq \begin{cases} 10^{14} \\ 10^{8} \\ 10^{4} \end{cases} \text{GeV}$$

$$(\delta^{\ell}_{12})_{12} \simeq 4.8 \times 10^{-3} \, \Delta a_L$$

$$< 10^{-5} \left(\frac{100\,\text{GeV}}{\tilde{m}_\ell} \right)^2$$

Down quark sector (LR):

EDM

$$(\delta^d_{11})_{LR} \simeq \Big[-1.6\tilde{A}^d_1 + 2.3\tilde{A}^d_2 - I\ 0.6(\tilde{A}^d_1 - \tilde{A}^d_2) - I\ 0.5(\tilde{A}^d_2 + \tilde{A}^d_3 - \tilde{A}^d_4 - \tilde{A}^d_5)\ \Big] \times 10^{-5}\ \left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right),$$

$$(\delta^d_{22})_{LR} \simeq \Big[-(1.1 + I\ 1.1)\tilde{A}^d_1 - (0.8 + I\ 1.8)\tilde{A}^d_2 - (15 + I\ 2.9)(\tilde{A}^d_3 - \tilde{A}^d_4) + (20 + I\ 2.9) \times 10^{-5}\ \left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right),$$

$$(\delta^d_{33})_{LR} \simeq \Big[(4.7 + I\ 3.4)\tilde{A}^d_1 + (8.0 + I\ 5.5)\tilde{A}^d_2 + (4.3 \times 10^{+4} + I\ 5.0)\tilde{A}^d_3 + (1.5 \times 10^{+4} - I\ 7.8)\tilde{A}^d_4 + (2.2 \times 10^{+2} - I\ 8.9)\tilde{A}^d_5\ \Big] \times 10^{-5}\ \left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right),$$

K^0-$\bar{K}^0$

$$(\delta^d_{12})_{LR} \simeq \Big[\ (2.4 + 1.7\ I)\left(\tilde{A}^d_2 - \tilde{A}^d_5\right) + (2.2 + 1.9\ I)\left(\tilde{A}^d_3 - \tilde{A}^d_4\right)\Big] \times 10^{-5}\ \left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right),$$

$$(\delta^d_{21})_{LR} \simeq \Big[\ (2.7 + 1.9\ I)\left(\tilde{A}^d_2 - \tilde{A}^d_5\right) + (2.0 + 1.7\ I)\left(\tilde{A}^d_3 - \tilde{A}^d_4\right)\Big] \times 10^{-5}\left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right),$$

$$(\delta^d_{13})_{LR} \simeq -\Big[\ (4.1 + 2.9\ I)\left(\tilde{A}^d_3 - \tilde{A}^d_5\right) + (1.4 + 0.1\ I)\left(\tilde{A}^d_2 - \tilde{A}^d_4\right)\ \Big] \times 10^{-5}\left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right),$$

$$(\delta^d_{31})_{LR} \simeq -(3.4 + 2.4\ I) \times 10^{-4}\left(\tilde{A}^d_3 - \tilde{A}^d_4\right)\ \left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right),$$

$$(\delta^d_{23})_{LR} \simeq \Big[\ -2.6 \times 10^{+1}\left(\tilde{A}^d_3 - \tilde{A}^d_4\right) + 3.4 \times 10^{+1}\left(\tilde{A}^d_4 - \tilde{A}^d_5\right) + I\ \left(\tilde{A}^d_1 + \tilde{A}^d_2 - 3\tilde{A}^d_3 - \tilde{A}^d_4 + 2\tilde{A}^d_5\right)\ \Big] \times 10^{-5}\left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right),$$

$$(\delta^d_{32})_{LR} \simeq (2.5 + 0.2\ I) \times 10^{-3}\left(\tilde{A}^d_3 - \tilde{A}^d_4\right)\ \left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}}}\right).$$

Severe Constraints

$\mu \to e\gamma$:

$$\tilde{A}_2^\ell - \tilde{A}_4^\ell \lesssim O(10^{-1}) \;\;,\;\; \Delta a_L^\ell \lesssim O(10^{-2})$$

EDMs:

$$|\mathrm{Im}(\tilde{A}_1^u)| \lesssim O(10^{-4}) \;\;,\;\; |\mathrm{Im}(\tilde{A}_3^u)| \lesssim O(10^{-2}) \;\;,$$
$$|\mathrm{Im}(\tilde{A}_1^d)| \lesssim O(10^{-3}) \;\;,\;\; |\mathrm{Im}(\tilde{A}_3^d)| \lesssim O(10^{-2}) \;\;,$$
$$\mathrm{Re}(\tilde{A}_i^d) - \mathrm{Re}(\tilde{A}_j^d) \lesssim O(10^{-2}) \;\; (i,j = 1 \sim 5),$$
$$\Delta a_L^u \Delta a_R^u \lesssim O(10^{-1}) \;\;,\;\; \Delta a_L^d \Delta a_R^d \lesssim O(10^{-2}) \;\;,$$

Most of the constraints are satisfied if $\Delta a, \tilde{A} \sim O(1)$, except for

Q_6 model

(Babu + Kubo, Nov. 204)

Motivations

1. To derive a successful mass matrix texture solely from a symmetry.

2. To suppress $\not{CP}$ in the SSB sector.

Modified Fritzsch mass matrices are very sucsessful.

$$M_{Ftz} \sim \begin{pmatrix} 0 & C & 0 \\ -C & 0 & B \\ 0 & B' & A \end{pmatrix}$$

In the quark sector

$8 + 2 = 10$ parameters

What are the discrete symmetries that yield M_{Ftz}? (Babu + Kubo)

Two conditions

1. Both real and pseudo real irrps should exist.
2. Both up-type and down-type Higgses should exist.

Construct a Q_6 invariant SUSY model with spontaneously induced CP phases.

$\Downarrow$

8+1 parameters to describe 10 parameters in the hadronic sector: m_q's + V_{CKM}

$\Downarrow$

One prediction

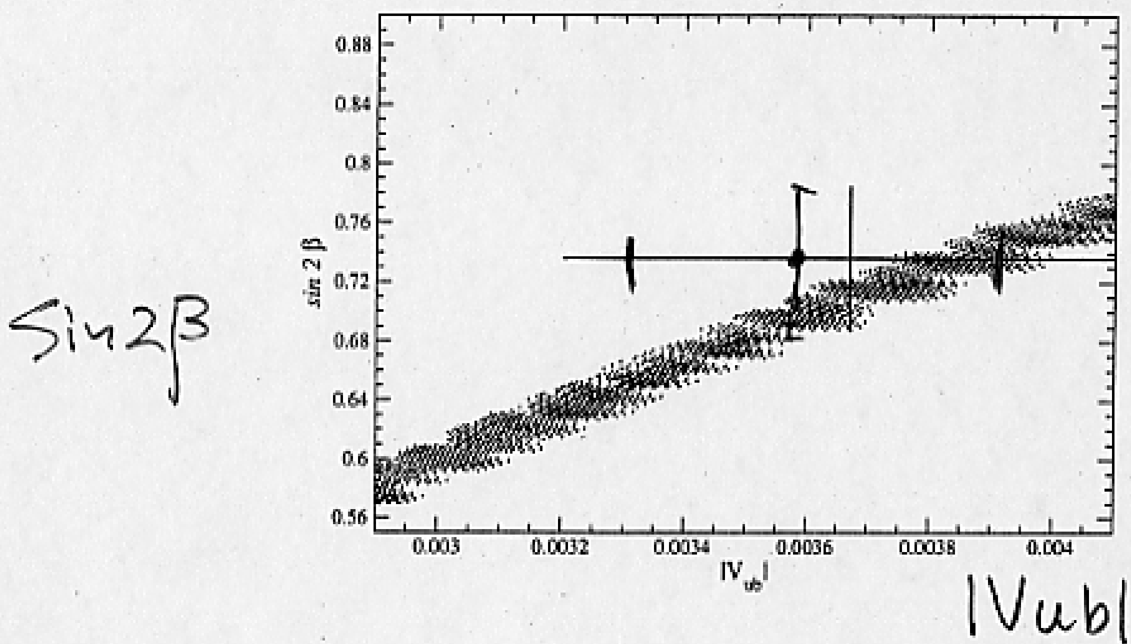
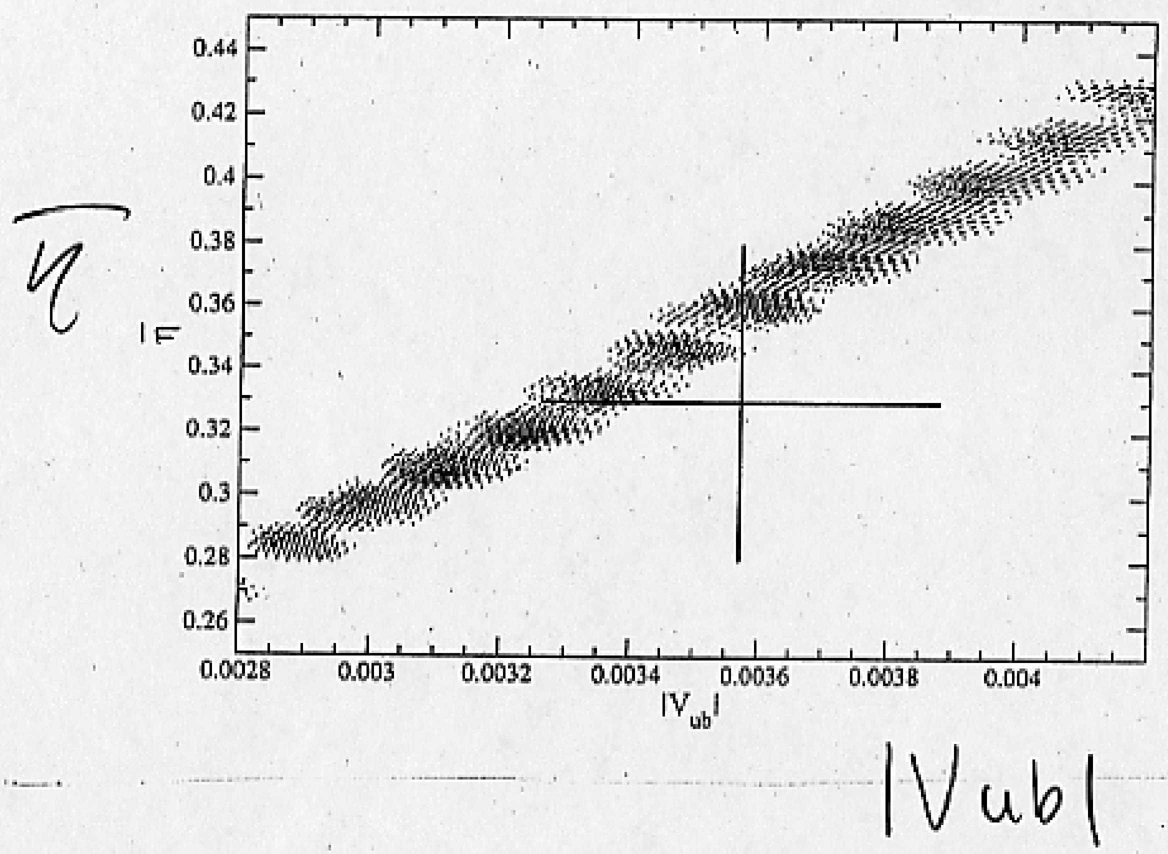
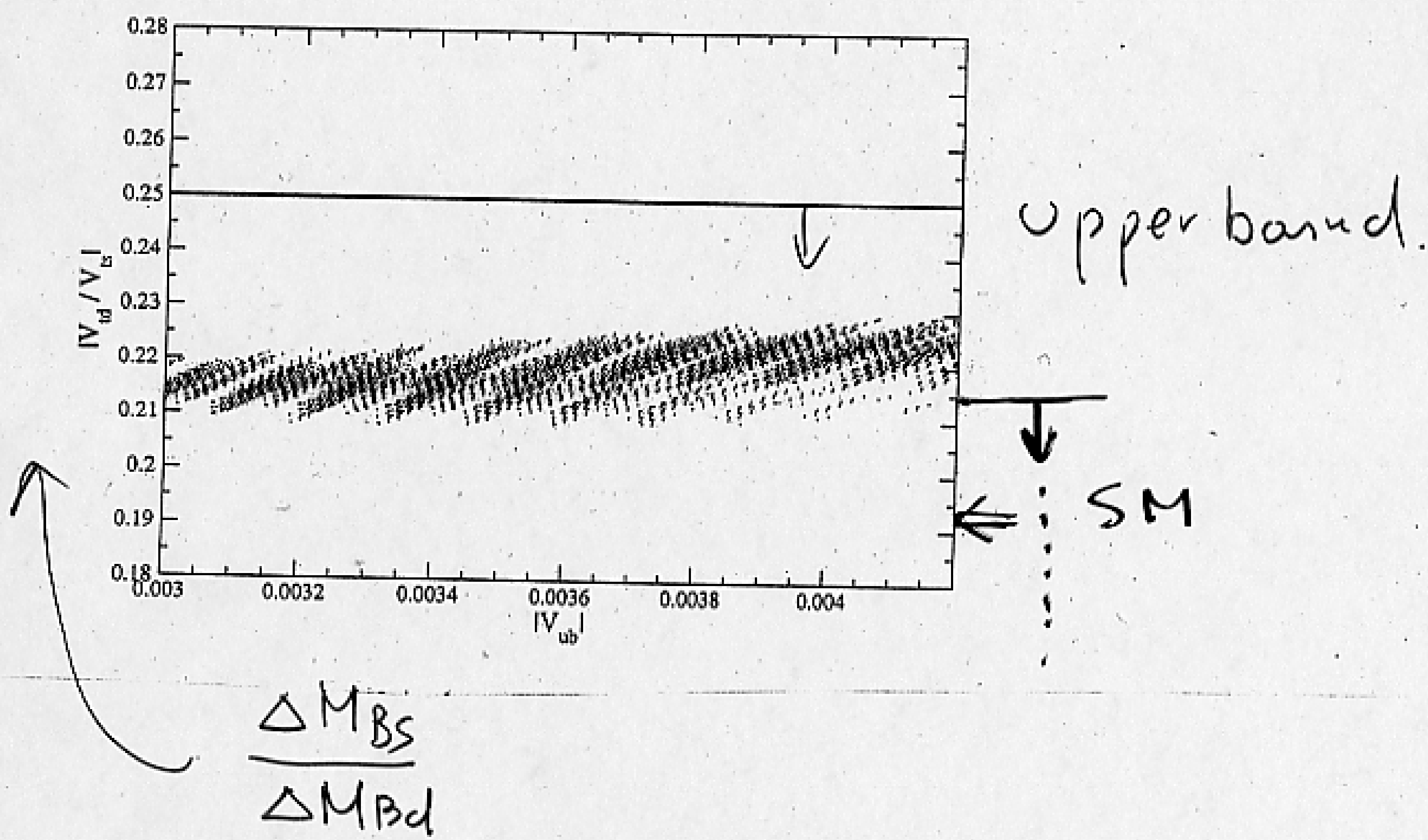


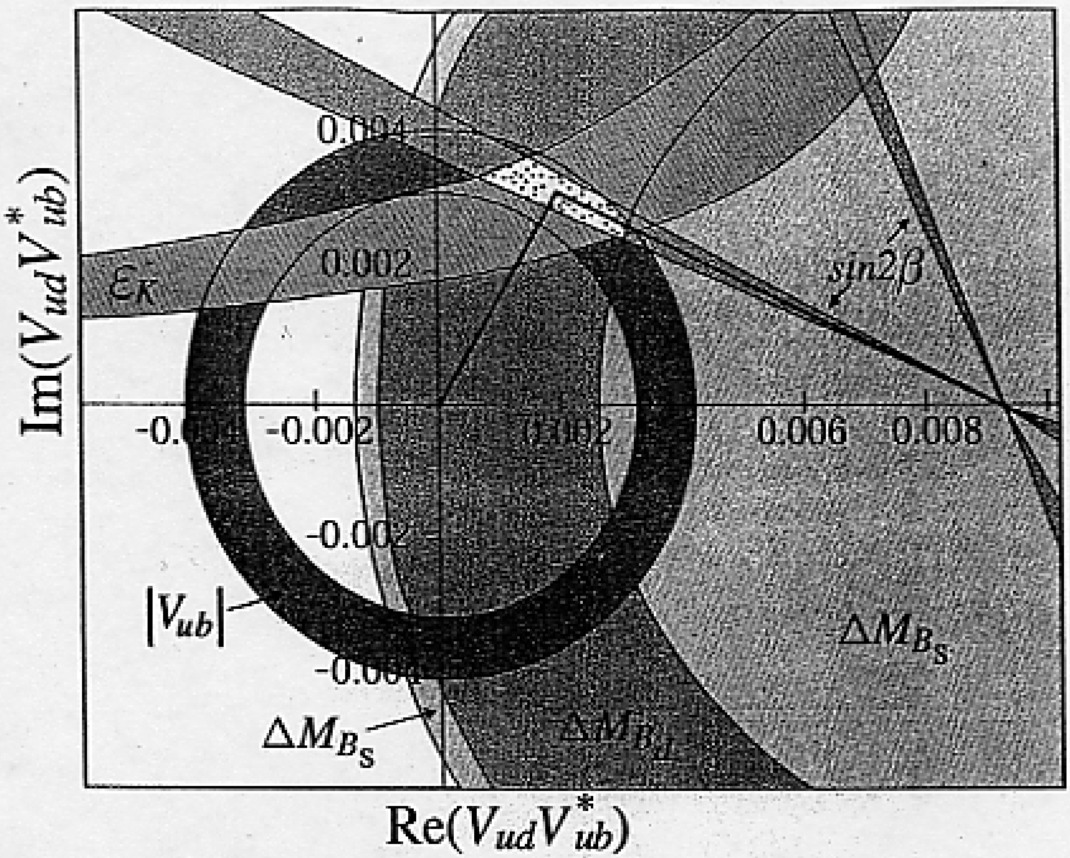
FIG. 2: Predictions in the $|V_{ub}| - \sin 2\beta$ plane. The vertical and horizontal lines correspond to the experimental values $|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3}$ and $\sin 2\beta = 0.736 \pm 0.049$.



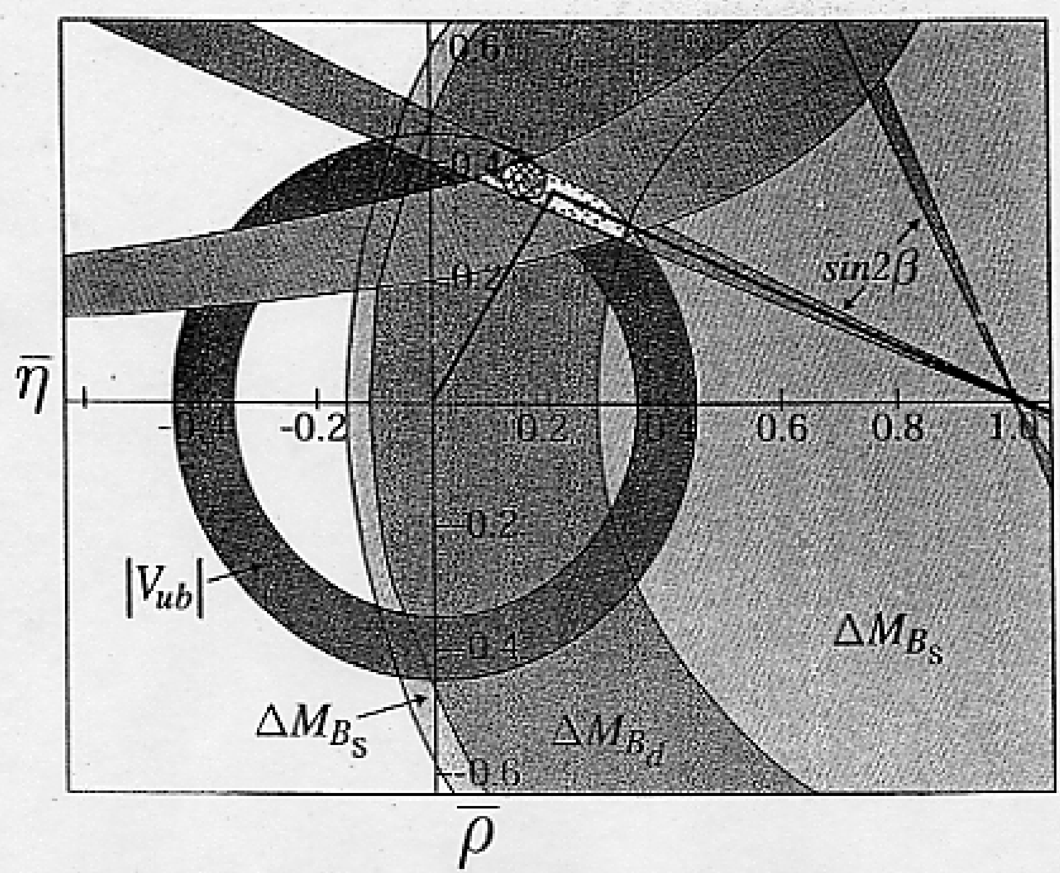
0.44
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
$\bar{\eta}$
0.0028
0.003
0.0032
0.0034
0.0036
0.0038
0.004
$|V_{ub}|$



0.28
0.27
0.26
0.25
0.24
0.23
0.22
0.21
0.2
0.19
0.18
$|V_{td}/V_{ts}|$
0.003
0.0032
0.0034
0.0036
0.0038
0.004
$|V_{ub}|$
upper band.
SM
$\frac{\Delta M_{Bs}}{\Delta M_{Bd}}$



0.004
0.002
-0.002
-0.004
0.002
0.006
0.008
ε_K
$sin2\beta$
$|V_{ub}|$
ΔM_{B_s}
ΔM_{B_s}
ΔM_{B_d}
$\mathrm{Im}(V_{ud}V_{ub}^*)$
$\mathrm{Re}(V_{ud}V_{ub}^*)$



0.6
0.4
0.2
-0.4
-0.2
0.2
0.4
0.6
0.8
1.0
-0.2
-0.4
-0.6
$sin2\beta$
$|V_{ub}|$
ΔM_{B_s}
ΔM_{B_s}
ΔM_{B_d}
$\bar{\eta}$
$\bar{\rho}$

Q_6 + spontaneous $\not{CP}$

$\Downarrow$

Phase Aligment.

$$U_L^\dagger \, m_q \, U_R = \text{diag.}$$

$$U_L^\dagger \, \tilde{m}_{q,LR} \, U_R$$

$=$ <u>real</u> matrix

The A-terms have no phase!

Note that

all the EDM constraints are satified in the Q_6 model.

Phase Alignment

$\Uparrow$

Flavor symmetry + spontaneous $\not{CP}$

It will not work with S_3!

Two choices for
the leptonic sector

I. appropriate for unification of quarks and leptons.

II: No extra Z_2 is necessary to reproduce the leptonic sector of the $S_3 \times Z_2$ model.

Conclusion

Flavor symmetry
(non-abelian and discrete)

⇓

Reduction of the redundant parameters of the SM

⇙ ⇓

Flavor structure of the SM

Softening the SUSY Flavor Problem

An important consequence.

More than one $SU(2)_L$ Higgs doublets

S_3 ($\times Z_2$) relates
the suppression of $\mu \to e\gamma$
with the smallness of $|V_{e3}|$.

Q_6 relates
the suppression of EDM
with one prediction in
the quark sector. (V_{CKM}).