Neutrino Mixing as a probe of Grand Unification

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Sin Kyu Kang

(Seoul National University)

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collaborated with C.S. Kim & J. Lee

1. Current Neutrino Experimental Results.

- (Accumulated neutrino oscillation data
  (measurement of CKM mixing matrix

  ⇒ so precise that we can catch some indication on a possible unifying theory for the mystery of the flavor structure.

- Global fit of Solar neutrino exp. & KamLAND data

$$\theta_{sun} = 32.3^\circ \pm 2.4^\circ \quad (1\sigma)$$
$$\Delta M^2_{sun} = (6.5 - 9) \times 10^{-5}\ eV^2$$

- Global analysis of the atmospheric neutrino exp.

$$\sin^2 2\theta_{atm} \gtrsim 0.94 \quad (90\%\ CL)$$
$$\Delta m^2_{atm} = (1.3 - 3.0) \times 10^{-3}\ eV^2$$
$$(\text{best fit} : \sin^2 2\theta_{atm} \simeq 1.0)$$

- CHOOZ Experiment

$$\sin^2\theta_{13} \lesssim 0.03 \quad (90\%\ CL)$$

2. Quark - Lepton Complementarity

- Taking the Cabibbo angle :

$$\theta_c = 12.8^\circ \pm 0.15^\circ$$

$$\Rightarrow \qquad \theta_{sun} + \theta_c = 45.1^\circ \pm 2.4^\circ \quad (1\sigma)$$

QLC relation : $\theta_{sun} + \theta_c = \frac{\pi}{4}$

- Is this relation signal of quark-lepton symmetry or quark-lepton unification ?

(Raidal '04, Minakata & Smirnov '04, Frampton & Mohapatra '04, Ferrandis & Pakvasa '04. Kang, Kim & Lee '05)

※ For (2.3) mixing ,

$$\theta_{atm} + \theta_{23}^{CKM} \simeq \frac{\pi}{4}$$

in good agreement with experimental data

3. QLC relation in GUT

- $\Delta I_W = 1/2$ Higgs doublet breaking of EW symmetry

  $\Rightarrow$ yields the quark Yukawa Matrices

$$M^{(2/3)} = U_{2/3} \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} V^{+}_{2/3}, \quad M^{(-1/3)} = U_{(-1/3)} \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} V^{+}_{(-1/3)}$$

$\rightarrow$ observable quark mixing : $U_{CKM} = U^{+}_{2/3} U_{-1/3}$

- The charged lepton Yukawa Matrix :

$$M^{(-1)} = U_{-1} \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} V^{+}_{-1}$$

- To obtain neutrino mass, we consider seesaw mechanism

$$M_\nu = M_{Dirac} \frac{1}{M_R} M^{T}_{Dirac}$$

$$= \overbrace{U_0 \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V_0^{+}} \frac{1}{M_R} V_0^{*} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U_0^{T}$$

$$\equiv U_0 V_M M_\nu^{diag} V_M^{T} U_0^{T}$$

where

$V_M \rightarrow$ rotation of $\quad M^{diag}_{Dirac} V_0^{+} \frac{1}{M_R} V_0^{*} M^{diag}_{Dirac}$

- Similar to quark mixing, the observable lepton mixing matrix can be written as :

$$U_{PMNS} = U_{-1}^{+} U_0 V_M$$

(A) Minimal GUT

$$M_{-1} = M^T_{-1/3} \quad , \quad M_{2/3} = M_{Dirac} \text{ (symmetric)}$$

$$\Rightarrow \quad U_{-1} = V^*_{-1/3} \quad ; \quad U_0 = U_{2/3}$$

$$\begin{aligned} U_{PMNS} &= U_{-1}^\dagger U_0 V_M \\ &= V^T_{-1/3} U_{2/3} V_M \\ &= V^T_{-1/3} U_{-1/3} \underline{\underline{U^\dagger_{CKM}}} V_M \end{aligned} \quad \Big\downarrow \; U_{CKM} = U^\dagger_{2/3} U_{-1/3}$$

① For symmetric M_{-1}, $M_{-1/3}$

$$U_{PMNS} = U^\dagger_{CKM} V_M$$

(→ this form of U_{PMNS} has been suggested by several authors in phenomenological way)

V_M should have two almost maximal mixing so as to account for the solar & atmospheric ν oscillations

- Can U_{PMNS} lead to QLC relation ?

$$U_{PMNS} = U^\dagger_{CKM} U^m_{23} U^m_{12} \equiv U_{23}(\theta_{23}) U_{13}(\theta_{13}) U_{12}(\tfrac{\pi}{4} - \theta_{12})$$

then, θ_{ij} can be presented in terms of λ

$$\sin\theta_{12} \simeq \frac{1}{\sqrt{2}}\lambda + O(\lambda^3) \; , \quad \sin\theta_{23} \simeq -\frac{1}{\sqrt{2}}(1 - \tfrac{1}{4}\lambda^2 - A\lambda^2)$$

$$\sin\theta_{13} \simeq -\frac{1}{\sqrt{2}}\lambda \qquad (※ \tfrac{1}{\sqrt{2}}\lambda \sim 0.16 \Rightarrow \theta \sim 9°)$$

$$\sin\theta_{sol} = \sin(\tfrac{\pi}{4} - \theta_c) + \underline{\tfrac{\lambda}{2}(\sqrt{2} - 1)}$$

↳ leads to $\delta\theta_{sun} \simeq 3.5°$

(3) Realistic quark-lepton unification

- Although the minimal GUT leads to an elegant relation between U_{PMNS} & U_{CKM}, it indicates undesirable mass relations between quarks & leptons at GUT scale:

$$m_d = m_\ell$$

Thus, we need to modify the siple relations between quarks & leptons Yukawa matrices

- A well known empirical relation:

$$|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \simeq 3\sqrt{\frac{m_e}{m_\mu}}$$

- As known from the '70's, if m_d is generated from the mixing between 1st & 2nd families, the relation between θ_c & (m_d/m_s) can be simply explained:

$$\hat{M}_{-1/3} = \begin{pmatrix} 0 & \sqrt{\frac{m_s m_d}{m_b^2}} & O(\lambda^3) \\ \sqrt{\frac{m_s m_d}{m_b^2}} & \frac{m_s}{m_b} & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

(in the basis where $M_{2/3}$ is real & diagonal)

- Analogously,

$$\hat{M}_{-1} = \begin{pmatrix} 0 & \sqrt{\frac{m_\mu m_e}{m_\tau^2}} & O(\lambda^3) \\ \sqrt{\frac{m_\mu m_e}{m_\tau^2}} & \frac{m_\mu}{m_\tau} & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

(in the basis where M_{Dirac} is diagonal)

$$U_{-1}^{+}U_0 \cong \begin{pmatrix} 1 & -\frac{\lambda}{3} & \frac{1}{3}\theta\lambda^2 \\ \frac{\lambda}{3} & 1 & 2\theta\lambda^2 \\ -\theta\lambda^2 & -2\theta\lambda & 1 \end{pmatrix}$$

(where $\theta \sim 0.07$)

→ This form of mixing matrix can be obtained by introducing the Higgs sector transforming under the rep. $\underline{45}$ of $SU(5)$ (Georgi & Jarlskog) or $\underline{126}$ of $SO(10)$ (Dimoplous et al. Mohapatra et al.)

- Can this case lead to QLC relation ?
  - θ_{ij} in U_{PMNS} presented in terms of λ:

$$\sin\theta_{12} \simeq \frac{\lambda}{3\sqrt{2}}, \qquad \sin\theta_{23} \simeq -\frac{1}{\sqrt{2}}(1-2\theta\lambda)$$

$$\sin\theta_{13} \simeq -\frac{\lambda}{3\sqrt{2}}$$

$$\Rightarrow \quad \sin\theta_{sun} \simeq \sin\left(\frac{\pi}{4}-\theta_c\right) + \underbrace{\frac{\lambda}{2}\left(\sqrt{2}-\frac{1}{3}\right)}_{\hookrightarrow\ \delta\theta_{sun} \sim 8^\circ}$$

- Is there any way to diminish the corrections to QLC relation ?

* A simple example of SU(5) Yukawa terms that lead to the empirical relation :

$$h_{33}\, 10_3 \bar{5}_3 \bar{5}_H + h_{22}\, 10_2 \bar{5}_2 \overline{45}_H + h_{12} (10_1 \bar{5}_2 + 10_2 \bar{5}_1) \bar{5}_H$$

$$\Rightarrow \quad Y_d = \begin{pmatrix} 0 & C & 0 \\ C & B & 0 \\ 0 & 0 & A \end{pmatrix}, \quad Y_\ell = \begin{pmatrix} 0 & C & 0 \\ C & -3B & 0 \\ 0 & 0 & A \end{pmatrix}$$

→ Georgi & Jarlskog

* The generalization to SO(10) of the simple Yukawa operators that give rise to the above Eq.

$$h_{33}\, 16_3 16_3 10_H + h_{22}\, 16_2 16_2 \overline{126}_H + h_{12}\, 16_1 16_2 10_H$$

(A) Non-symmetric $M_{-1/3}$, M_{-1}

$$U_{PMNS} = \underline{V_{-1/3}^T U_{-1/3}} U_{CKM}^\dagger V_{M}$$

↳ (responsible for the correction that can diminish the correction to QLC)

when

$$V_{-1/3}^T U_{-1/3} \simeq \begin{pmatrix} 1 & (1-\sqrt{2})\lambda & -(1-\sqrt{2})\lambda \\ -(1-\sqrt{2})\lambda & 1 & 0 \\ (1-\sqrt{2})\lambda & 0 & 1 \end{pmatrix}$$

- we obtain $\sin\theta_{sun} \simeq \sin(\frac{\pi}{4}-\theta_C) \longrightarrow$ QLC for the minimal GUT.

  For the realistic GUT, $\lambda \to \frac{\lambda}{3}$

(B) Renormalization Effects

- RG to M_ν :

$$M_\nu \equiv I \cdot M_\nu^0 \cdot I$$
$$= I \cdot U_{CKM}^T V_M^* M_\nu^{diag} V_M^\dagger U_{CKM} \cdot I$$

where $I \equiv I_A \delta_{AB}$ $(A,B = e,\mu,\tau)$

$= I^{RG} + I^{TH}$

※ sizable RG evolution from M_{seesaw} to M_{EW} enhances the size of θ_{12} → problem!

- In SUSY, sizable RG effects ⇒ (large $\tan\beta$, degenerate m_{ν_i})

- Taking $|I_e^{TH}| \gg |I_{M,\tau}^{TH}|$

$$M_\nu \simeq U_{CKM}^T U_{23}^{m*} [I_D + I_e \Lambda] M_{D12} [I_D + I_e \Lambda^\dagger] U_{23}^{m\dagger} U_{CKM}$$

$$\Lambda \simeq \begin{pmatrix} 1 & -\frac{\lambda}{\sqrt{2}} & -\frac{\lambda}{\sqrt{2}} \\ -\frac{\lambda}{\sqrt{2}} & \frac{\lambda^2}{2} & \frac{\lambda^2}{2} \\ -\frac{\lambda}{\sqrt{2}} & \frac{\lambda^2}{2} & \frac{\lambda^2}{2} \end{pmatrix}$$

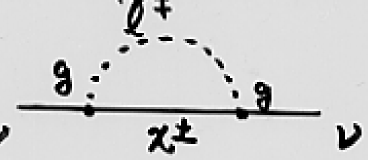
$$M_{D12} \equiv U_{12}^m M_\nu^{diag} U_{12}^{m\dagger}$$

- Varying I_e and m_{ν_1}, we find parameter set (I_e, m_{ν_1})

(e.g. For $m_{\nu_1} \simeq 0.15\,\mathrm{eV}$, $I_e \simeq -4.0\times10^{-5}$
$0.1\,\mathrm{eV}$, $I_e \simeq -8.5\times10^{-5}$
$0.05\,\mathrm{eV}$, $I_e \simeq -3.4\times10^{-4}$)

- How can we obtain such a value of I_e while keeping $|I_e| \gg |I_{m,\tau}|$?

it can be realized by taking into account the contribution of chargino/slepton in SUSY



$$I_e \sim \frac{g^2}{32\pi^2}\left[-\frac{1}{x_e} + \frac{x_e^2-1}{x_e^2}\ln(1-x_e)\right], \quad x_e = 1-\left(\frac{M_{\tilde e}}{\tilde m}\right)^2$$

(e.g. $x_e \sim 0.7 \Rightarrow I_e \sim -8\times10^{-5}$, & we need $M_{\tilde e} \gtrsim 2M_{\tilde\chi}$)

* Summary

- The recent experimental measurement of θ_{sol} and θ_c reveal a surprising relation $\theta_s + \theta_c = \frac{\pi}{4}$.

- This empirical relation can be interpreted as a support of the idea of Grand Unification.

- It can also be a coincidence in the sense that reproducing the relation at high energy in the framework of GUT depends on the renormalization effects whose size can vary with the choice of parameter space.

- While RG effects generally lead to additive contribution on top of the deviation from QLC, we show that the threshold corrections which may exist in SUSY diminish the deviation, so we can achieve QLC relation at low energy.