2005年1月12日　　京都大学基礎物理学研究所　研究会

CPの破れと物質生成

Protecting the primordial baryon asymmetry in the $SU(2)_L$ triplet Higgs model compatible with KamLAND and WMAP

Reference

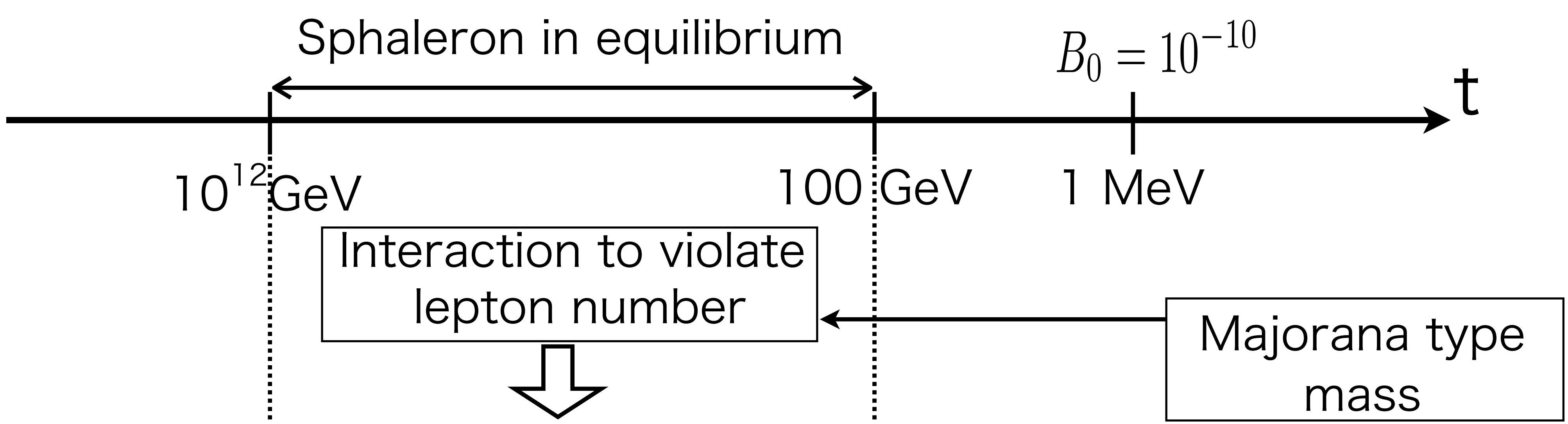
K. Hasegawa, Physical Review D 70, 054002(2004)
(hep-ph/0403272)

Kouhei Hasegawa (Kobe University)

1. Introduction

- Neutrino oscillation experiments (S-K, SNO, KamLAND, ・・・) ⇒ Neutrinos have masses $M_\nu \simeq 0.1 \text{ eV}$
- Majorana type mass ⇒ Lepton number violation ⇒ Influence upon baryon number

Sphaleron $\Delta B = \Delta L \neq 0$, Equilibrium (100 GeV ～10^{12} GeV)



Possibility to wash out any initial baryon number

- In order to protect the initial baryon number, it is necessary that the lepton number violating processes are out of equilibrium in the equilibrium region of sphaleron.

(Time interval during which a lepton number violating process occurs once) > (Age of universe)

$$\frac{1}{\Gamma_{\not{L}}} > \frac{1}{H}$$

$\Gamma_{\not{L}}$: Interaction rate of the lepton number violating process

H : Hubble parameter

$$\Leftrightarrow \boxed{\Gamma_{\not{L}} < H}$$

: Condition to protect the initial baryon number

Today's talk

goal of this research

1. What is the condition to protect the initial baryon number in the $SU(2)_L$ triplet Higgs model ?

(2. Can the obtained condition effectively constraint the model in addition to the results of the neutrino oscillation experiments and WMAP.)

Strategy : Solving the Boltzmann equation

Table of contents of the following talk

2. $SU(2)_L$ triplet Higgs model

○ We extend the standard model within the framework of $SU(2)_L \times U(1)_Y$ gauge theory

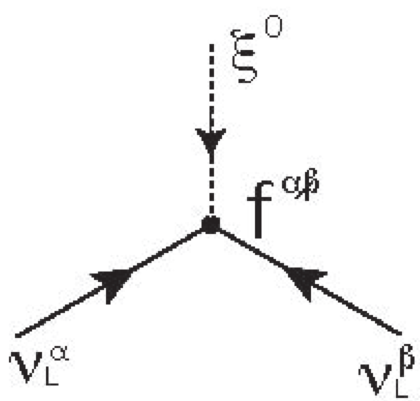
○ $SU(2)_L$ triplet Higgs fields are newly introduced

$$\Delta \equiv \begin{pmatrix} \xi^+/\sqrt{2} & \xi^{++} \\ \xi^0 & -\xi^+/\sqrt{2} \end{pmatrix}$$

○ We newly introduce two kinds of interactions.

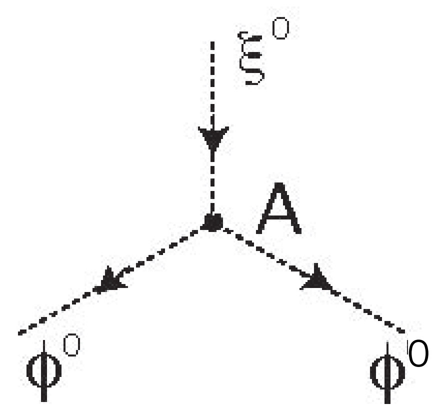
$$\mathcal{L}_\nu^{yukawa} = -\frac{1}{2} f^{\alpha\beta} Tr[T_{l^\alpha,l^\beta}\Delta]$$

$$T_{l^\alpha,l^\beta} \equiv \begin{pmatrix} -\overline{(\nu_L^\alpha)^c} e_L^\beta & \overline{(\nu_L^\alpha)^c}\nu_L^\beta \\ -\overline{(e_L^\alpha)^c} e_L^\beta & \overline{(\nu_L^\alpha)^c} e_L^\beta \end{pmatrix}$$



$$\mathcal{L}^{cubic} = -\frac{1}{2} A Tr[T_{\Phi,\Phi}\Delta^\dagger]$$

$$T_{\Phi,\Phi} \equiv \begin{pmatrix} -\phi^+\phi^0 & \phi^+\phi^+ \\ -\phi^0\phi^0 & \phi^+\phi^0 \end{pmatrix}$$



○ Higgs potential

$$V(\Phi,\Delta) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 + M^2 Tr[\Delta^\dagger\Delta] + \frac{1}{2}\left(A\ Tr[T_{\Phi,\Phi}\Delta^\dagger] + \text{h.c.}\right)$$

$$(\mu^2 > 0, M^2 > 0)$$

○ Vacuum expectation value of triplet Higgs field : $\langle \xi^0 \rangle \equiv \frac{v_\Delta}{\sqrt{2}} = \frac{Av^2}{4M^2}$

○ Neutrino mass matrix

: $m^{\alpha\beta} = f^{\alpha\beta}\frac{v_\Delta}{\sqrt{2}}$

○ Constraint on ρ parameter

$$\frac{v_\Delta}{v} = \frac{Av}{2\sqrt{2}M^2} \leq 0.03 \quad \text{(LEP)}$$

3.Condition to protect the baryon number

There exists an approximately conserved number including baryon number

○ Exact conserved number

$$\left\{ \begin{array}{l} A = 0 \ \Rightarrow \ P = B - L + 2\Delta \\ f^{\alpha\beta} = 0 \Rightarrow \ P = B - L \end{array} \right\} \Rightarrow \ B_{fin} \propto P_{ini}$$

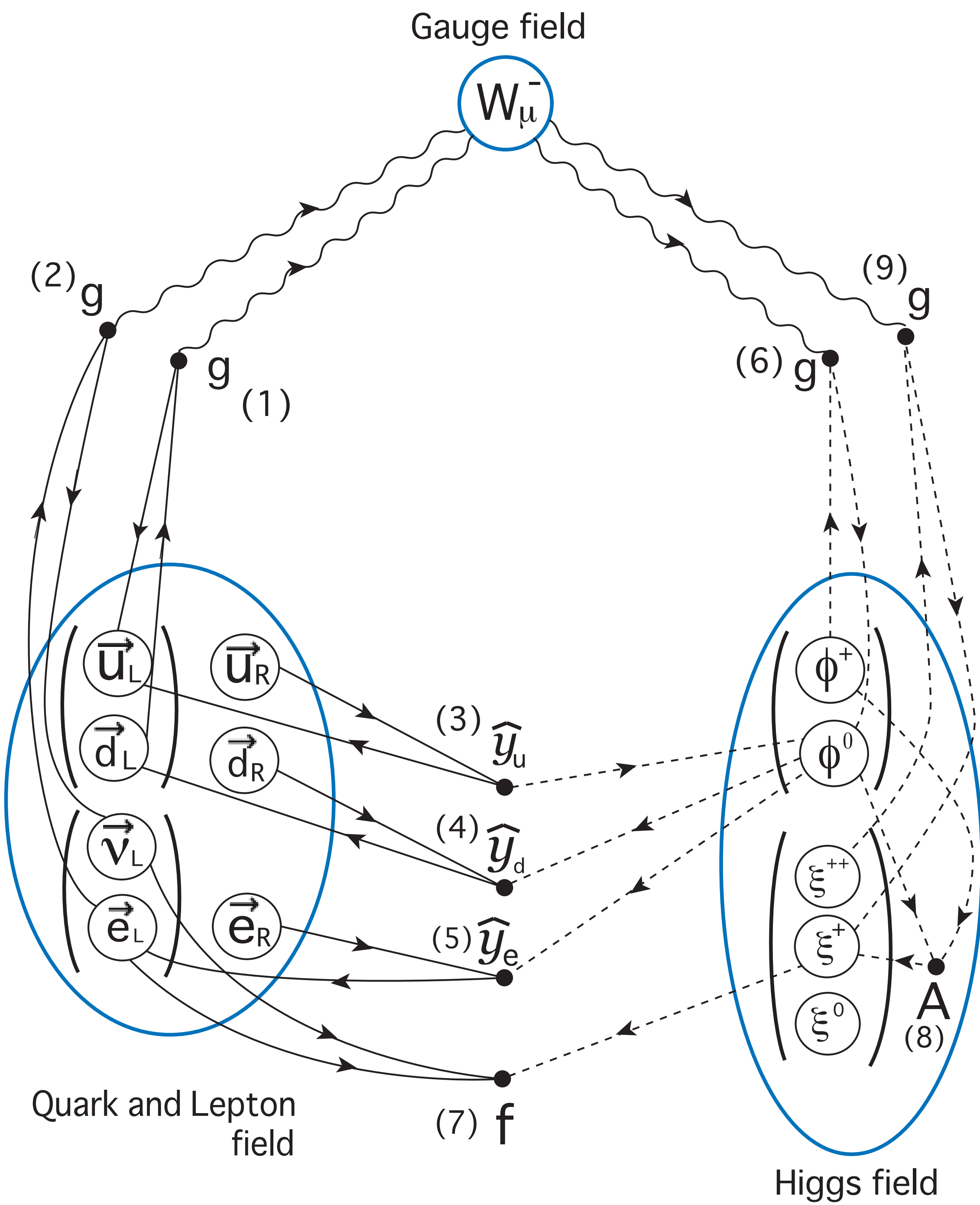
- Condition to protect the initial baryon number

$$\Gamma_A \leq H \text{ or } \Gamma_f \leq H$$

$$\Gamma_A \equiv \Gamma(\xi^0 \to \phi^{0*}\phi^{0*}), \quad \Gamma_f \equiv \Gamma(\xi^0 \to \bar{\nu}_L^\alpha \bar{\nu}_L^\beta)$$

We can obtain this condition by solving the Boltzmann equation

● Interactions in the $SU(2)_L$ triplet Higgs model



○ Interactions, (1)〜(6) exist in the standard model

○ Interactions, (7)〜(9) is newly introduced

g : $SU(2)_L$ gauge coupling constant

$\hat{y}, f$: Yukawa coupling constant

A : cubic coupling constant of Higgs fields

- Definition of (particle number — antiparticle number)

Baryon number : $B \equiv \frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s}$

Lepton number : $L \equiv \frac{n_L}{s} = \frac{n_l - n_{\bar{l}}}{s}$

Triplet Higgs number : $\Delta \equiv \frac{n_\Delta}{s} = \frac{n_\delta - n_{\bar{\delta}}}{s}$

(n_X : density of particle number s : density of enrtopy)

- Relation between the particle number and the chemical potential

$$n_+ - n_- = \frac{1}{6} g \mu T^2 \times \begin{cases} 1 & \text{(fermion)} \\ 2 & \text{(boson)} \end{cases}$$

- We assume that all particles obey the Maxwell-Boltzmannn distribution

$$f(E_X) = \exp\left[-\frac{E_X - \mu_X}{T} \right]$$

- Boltzmann equation for the Lepton number

$$s\frac{dL}{dt} = \bigcap \cdot e^{-\frac{E}{T}} \cdot \frac{2}{T}$$

$$+ \sum_{\alpha=e,\mu,\tau} |M_f(f^{\alpha\alpha})|^2(-\mu_{\xi^0} - 2\mu_{\nu_\alpha}) + 2|M_f(f^{e\mu})|^2(-\mu_{\xi^0} - \mu_{\nu_e} - \mu_{\nu_\mu})$$
$$+2|M_f(f^{e\tau})|^2(-\mu_{\xi^0} - \mu_{\nu_e} - \mu_{\nu_\tau}) + 2|M_f(f^{\mu\tau})|^2(-\mu_{\xi^0} - \mu_{\nu_\mu} - \mu_{\nu_\tau})$$

$$+ \sum_{\alpha=e,\mu,\tau} |M_f(f^{\alpha\alpha})|^2(\mu_{\xi^-} - \mu_{\nu_\alpha} - \mu_{\alpha L}) + |M_f(f^{e\mu})|^2(\mu_{\xi^-} - \mu_{\nu_e} - \mu_{\mu L})$$
$$+|M_f(f^{e\tau})|^2(\mu_{\xi^-} - \mu_{\nu_e} - \mu_{\tau_L}) + |M_f(f^{\mu\tau})|^2(\mu_{\xi^-} - \mu_{\nu_\mu} - \mu_{\tau_L})$$
$$+|M_f(f^{e\mu})|^2(\mu_{\xi^-} - \mu_{e_L} - \mu_{\nu_\mu}) + |M_f(f^{e\tau})|^2(\mu_{\xi^-} - \mu_{e_L} - \mu_{\nu_\tau})$$
$$+|M_f(f^{\mu\tau})|^2(\mu_{\xi^-} - \mu_{\mu_L} - \mu_{\nu_\tau})$$

$$+ \sum_{\alpha=e,\mu,\tau} |M_f(f^{\alpha\alpha})|^2(\mu_{\xi^{--}} - 2\mu_{\alpha_L}) + 2|M_f(f^{e\mu})|^2(\mu_{\xi^{--}} - \mu_{e_L} - \mu_{\mu L})$$
$$+2|M_f(f^{e\tau})|^2(\mu_{\xi^{--}} - \mu_{e_L} - \mu_{\tau_L}) + 2|M_f(f^{\mu\tau})|^2(\mu_{\xi^{--}} - \mu_{\mu_L} - \mu_{\tau_L})$$

$$+ \int d\Pi_3 \frac{\delta^4(p_X - p_1 - p_2 - p_3)}{\delta^4(p_X - p_1 - p_2)}$$
$$\sum_{\alpha=e,\mu,\tau} \frac{-1}{2}|M_s|^2(2\mu_{u_L} + \mu_{d_L} + \mu_{\alpha_L} + \mu_{u_L} + 2\mu_{d_L} + \mu_{\nu_\alpha})$$

$$(\bigcap \equiv \int d\Pi_X d\Pi_1 d\Pi_2 (2\pi)^4 \delta^4(p_X - p_1 - p_2))$$

- Boltzmann equations for (B, L, W, Φ, Δ)

$$s\frac{d}{dt}\begin{pmatrix} B \\ L \\ W \\ \Phi \\ \Delta \end{pmatrix} = M(t)\begin{pmatrix} B \\ L \\ W \\ \Phi \\ \Delta \end{pmatrix}$$

- Conditions which should be satisfied

I. $Q^{total} = 0$ and $I_3^{total} = 0$

II. Interactions, (1)~(6) and (9), are in equilibrium

III. Sphaleron process are in equilibrium

IV. We don't distinguish lepton flavor (e, μ, τ)

- Boltzmann equation for (L, Δ)

$$\boxed{\frac{d}{dT}\vec{N} = f(T) \cdot M\vec{N}}$$

$$\vec{N} \equiv \begin{pmatrix} L \\ \Delta \end{pmatrix}, \qquad f(T) \equiv \begin{cases} 2.3\frac{M^3}{T^4} & (T > M) \\ 2.8\frac{M^{7/2}}{T^{9/2}}e^{-\frac{M}{T}} & (T < M) \end{cases}$$

$$M \equiv \begin{pmatrix} 2K_f & \frac{11}{14}K_f \\ K_f - \frac{4}{63}K_A & \frac{11}{28}K_f + \frac{5}{36}K_A \end{pmatrix} \equiv \begin{pmatrix} a & d \\ c & b \end{pmatrix}$$

$$K_f \equiv \left.\frac{\Gamma_f}{H}\right|_{T=M}, \qquad K_A \equiv \left.\frac{\Gamma_A}{H}\right|_{T=M}$$

$$(\Gamma_f \equiv \Gamma(\xi^0 \leftrightarrow \nu_\alpha \nu_\beta), \quad \Gamma_A \equiv \Gamma(\xi^0 \leftrightarrow \phi^0 \phi^0))$$

$$B = -\frac{28}{51}L + \frac{2}{17}\Delta$$

- We solve the Boltzmann equation for (L, Δ)

$$\vec{N}' \equiv \begin{pmatrix} X \\ Y \end{pmatrix} = V^{-1}\vec{N}, \ \hat{M} = V^{-1}MV \equiv \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \qquad (\lambda_1 > \lambda_2)$$

$$V \equiv \begin{pmatrix} 2d & -2d \\ D+b-a & D+a-b \end{pmatrix} \qquad (D \equiv \sqrt{(a-b)^2 + 4cd})$$

$$\boxed{\frac{d}{dT}\vec{N}' = f(T) \cdot \hat{M}\vec{N}'}$$

$$\Rightarrow \quad \vec{N}'_{fin} = e^{-r\hat{M}}\vec{N}'_{ini} \quad (r \sim 9.8)$$

$$\boxed{\begin{pmatrix} L_{fin} \\ \Delta_{fin} \end{pmatrix} = V \begin{pmatrix} e^{-r\lambda_1} & 0 \\ 0 & e^{-r\lambda_2} \end{pmatrix} V^{-1} \begin{pmatrix} L_{ini} \\ \Delta_{ini} \end{pmatrix}}$$

- Final baryon number

$$B_{fin} = F(L_{ini}, \Delta_{ini}, K_f, K_A)$$

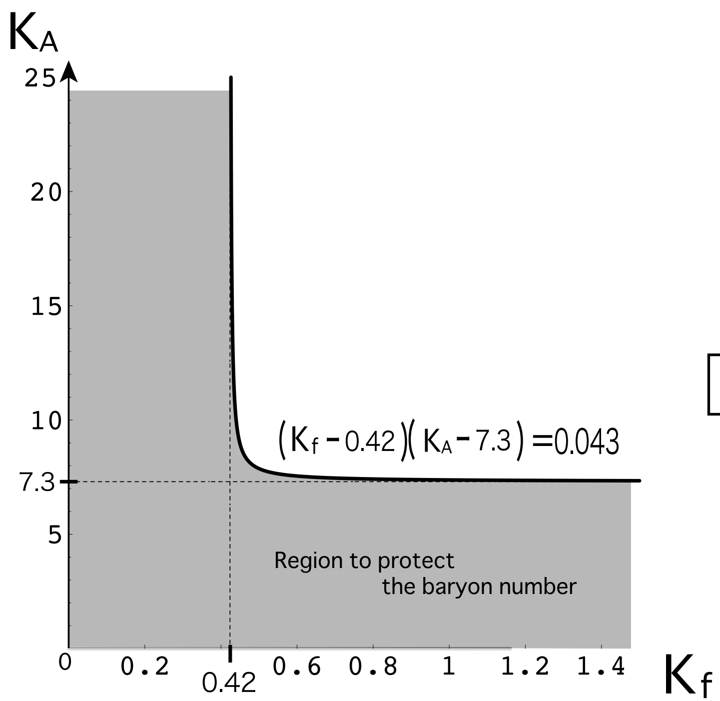
- Condition to protect the initial baryon number

$$\lambda_2 = 0 \Leftrightarrow \text{Existense of the conserved quantity, P=Y} \Leftrightarrow B_f \propto P_i \neq 0$$

$$\Leftrightarrow det[M] = 0 \Leftrightarrow K_f = 0 \text{ or } K_A = 0$$

(Because $det[M] = \frac{289}{882} \cdot K_f \cdot K_A$)

- Even if $\lambda_2 \leq 1$, Eigen mode Y is approximately conserved

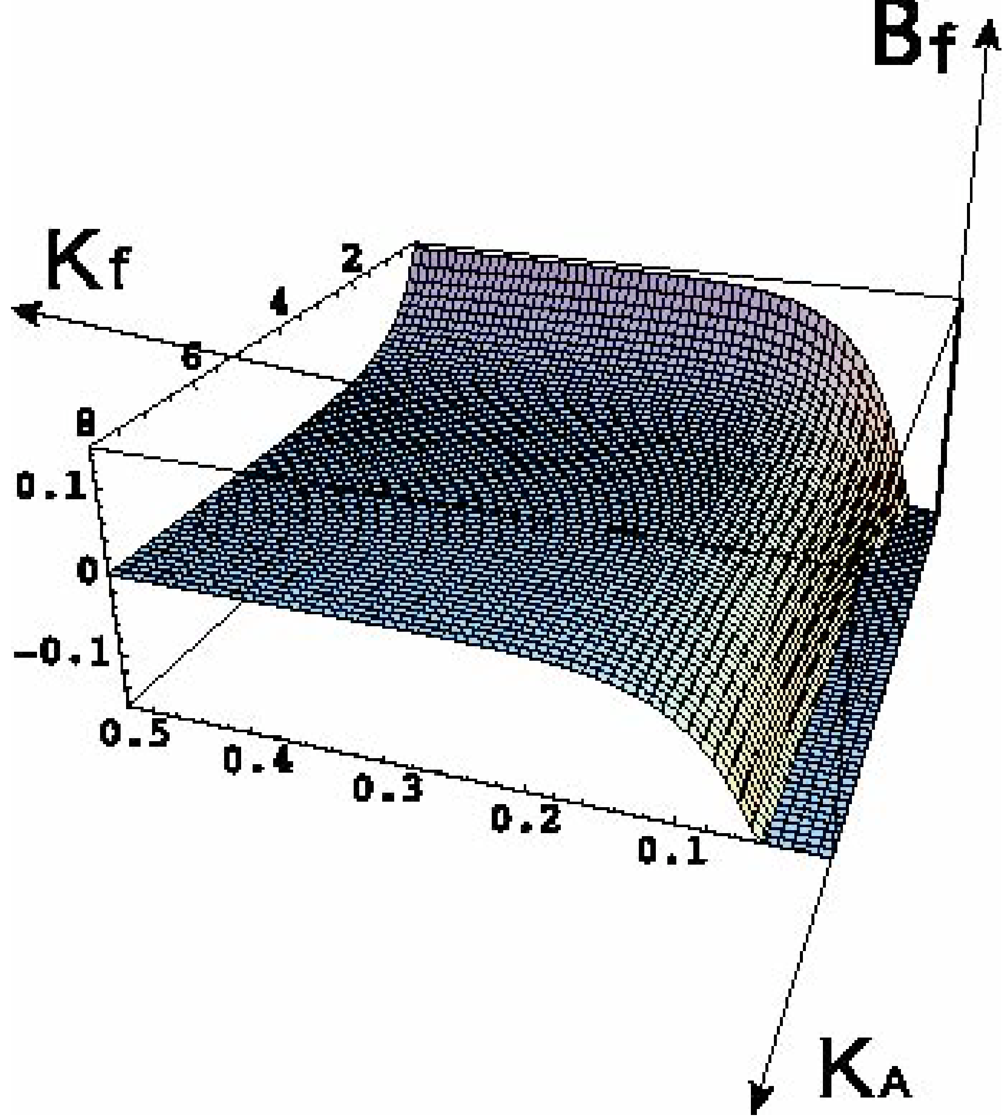


Condition to protect the baryon number

$$\Gamma_A < H\big|_{T=M} \quad \text{or} \quad \Gamma_f < H\big|_{T=M}$$

- We observe final baryon number under the properly fixed initial condition, (L_{ini}, Δ_{ini})

$$(L_{ini}, \Delta_{ini}) = (1, 1)$$



- In the case that three lepton flavor (e, μ, τ) is distinguished

Sphaleron processes conserve $\left(\frac{B}{3} - L_e,\ \frac{B}{3} - L_\mu,\ \frac{B}{3} - L_\tau\right)$

	Approximately conserved number	Condition
(0)	$P_0 = B - L + 2\Delta$	$K_A < 1$
(1)	$P_1^a = B/3 - L_e$	$K_{L_e} < 1$
(2)	$P_1^a = B/3 - L_\mu$	$K_{L_\mu} < 1$
(3)	$P_1^a = B/3 - L_\tau$	$K_{L_\tau} < 1$
(4)	$P_2^a = L_e - L_\mu$	$K_{L_e - L_\mu} < 1$
(5)	$P_2^b = L_e - L_\tau$	$K_{L_e - L_\tau} < 1$
(6)	$P_2^c = L_\mu - L_\tau$	$K_{L_\mu - L_\tau} < 1$
(7)	$P_3^a = B/3 + L_{e\mu}$	$K_{L_{e\mu}} < 1$
(8)	$P_3^b = B/3 + L_{e\tau}$	$K_{L_{e\tau}} < 1$
(9)	$P_3^c = B/3 + L_{\mu\tau}$	$K_{L_{\mu\tau}} < 1$

Condition to protect the baryon number

At least one of the ten conditions is satisfied

4. Conclusion

1 Condition to protect the baryon number in $SU(2)_L$ triplet Higgs model

$$\Gamma_A < H\big|_{T=M} \quad \text{or} \quad \Gamma_f < H\big|_{T=M}$$

2 The obtained conditions effectively constrain $SU(2)_L$ triplet Higgs model in addition to the results of the neutrino oscillation experiments and WMAP

- Problems which should be solved or considered

① Estimation of the Interaction rate, $\Gamma_A \equiv \Gamma(\xi^0 \to \phi^{0*}\phi^{0*})$

$$2\Gamma(\xi^0 \to \phi^0\phi^0) = \frac{1}{8\pi EM}\sqrt{\frac{M^2}{4} - m_{\phi^0}^2}\,|A|^2 \simeq \frac{1}{16\pi}\frac{|A|^2}{E}$$

② Four body processes, $ll \leftrightarrow ll,\ ll \leftrightarrow \phi\phi$, are not considered

- I research the condition to protect the initial baryon number in other models compatible with the results of the neutrino oscillation experiments and WMAP

(i) Zee model

K. Hasegawa, C. S. Lim, K. Ogure,
Physical Revierw D68, 053006(2003)

(ii) Seesaw model

K. Hasegawa,
Physical Revierw D69, 013002(2004)

Allowed region

1 Condition to protect the baryon number

$$K_A < 1 \Leftrightarrow \ |A| < 12 \times \left(\frac{M}{\mathrm{GeV}}\right)^{\frac{3}{2}} \mathrm{eV} \quad \text{or} \quad K_f < 1 \Leftrightarrow |f^{\alpha\beta}| \ < \ 4.3 \times 10^{-9} \times \left(\frac{M}{GeV}\right)^{\frac{1}{2}}$$

2 Constraint on ρ parameter

$$|A| < 0.03 \times \frac{2\sqrt{2}M^2}{v}$$

3 Results of the neutrino oscillation experiments

$$\Delta_a = |m_3^2 - m_2^2|, \ \Delta_s = m_2^2 - m_1^2, \quad \theta_{atm} = \theta_{23}, \ \theta_{13}, \ \theta_{\odot} = \theta_{12}$$

4 Results of WMAP

$$\sum_i |m_i| < 0.71[\mathrm{eV}]$$

- I find that the parameter region, $K_A < 1$ and $K_f < 1$, can not satisfy the both of 3 and 4

- Allowed region in the case that the mass of the triplet Higgs fields are fixed at M=100GeV