Neutrino Mass Matrix from Non-Abelian Discrete symmetry

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W.Grimus, A.Joshipura, S.K., L.Lavoura, H.Sawanaka and M.Tanimoto, hep-ph/0408123

M. Frigerio, S. K., E. Ma, M. Tanimoto, hep-ph/0409187.

I. Introduction

This is a study on **predictive neutrino models** which is based on **non-Abelian discrete symmetries**.

today's talk → D_4 symmetry

Simple realization of flavor structure of leptons is obtained.

Some other interesting issues :

ex.) SUSY flavor problem, quark sector . . . .

→ J. Kubo san's talk

$\sin^2 2\theta_{atm} > 0.92$ (90% C.L.) $\theta_{23} = 45^\circ \pm 8^\circ$

$\tan^2\theta_{sol} > 0.33 - 0.49$ (90% C.L.) $\theta_{12} = 33^\circ \pm 4^\circ$

$\sin^2\theta_{CHOOZ} < 0.057$ (3σ) $\theta_{13} < 13^\circ$

Does these results reflect some flavor symmetry ?

There are two ways to be investigated :

(1) The structure of mass matrix is assumed.
→ What symmetry can generate such mass matrix ?

(2) The flavor symmetry is assumed.
→ What mass matrix can be generated ?

Today's talk → Scenerio (1)

Non-Abelian discrete groups

order : number of elements

order	6	8	10	12	14	...
S_N	S_3					...
D_N	$D_3(=S_3)$	D_4	D_5	D_6	D_7	...
Q_N		$Q_8(Q_4)$		$Q_{12}(Q_6)$		...
T				$T(A_4)$		...

Geometrical object :

$D_3(=S_3)$: rotations and reflections of △

D_4 : rotations and reflections of □

A_4 : rotations and reflections of tetrahedron

Models based on non-Abelian discrete groups

S_3 S.Pakvasa and H.Sugawara, PLB 73(1978)61.
J.Kubo, A.Modragon, M.Mondragon and E.Rodrigues-Jauregui, Prog.Theor.Phys.109(2003)795.

D_4 W.Grimus and L.Lavoura, PLB 572 (2003) 189.
Grimus, Joshipura, S.K., Lavoura, Sawanaka and Tanimoto hep-ph/0408123

$Q_8(Q_4)$ M. Frigerio, S. K., E. Ma and M. Tanimoto, hep-ph/0409187.

A_4 E.Ma and G.Rajasekaran, PRD 64 (2001) 113012.
K.S.Babu, E.Ma and J.W.F.Valle, PLB 552 (2003) 207.

$Q_{12}(Q_6)$ K.S.Babu and J.Kubo, hep-ph/0411226.

Plan of the talk

1. Introduction

2. The model based on D_4

   Grimus and Lavoura PLB('03) ...

3. Symmetry breaking and neutrino mixing angles

   Grimus, Joshipura, S.K., Lavoura, Sawanaka and Tanimoto hep-ph/0408123

4. Summary and Discussion

2. The model based on D_4

Neutrino mass matrix

$\sin^2 2\theta_{atm} > 0.92$ (90% C.L.) $\theta_{23} = 45^\circ \pm 8^\circ$

$\tan^2\theta_{sol} > 0.33 - 0.49$ (90% C.L.) $\theta_{12} = 33^\circ \pm 4^\circ$

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$$M_\nu = U_{\rm MNS} M_\nu^{\rm diag} U_{\rm MNS}^T, \quad M_e = M_e^{\rm diag}$$

$$U_{\rm MNS} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric } \nu, \text{ K2K}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{reactor } \nu, \text{ CP violation}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar } \nu, \text{ KamLAND}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & e^{i\sigma} \end{pmatrix}}_{\text{Majorana phases}}$$

Normal : $m_1 < m_2 < m_3$ $(\Delta m^2_{sol} = m_2^2 - m_1^2, \Delta m^2_{atm} = m_3^2 - m_1^2)$

Inverted : $m_3 < m_1 < m_2$ $(\Delta m^2_{sol} = m_2^2 - m_1^2, \Delta m^2_{atm} = m_1^2 - m_3^2)$

Quasi-degenerate : $m_1 \sim m_2 \sim m_3$

Neutrino mass matrix

W.Grimus and L.Lavoura(2003)

Assumption : $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$

Framework : SM + 3 ν_R (seesaw model)

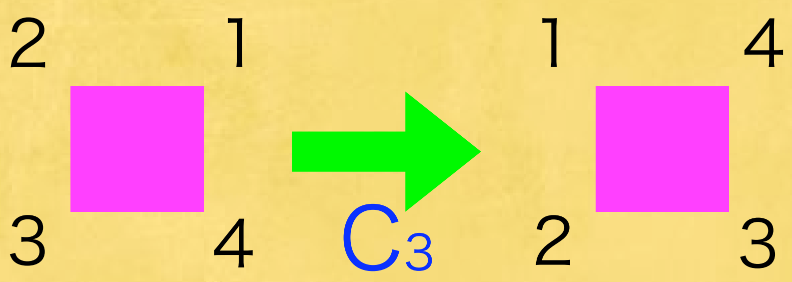
$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} , \quad M_e = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} .$$

$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} \quad ; \; S\, M_\nu\, S = M_\nu \; , \quad S = \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$$

$$\begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = (z-w) \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \;\Rightarrow\; U = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta}{\sqrt{2}} & -\frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

θ_{12} = arbitrary, no Dirac phase, two Majorana phases

Group D4



class	transf.	(1234)→	2 dim. reps.	Tr
C_1	unit	(1234)	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	2
C_2	180° rot.	(3412)	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	-2
C_3	90° rot.	(4123)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	0
C_3	270° rot.	(2341)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	0
C_4	reflection ($y = x$ 軸)	(1432)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	0
C_4	reflection ($y = -x$ 軸)	(3214)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	0
C_5	reflection ($y = 0$ 軸)	(4321)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	0
C_5	reflection ($x = 0$ 軸)	(2143)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	0

Σ(dim. of reps.)^2
= # of elements

# of reps.
= # of classes

n : # of elements

h : order of any elements in that class(g^h=1)

class	n	h	χ_{++}	χ_{+-}	χ_{-+}	χ_{--}	χ_2
C_1	1	1	1	1	1	1	2
C_2	1	2	1	1	1	1	-2
C_3	2	4	1	-1	-1	1	0
C_4	2	2(4)	1	1	-1	-1	0
C_5	2	2(4)	1	-1	1	-1	0
rep.			$\mathbf{1}^{++}$	$\mathbf{1}^{+-}$	$\mathbf{1}^{-+}$	$\mathbf{1}^{--}$	$\mathbf{2}$

2×2 Decompositions of D4

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \times \begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = \mathbf{1}^{++} + \mathbf{1}^{+-} + \mathbf{1}^{-+} + \mathbf{1}^{--}$$

group	$\alpha\alpha' + \beta\beta'$	$\alpha\beta' + \beta\alpha'$	$\alpha\alpha' - \beta\beta'$	$\alpha\beta' - \beta\alpha'$
D_4	$\mathbf{1}^{++}$	$\mathbf{1}^{+-}$	$\mathbf{1}^{-+}$	$\mathbf{1}^{--}$
Q_8	$\mathbf{1}^{--}$	$\mathbf{1}^{-+}$	$\mathbf{1}^{+-}$	$\mathbf{1}^{++}$

Mass matrix in D4 doublet basis

lepton : $(l_{L1},\ l_{L2}),\ (l_{R1},\ l_{R2})$

Higgs : $H_1,\ H_2$

$$H_1(\mathbf{1}^{++}),\ H_2(\mathbf{1}^{+-})\ :\ \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

$$H_1(\mathbf{1}^{++}),\ H_2(\mathbf{1}^{-+})\ :\ \begin{pmatrix} a+b & 0 \\ 0 & a-b \end{pmatrix}$$

$$H_1(\mathbf{1}^{++}),\ H_2(\mathbf{1}^{--})\ :\ \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$$H_1(\mathbf{1}^{+-}),\ H_2(\mathbf{1}^{-+})\ :\ \begin{pmatrix} a & b \\ b & -a \end{pmatrix}$$

$$H_1(\mathbf{1}^{+-}),\ H_2(\mathbf{1}^{--})\ :\ \begin{pmatrix} 0 & a+b \\ a-b & 0 \end{pmatrix}$$

$$H_1(\mathbf{1}^{-+}),\ H_2(\mathbf{1}^{--})\ :\ \begin{pmatrix} a & b \\ -b & -a \end{pmatrix}$$

$D_4 \times Z_2$ model   SM + 3 ν_R + 3 ϕ + 2 χ

ϕ : gauge doublet Higgs   W.Grimus and L.Lavoura (2003)

χ : gauge singlet Higgs

Charge assignment of D_4 :

$$D_\ell \equiv (\nu_\ell, \ell)^T \quad : \quad \ell = e, \mu, \tau$$

$$\begin{aligned} D_{eL} &: \mathbf{1}^{++} \quad , \quad (D_{eL}, D_{eL}) : \mathbf{2} \\ e_R &: \mathbf{1}^{++} \quad , \quad (\mu_R, \tau_R) : \mathbf{2} \\ \nu_{eR} &: \mathbf{1}^{++} \quad , \quad (\nu_{\mu R}, \nu_{\tau R}) : \mathbf{2} \end{aligned}$$

Higgs doublets $\Phi_1 : \mathbf{1}^{++}, \ \Phi_2 : \mathbf{1}^{++}, \ \Phi_3 : \mathbf{1}^{-+}$

Higgs singlets $(\chi_1, \chi_2) : \mathbf{2}$

$$M_{D,\ell} = \begin{array}{c|ccc} & \mathbf{1}^{++} & (1 & 2) \\ \hline \mathbf{1}^{++} & y_{1,6}\Phi_1 + y_{2,7}\Phi_2 & 0 & 0 \\ 1 & 0 & y_{3,8}\Phi_1 + y_{4,9}\Phi_2 + y_{5,10}\Phi_3 & 0 \\ 2 & 0 & 0 & y_{3,8}\Phi_1 + y_{4,9}\Phi_2 - y_{5,10}\Phi_3 \end{array} ,$$

$$M_R = \begin{array}{c|ccc} & \mathbf{1}^{++} & (1 & 2) \\ \hline \mathbf{1}^{++} & M & y_\chi \chi_1 & y_\chi \chi_2 \\ 1 & y_\chi \chi_1 & M' & 0 \\ 2 & y_\chi \chi_2 & 0 & M' \end{array}$$

D4 × Z2 model

- $e_R, \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}, \Phi_1$ transforms as odd under an extra Z_2

$$M_D = \left(\begin{array}{c|ccc} & \mathbf{1}^{++} & (1 & 2) \\ \hline \mathbf{1}^{++} & y_1\Phi_1 & 0 & 0 \\ \frown 1 & 0 & y_2\Phi_1 & 0 \\ 2 \smile & 0 & 0 & y_2\Phi_1 \end{array}\right), \quad M_R = \left(\begin{array}{c|ccc} & \mathbf{1}^{++} & (1 & 2) \\ \hline \mathbf{1}^{++} & M & y_\chi\chi_1 & y_\chi\chi_2 \\ \frown 1 & y_\chi\chi_1 & M' & 0 \\ 2 \smile & y_\chi\chi_2 & 0 & M' \end{array}\right).$$

$$M_\ell = \left(\begin{array}{c|ccc} & \mathbf{1}^{++} & (1 & 2) \\ \hline \mathbf{1}^{++} & y_3\Phi_1 & 0 & 0 \\ \frown 1 & 0 & y_4\Phi_2 + y_5\Phi_3 & 0 \\ 2 \smile & 0 & 0 & y_4\Phi_2 - y_5\Phi_3 \end{array}\right)$$

origin of lepton mixings

Neutrino mass matrix : $M_D \equiv diag(A, B, B), \quad M_R \equiv \begin{pmatrix} d & e & f \\ e & g & 0 \\ f & 0 & g \end{pmatrix}$

$$M_\nu = M_D M_R{}^{-1} M_D^T \propto \begin{pmatrix} A^2g^2 & -ABeg & -ABfg \\ -ABeg & B^2(dg - f^2) & B^2ef \\ -ABfg & B^2ef & B^2(dg - e^2) \end{pmatrix}$$

If we assume $\langle\chi_2\rangle = \langle\chi_2\rangle = \frac{W}{2}$; $W \gg v$, $M_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}$

3. Symmetry breaking and neutrino mixing angles

Small perturbation to M_ν by ε and ε'.

W.Grimus, A.S.Joshipura, S.K., L.Lavoura and M.Tanimoto ('04)

Assumption : $D_4 \times Z_2$ symmetry is broken at low-energy.

$$M_\nu \;\Rightarrow\; M_\nu + \delta M_\nu \;=\; \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} + \begin{pmatrix} 0 & \epsilon_1 & -\epsilon_1 \\ \epsilon_1 & \epsilon_2 & 0 \\ -\epsilon_1 & 0 & -\epsilon_2 \end{pmatrix}$$

$$\equiv \begin{pmatrix} x & y(1+\epsilon) & y(1-\epsilon) \\ y(1+\epsilon) & z(1+\epsilon') & w \\ y(1-\epsilon) & w & z(1-\epsilon') \end{pmatrix}$$

$\epsilon_1 \equiv \epsilon y, \;\; \epsilon_2 \equiv \epsilon' z$

$|\varepsilon|$,$|\varepsilon'|$ are constrained by experiments.

$|U_{e3}| < 0.2$, $|\cos 2\theta_{23}| < 0.28$, $0.33 < \tan^2\theta_{atm} < 0.49$,

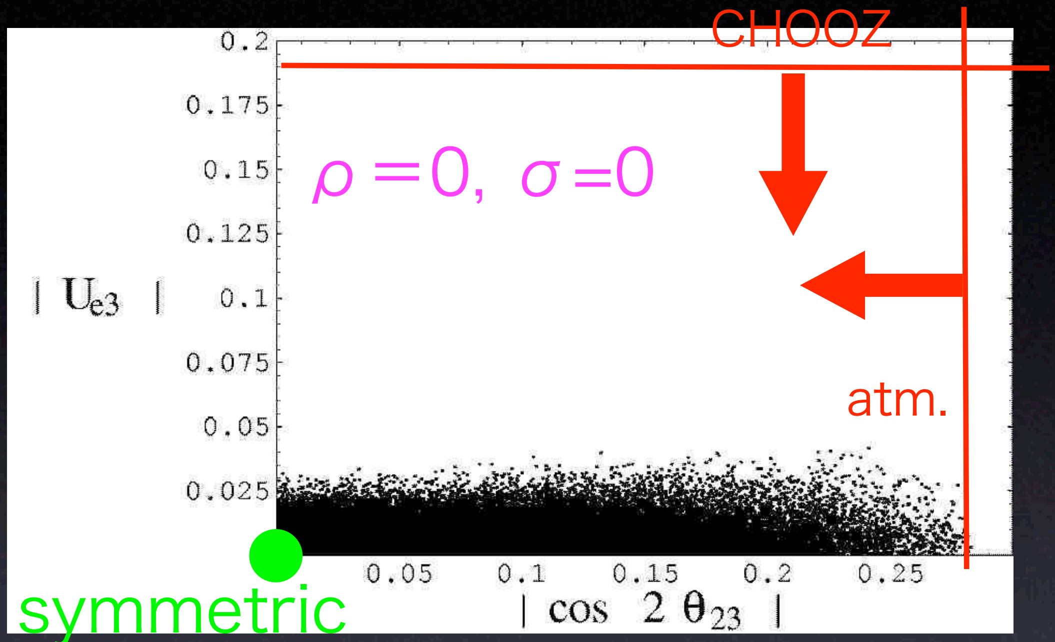
$7.7\times10^{-5} < \Delta m_{sol} < 8.8\times10^{-5}\,\mathrm{eV}^2$, $1.5\times10^{-3} < \Delta m_{atm} < 3.4\times10^{-3}\,\mathrm{eV}^2$

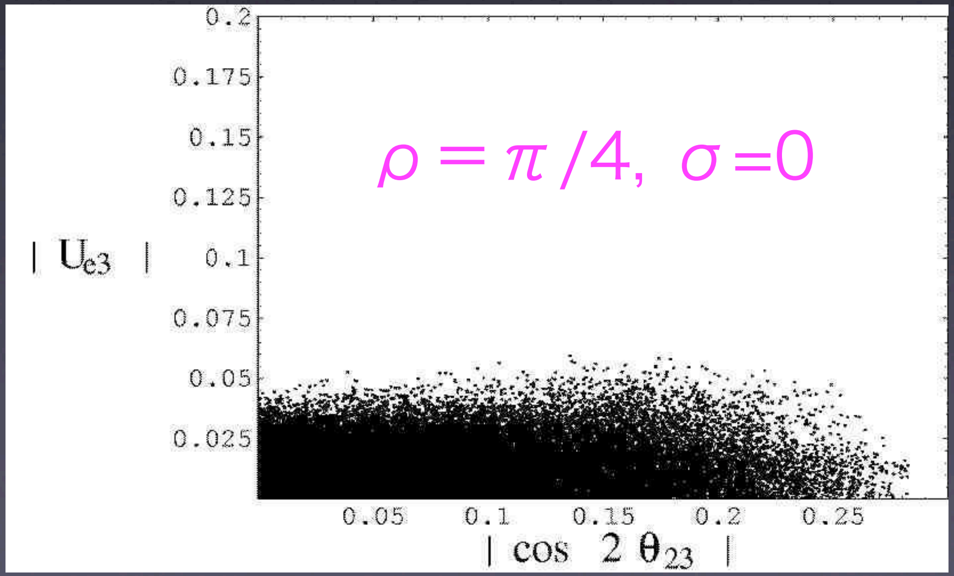
Normal hierarcy of ν mass

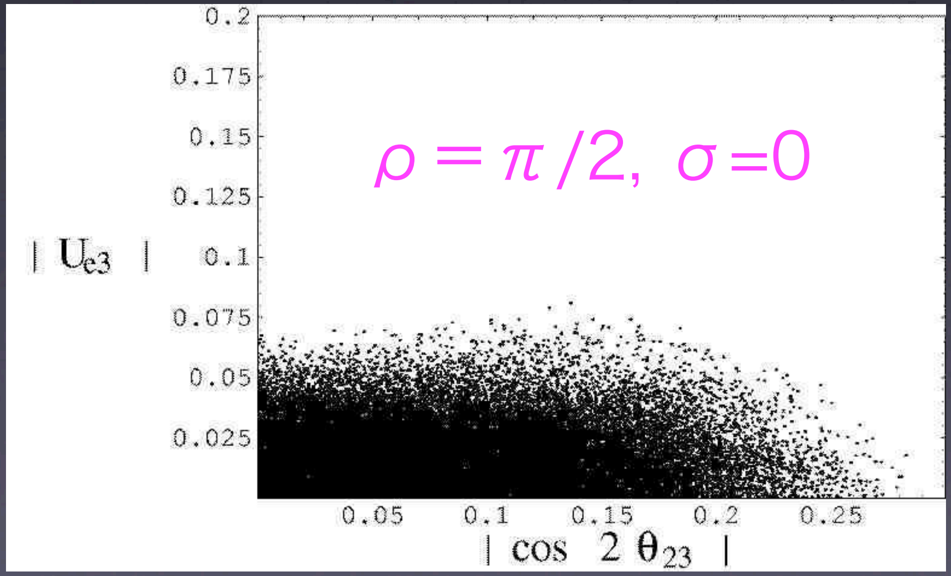
$|\varepsilon|, |\varepsilon'| < 0.3$

$|U_{e3}| < 0.2$

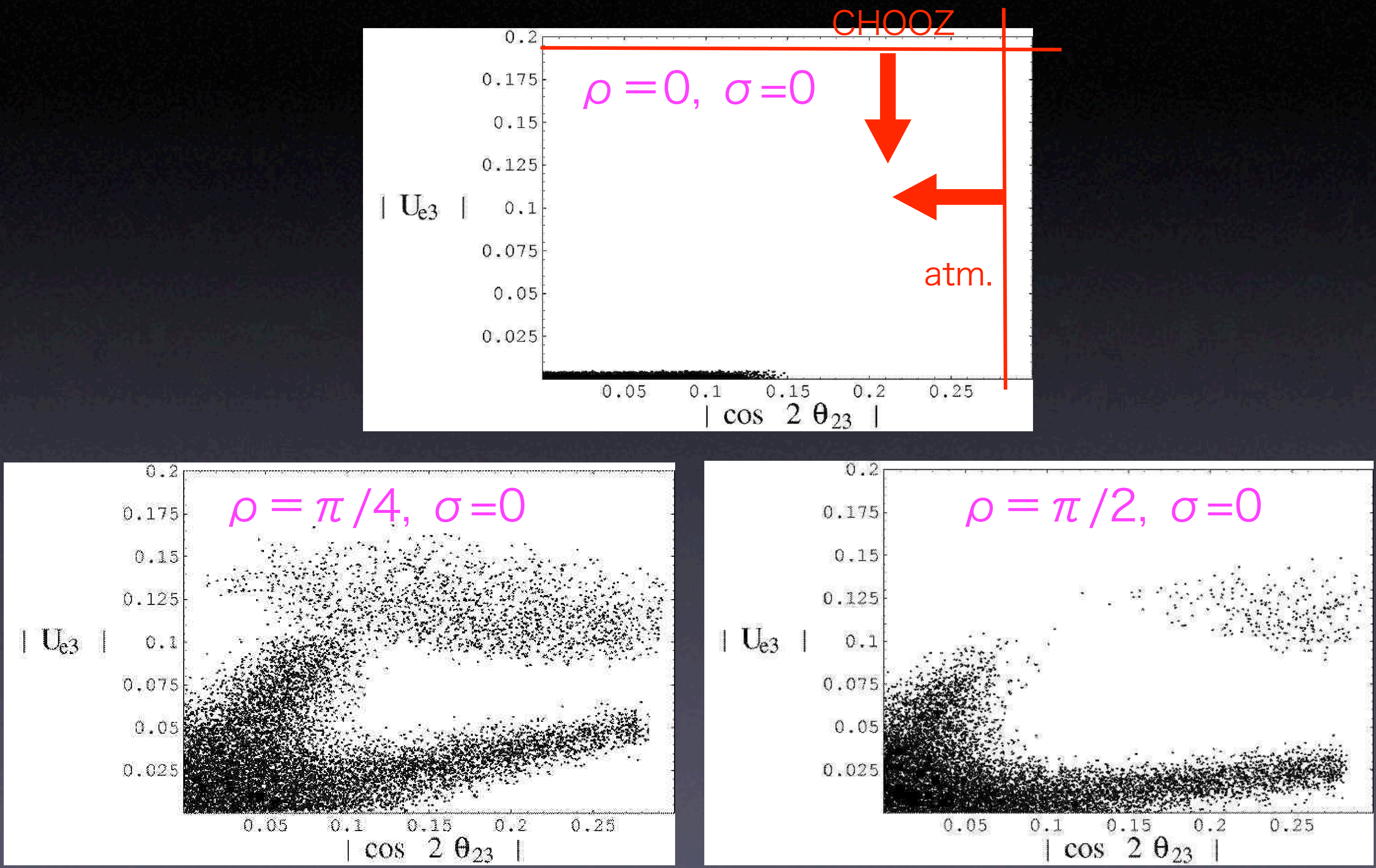
$|\cos 2\theta_{23}| < 0.28$





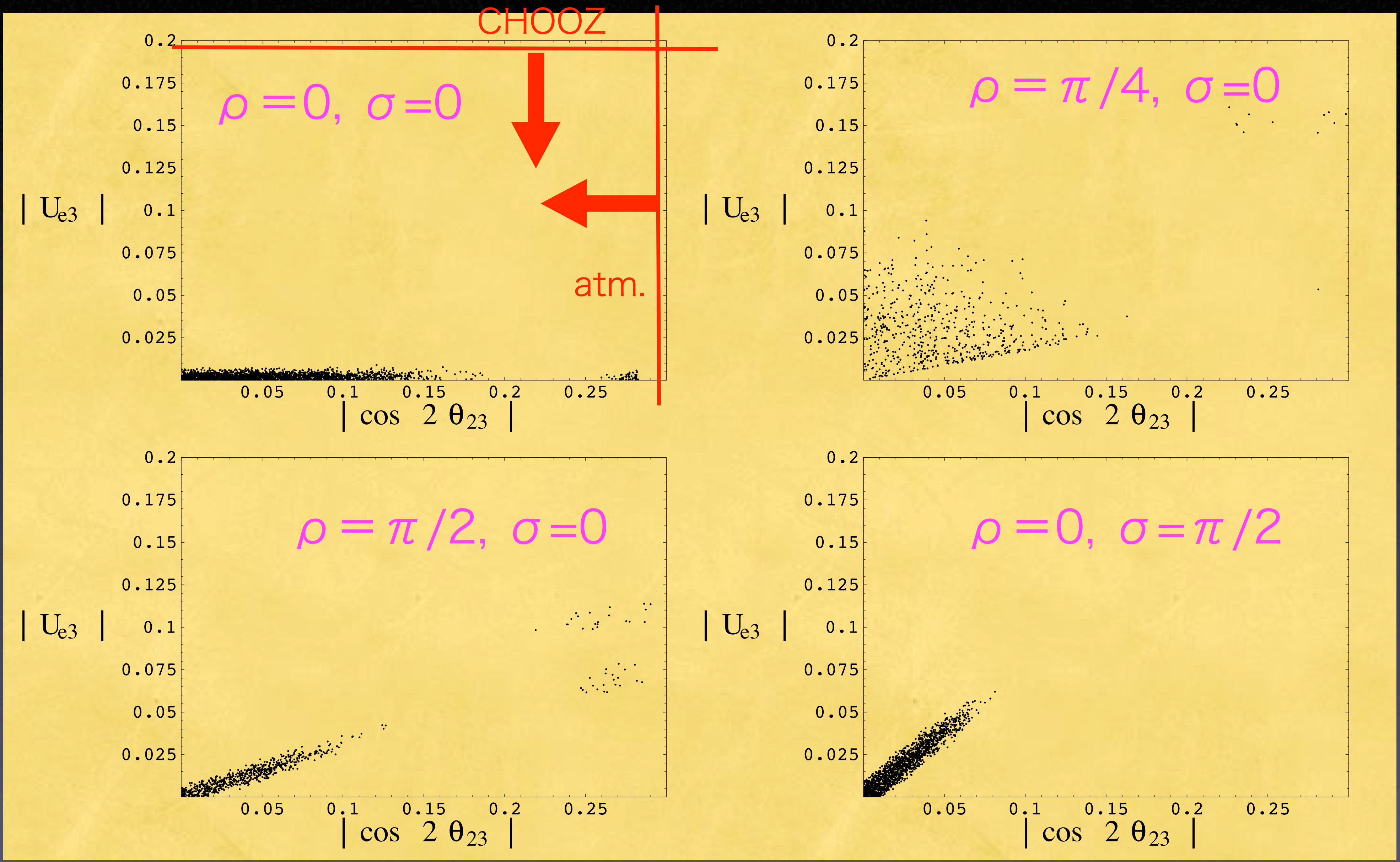


Inverted hierarcy of ν mass



Quasi-degenerate of ν mass

$|\varepsilon| < 0.3$, $|\varepsilon'| < 0.03$, $m = 0.3$ eV



1. Normal ν mass hierarchy :

small deviation of $|U_{e3}|$, large deviation of $|\cos 2\theta_{23}|$

2. Inverted ν mass hierarchy :

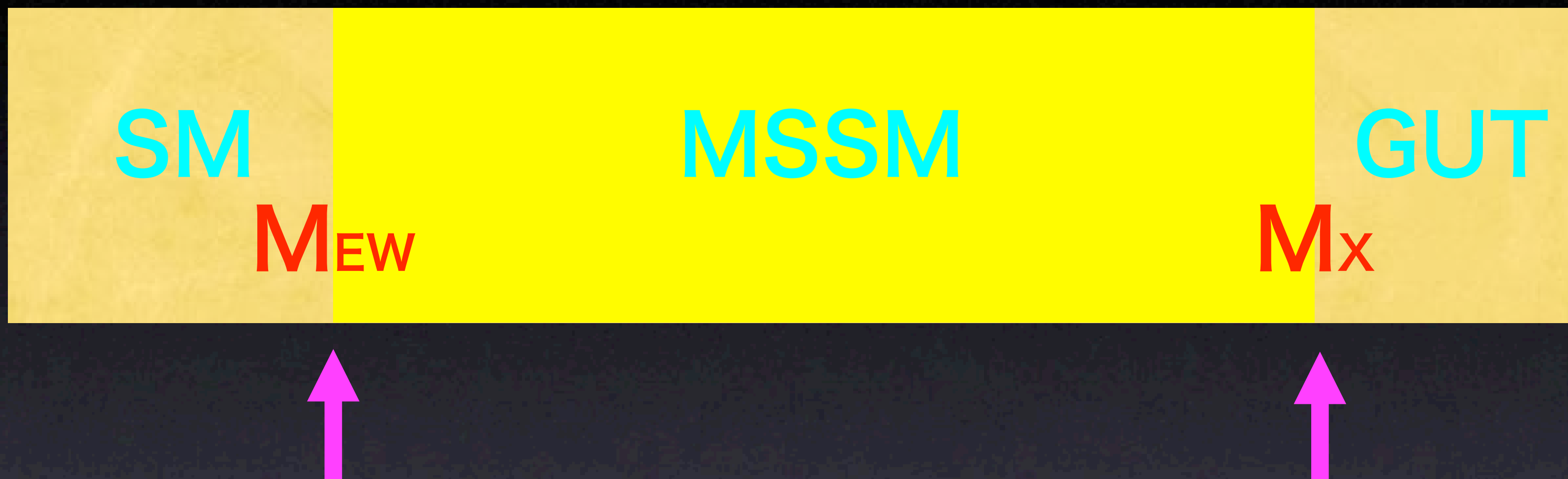
small/large deviation of $|U_{e3}|$, large deviation of $|\cos 2\theta_{23}|$

3. Quasi-degenerate ν mass hierarchy :

small/large deviation of $|U_{e3}|$, small deviation of $|\cos 2\theta_{23}|$

Example Radiatively generated Ue3 and cos2θatm

Assumption : ε , ε ’ are generated due to radiative corrections.



D_4 is broken

$U_{e3} \neq 0$, $\cos 2\theta_{23} \neq 0$

D_4 symmetric

$U_{e3} = \cos 2\theta_{23} = 0$

M_ν at low-scale is given by

$$M_\nu(M_Z) \propto I\, M_\nu(M_X)\, I \quad ,$$

where I is flavor dependent matrix :

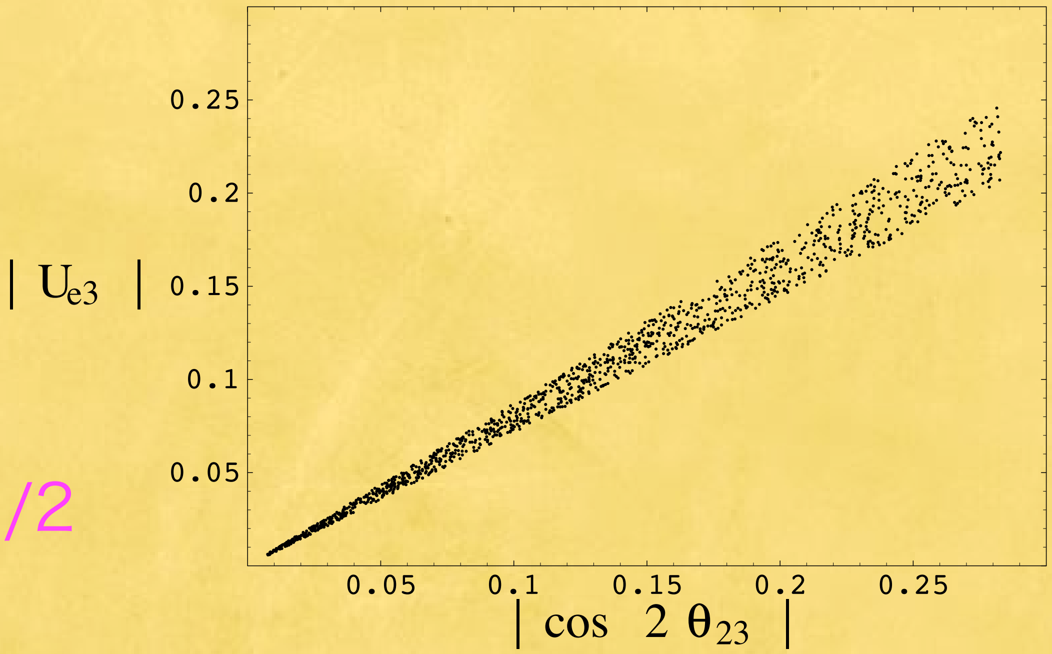
$$I \simeq diag(1+\delta_e . 1+\delta_\mu, 1+\delta_\tau) \quad \text{with} \quad \delta_\alpha \simeq c \left(\frac{m_\alpha}{4\pi v}\right) \ln \frac{M_X}{M_Z}$$

where $c = 3/2, -1/\cos\beta$ in the case of SM, MSSM, respectively.

$\tan\beta$ is constrained :

$\tan\beta \lesssim 23$

m = 0.3 eV, $\rho = 0$, $\sigma = \pi/2$



4. Summary and discussions

Q8 model : SM + triplet Higgs ξ

M. Frigerio, S. K., E. Ma and M. Tanimoto, ('04)

$D_\ell \equiv (\nu_\ell, \ell)^T \; : \; \ell = e, \mu, \tau$

$D_{eL} \; : \; \mathbf{1}^{++} \quad , \quad (D_{eL}, D_{eL}) \; : \; \mathbf{2}$

$e_R \; : \; \mathbf{1}^{++} \quad , \quad (\mu_R, \tau_R) \; : \; \mathbf{2}$

Higgs doublets $\Phi_1 \; : \; \mathbf{1}^{++}$, $\underline{\Phi_2 \; : \; \textbf{many possibilities}}$

Higgs triplets $\underline{\xi_1, \xi_2 \; : \; \textbf{many possibilities}}$, $(\xi_3, \xi_4) \; : \; \mathbf{2}$

			1	2	3
	ξ_1	ξ_2	$\phi_2 \sim \mathbf{1}^{+-}$	$\phi_2 \sim \mathbf{1}^{-+}$	$\phi_2 \sim \mathbf{1}^{--}$
A	$\mathbf{1}^{++}$	$\mathbf{1}^{+-}$	(1)	(2)	(2)
B	$\mathbf{1}^{++}$	$\mathbf{1}^{-+}$	(2)	(1)	(2)
C	$\mathbf{1}^{++}$	$\mathbf{1}^{--}$	(2)	(2)	(1)
D	$\mathbf{1}^{+-}$	$\mathbf{1}^{-+}$	(3)	(3)	(4)
E	$\mathbf{1}^{+-}$	$\mathbf{1}^{--}$	(3)	(4)	(3)
F	$\mathbf{1}^{-+}$	$\mathbf{1}^{--}$	(4)	(3)	(3)

Basis : charged lepton mass matrix is diagonal.

$$(1) \; : \; M_\nu = \begin{pmatrix} a & c & d \\ c & 0 & b \\ d & b & 0 \end{pmatrix}$$

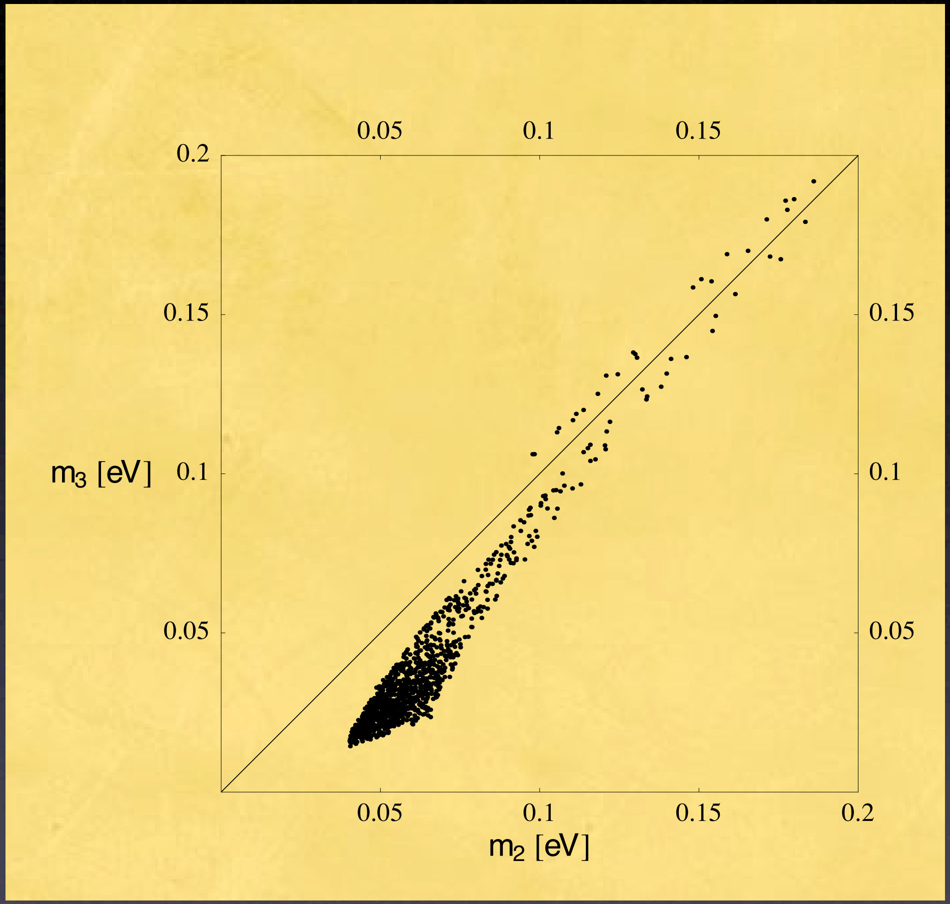
$$(2) \; : \; M_\nu = \begin{pmatrix} a & c & d \\ c & b & 0 \\ d & 0 & b \end{pmatrix}$$

$$(3) \; : \; M_\nu = \begin{pmatrix} 0 & c & d \\ c & a & b \\ d & b & a \end{pmatrix}$$

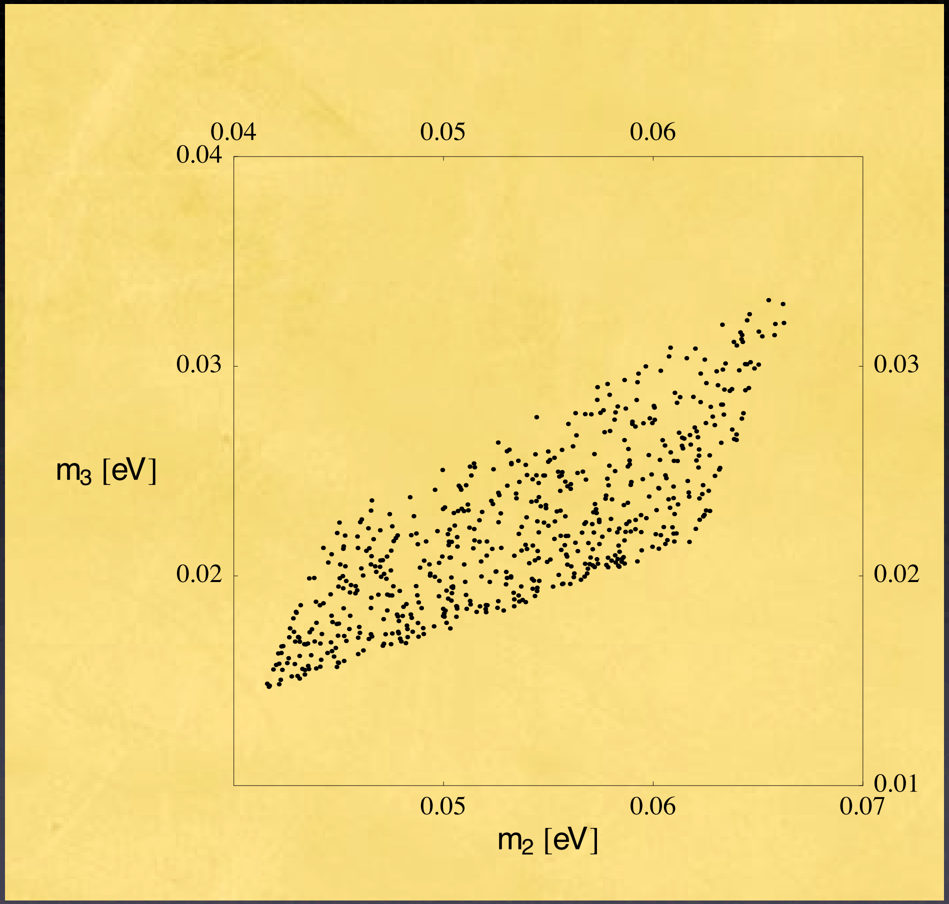
$$(4) \; : \; M_\nu = \begin{pmatrix} 0 & c & d \\ c & a & 0 \\ d & 0 & b \end{pmatrix}$$

~~(4)~~ excluded

scenario (1)



scenario (2)



Summary

Neutrino Models based on D_4 symmetry :

$\theta_{13} = 0°$, $\theta_{23} = 45°$ is predicted in symmetric limit

$|U_{e3}|$ and $|\cos 2\theta_{23}|$ are deviated from zero by small perturbation (ex. radiative correction).

$|U_{e3}|$ is strongly suppressed and $|\cos 2\theta_{23}|$ could be large near exp. bound. Both depend on Majorana phases : ρ, σ.

Neutrino Models based on Q_8 symmetry.

Questions

Is a predictive D_4 model possible without extra Z_2 ?

How about in quark sector ?

Supersymmetric model ?

Higgs potential ?