Neutrino Mixing as a probe of Grand Unification

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 collaborated with C.S. Kim & J. Lee
1. Current Neutrino Experimental Results

- Accumulated neutrino oscillation data
  (measurement of CKM mixing matrix
  \[ \Rightarrow \text{so precise that we can catch some indication on a possible unifying theory for the mystery of the flavor structure.} \]
- Global fit of solar neutrino exp. & KamLAND data
  \[ \Theta_{\text{sun}} = 32.3^\circ \pm 2.4^\circ \ (1\sigma) \]
  \[ \Delta M^2_{\text{sun}} = (6.5 - 9) \times 10^{-5} \text{ eV}^2 \]
- Global analysis of the atmospheric neutrino exp.
  \[ \sin^2 2\theta_{\text{atm}} \geq 0.94 \ (90\% \text{ CL}) \]
  \[ \Delta M^2_{\text{atm}} = (1.3 - 3.0) \times 10^{-3} \text{ eV}^2 \]
  (best fit : \[ \sin^2 2\theta_{\text{atm}} \approx 1.0 \]
- CHOOZ Experiment
  \[ \sin^2 \theta_{13} \leq 0.03 \ (90\% \text{ CL}) \]
2. Quark - Lepton Complementarity

- Taking the Cabibbo angle:
  \[ \Theta_c = 12.8^\circ \pm 0.15^\circ \]

\[ \Rightarrow \Theta_{\text{sun}} + \Theta_c = 45.1^\circ \pm 2.4^\circ \] (10)

QLC relation: \[ \Theta_{\text{sun}} + \Theta_c = \frac{\pi}{4} \]

- Is this relation signal of quark-lepton symmetry or quark-lepton unification?

(Raidal '04, Minakata & Smirnov '04, Firman et al. '04, Mohapatra '04, Ferrandis & Pakvasa '04, Kang, Kim & Lee '05)

* For (2,3) mixing, \[ \Theta_{\text{atm}} + \Theta_{23}^{\text{CKM}} = \frac{\pi}{4} \]

in good agreement with experimental data.
3. QLC relation in GUT

- $\Delta_{Iw} = \frac{1}{2}$ Higgs doublet breaking of EW symmetry
  $\Rightarrow$ yields the quark Yukawa Matrices

\[
M^{(u3)} = U^{(u3)} \begin{pmatrix} m_u & m_c & m_t \\ m_c & m_t & m_t \\ m_t & m_t & m_t \end{pmatrix} V_{3/2}^+, \quad M^{(d3)} = U^{(d3)} \begin{pmatrix} m_d & m_s & m_b \\ m_s & m_s & m_s \\ m_b & m_b & m_b \end{pmatrix} V_{3/2}^+
\]

- observable quark mixing : $U_{CKM} = U_{3/2}^+ U_{3/2}$

- The charged lepton Yukawa Matrix :

\[
M^{(e)} = U^{-1} \begin{pmatrix} m_e & m_e & m_e \\ m_e & m_e & m_e \\ m_e & m_e & m_e \end{pmatrix} V T
\]

- To obtain neutrino mass, we consider seesaw mechanism

\[
M_\nu = M_{\text{Dirac}} \frac{1}{M_R} M_{\text{Dirac}}^T
\]

\[
= U_0 \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V_0^+ \frac{1}{M_R} V_0^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_0^T
\]

\[
\equiv U_0 V_M M_{\text{dirac}} V_M^T U_0^T
\]

where

- $V_M \rightarrow$ rotation of $M_{\text{dirac}} V_0^+ \frac{1}{M_R} V_0^* M_{\text{dirac}}$

- Similar to quark mixing, the observable lepton mixing matrix can be written as

\[
U_{\text{PMNS}} = U_0^+ U_0 V_M
\]
(A) Minimal GUT

\[ M_{\nu_1} = M_{\nu_3}^T, \quad M_{\nu_3} = M_{\text{Dirac}} \text{ (symmetric)} \]

\[ \Rightarrow U_1 = V_{\nu_3}^*, \quad U_0 = U_{\nu_3} \]

\[ U_{\text{PMNS}} = U_{\nu_1}^T U_{\nu_0} V_M \]

\[ = V_{\nu_3}^T U_{\nu_3} V_M \]

\[ = V_{\nu_3} U_{\nu_3} U_{CKM} V_M \]

1. For symmetric \( M_{\nu_1}, M_{\nu_3} \)

\[ U_{\text{PMNS}} = U_{CKM} V_M \]

(→ this form of \( U_{\text{PMNS}} \) has been suggested by several authors in a phenomenological way)

\( V_M \) should have two almost maximal mixing so as to account for the solar & atmospheric \( \nu \) oscillations.

- Can \( U_{\text{PMNS}} \) lead to QHC relation?

\[ U_{\text{PMNS}} = U_{CKM} U_{\nu_3} U_{\nu_2} = U_{\nu_3} (\Theta_{23}) U_{\nu_2} (\Theta_{12}) U_{\nu_2} (\Theta_{12}) \]

then, \( \Theta_{ij} \) can be presented in terms of \( \lambda \)

\[ \sin \Theta_{12} = \frac{1}{\sqrt{2}} \lambda + O(\lambda^3) \]

\[ \sin \Theta_{23} \approx -\frac{1}{\sqrt{2}} (1 - \frac{1}{4} \lambda^2 - A \lambda^3) \]

\[ \sin \Theta_{13} = -\frac{1}{\sqrt{2}} \lambda \quad (\because \frac{1}{\sqrt{2}} \lambda \sim 0.16 \Rightarrow \theta \sim 90^\circ) \]

\[ \sin \Theta_{\text{sol}} = \sin(\frac{\pi}{4} - \Theta_{12}) + \frac{A}{2} (\sqrt{2} - 1) \]

\[ \Rightarrow \text{leads to } \delta \text{ of Sun } \approx 3.5^\circ \]
(i) Realistic quark-lepton unification

- Although the minimal GUT leads to an elegant relation between $U_{PMNS}$ & $U_{CKM}$, it indicates undesirable mass relations between quarks & leptons at GUT scale:

\[ M_d = M_e \]

Thus, we need to modify the simple relations between quarks & leptons Yukawa matrices.

- A well known empirical relation:

\[ |V_{us}| \approx \sqrt{\frac{M_d}{M_s}} \approx 3 \sqrt{\frac{M_e}{M_u}} \]

- As known from the '70s, if $M_d$ is generated from the mixing between 1st & 2nd families, the relation between $\Theta_{23}$ & $(M_3/M_1)$ can be simply explained:

\[
\hat{M}_{YM} = \begin{pmatrix}
0 & \sqrt{\frac{M_s M_d}{m_b^2}} & 0 (\alpha^3) \\
\sqrt{\frac{M_s M_d}{m_b^2}} & \frac{M_s}{M_b} & 0 (\alpha^2) \\
0 (\alpha^3) & 0 (\alpha^2) & 1
\end{pmatrix} \quad \text{(in the basis where $M_{YM}$ is real & diagonal)}
\]

- Analogously,

\[
\hat{M}_4 = \begin{pmatrix}
0 & \sqrt{\frac{m_u m_e}{m_t^2}} & 0 (\alpha^3) \\
\sqrt{\frac{m_u m_e}{m_t^2}} & \frac{m_u}{m_t} & 0 (\alpha^2) \\
0 (\alpha^3) & 0 (\alpha^2) & 1
\end{pmatrix} \quad \text{(in the basis where $M_4$ is diagonal)}
\]
\[ U_1^+ U_0 = \begin{pmatrix}
1 & -\frac{\Lambda}{3} & \frac{1}{3} \theta \lambda^2 \\
\frac{\Lambda}{3} & 1 & 2 \theta \lambda^2 \\
-\theta \lambda^2 & -2 \theta \lambda & 1
\end{pmatrix} \]

(where \( \theta \approx 0.09 \))

- This form of mixing matrix can be obtained by introducing the Higgs sector transforming under the rep. \( \mathbf{25} \) of SU(5) (Georgi & Jarlskog) or \( \mathbf{126} \) of SO(10) (Dimopoulos et al., Mohapatra et al.)

- Can this case lead to QLC relation?

- \( \theta_{ij} \) in PMNS presented in terms of \( \lambda \):

\[
\sin \theta_{12} = \frac{\Lambda}{2 \sqrt{2}}, \quad \sin \theta_{23} = -\frac{1}{\sqrt{2}} (1 - 2 \theta \lambda) \\
\sin \theta_{13} = -\frac{\Lambda}{2 \sqrt{2}}
\]

\[
\Rightarrow \sin \theta_{\text{sun}} = \sin \left( \frac{\pi}{4} - \theta \right) + \frac{\Lambda}{2} \left( \sqrt{2} - \frac{1}{3} \right) \\
\Rightarrow \delta \theta_{\text{sun}} \approx 8^\circ
\]

- Is there any way to diminish the corrections to QLC relation?
* A simple example of SU(5) Yukawa terms that lead to the empirical relation:

\[ h_{33} \bar{10}_3 \bar{5}_3 \bar{5}_H + h_{22} \bar{10}_2 \bar{5}_2 \bar{45}_H + h_{12} (\bar{10}_1 \bar{5}_2 + 10_2 \bar{5}_1) \bar{5}_H \]

\[ \Rightarrow \quad Y_d = \begin{pmatrix} 0 & C & 0 \\ C & 0 & 0 \\ 0 & 0 & A \end{pmatrix}, \quad Y_L = \begin{pmatrix} 0 & C & 0 \\ C & -3B & 0 \\ 0 & 0 & A \end{pmatrix} \]

\[ \quad \Rightarrow \text{Georgi & Jarlskog} \]

* The generalization to SO(10) of the simple Yukawa operators that give rise to the above Eq.

\[ h_{33} \bar{16}_3 \bar{16}_3 \bar{10}_H + h_{22} \bar{16}_2 \bar{16}_2 \bar{126}_H + h_{12} \bar{16}_1 \bar{16}_2 \bar{10}_H \]
(A) Non-symmetric $M_{V3}, M_1$

$$U_{PMNS} = V_{Y3}^T U_{Y3} U_{ckm} V_M$$

$\rightarrow$ responsible for the correction

(then can diminish the correction to QLC)

when

$$V_{Y3}^T U_{Y3} = \begin{pmatrix} 1 & (1-\frac{\lambda}{2}) & -\frac{1}{2} \frac{\lambda}{2} \\ -(1-\frac{\lambda}{2}) & 1 & 0 \\ (1-\frac{\lambda}{2}) & 0 & 1 \end{pmatrix}$$

we obtain $\sin \theta_{wsn} = \sin (\frac{\pi}{4} - \theta_C) \rightarrow QLC$ for the minimal GUT.

For the realistic GUT, $\lambda \rightarrow \frac{\lambda}{3}$

(B) Renormalization Effects

- RG to $M_\nu$:

$$M_\nu = I \cdot M_0 \cdot I$$

$$= I \cdot U_{ckm} V_M^* M_\nu^{diag} V_M U_{ckm} \cdot I$$

where $I = I_A \delta_{AB}$ (A,B = e,m,$\nu$)

$$= I_{RG} + I_{TH}$$

※ sizable RG evolution from MS to ME

enhances the size of $\theta_{12}$ → problem!

- In SUSY, sizable RG effects $\Rightarrow \begin{pmatrix} \text{large tan} \beta \\ \text{degenerate } M_{\nu_i} \end{pmatrix}$
Taking \( |Ie|^n >> |I_{m2}| \)

\[
M_\nu \approx U_{CKM} U_{3}^{m \nu} \left[ I_{0} + I e \Lambda \right] M_{12} \left[ I_{0} + I e \Lambda^{+} \right] U_{3}^{m \nu} U_{CKM} \\
\Lambda = \begin{pmatrix}
1 & -\frac{1}{\Delta_{3}} & -\frac{1}{\Delta_{3}} \\
-\frac{1}{\Delta_{3}} & \frac{\Delta_{2}}{2} & \frac{\Delta_{2}}{2} \\
-\frac{1}{\Delta_{3}} & \frac{\Delta_{2}}{2} & \frac{\Delta_{2}}{2}
\end{pmatrix}
\]

\[M_{D12} \equiv U_{li}^{m} M_{\nu}^{diag} U_{li}^{m+} \]

- Varying \( Ie \) and \( M_{\nu} \), we find parameter set \((Ie, M_{\nu})\)

\[(e.g. \text{ For } M_{\nu} \approx 0.15\text{eV, } Ie = -4.0 \times 10^{-5} \]
\[0.11\text{eV, } Ie = -9.5 \times 10^{-5} \]
\[0.05\text{eV, } Ie = -3.4 \times 10^{-4} \)

- How can we obtain such a value of \( Ie \) while keeping \( |Ie| >> |I_{m2}| \)?

it can be realized by taking into account the contribution of charged sleptons in SUSY

\[
Ie \sim \frac{g^2}{32\pi^2} \left[ -\frac{1}{\chi e} + \frac{\chi e^2 - 1}{\chi e^2} \ln(1 - \chi e) \right], \quad \chi e = 1 - \left( \frac{M_{\tilde{e}}}{M_{\nu}} \right)^2
\]

\[(e.g. \chi e \approx 0.7 \Rightarrow Ie \approx -8 \times 10^{-5}, \text{ we need } M_{\tilde{e}} \geq 2M_{\nu}\)
* Summary

- The recent experimental measurement of $\Theta_{sol}$ and $\Theta_c$ reveal a surprising relation $\Theta_s+\Theta_c=\frac{\pi}{4}$.

- This empirical relation can be interpreted as a support of the idea of Grand Unification.

- It can also be a coincidence in the sense that reproducing the relation at high energy in the framework of GUT depends on the renormalization effects whose size can vary with the choice of parameter space.

- While RG effects generally lead to additive contribution on top of the deviation from QLC, we show that the threshold corrections which may exist in SUSY diminish the deviation, so we can achieve QLC relation at low energy.