Parametric Resonance and Charge Generation

船久保　公一　（佐賀大理工）

2001年 7月10日 @基研

内容

§1. Introduction — Parametric Resonance とは

§2. Charge Generation

§3. Application

§ 1. Introduction

parametric resonace

= a realizaion of preheating after inflation

[Kofman, Linde, Starobinsky, PRL73 ('94)]

reheating after inflation

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{2}(\partial\chi)^2 + \bar{\psi} i \partial\!\!\!/ \psi$$
$$- \frac{1}{2} g^2 \phi^2 \chi^2 - f \bar{\psi} \psi \phi$$

EOM

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \cdots\right)$$
$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + V'(\phi) + \cdots = 0$$

uniform ϕ-dominant $\Longrightarrow$ damped-oscillating ϕ

decay of 0-momentum ϕ-particles

$$\Gamma_\phi = \Gamma(\phi \to \chi\chi) + \Gamma(\phi \to \psi\psi) = \frac{g^4 \langle\phi\rangle^2}{8\pi m_\phi} + \frac{f^2 m_\phi}{8\pi}$$

EOM for ϕ:

$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + \Gamma_\phi \dot{\phi}(t) + V'(\phi) = 0$$

entropy production $T_{\rm rh} \simeq \left(\Gamma_\phi V_{\rm ini}^{1/4}\right)^{1/2}$

現在の宇宙の粒子とエントロピーがinflation後に生成されたとすれば、**バリオン数もその時期に出来た**と考えるのは自然。

疑問点

(1) 「古典的場の崩壊」を**場の量子論**でどう取り扱うか？

(2) g^2 や f が小さくても inflaton 振幅が大きいときに摂動論が使えるか？ (e.g., $|g\phi| > m_\chi$ のとき)

⇓

古典的スカラー場 $\phi(t)$ を背景とする場の理論と考える

⇕

preheating

[Kofman, Linde, Starobinsky, PRD56('97)]

EOM for the inflaton with $V(\phi) \simeq \frac{1}{2}m^2\phi^2$:

$$\Rightarrow \phi(t) = \Phi(t)\sin(mt) \propto \frac{1}{t}\sin(mt)$$

mode equation for $\chi_k(t)$:

$$\ddot{\chi}_k(t) + 3H(t)\dot{\chi}_k(t) + \left(\frac{k^2}{a^2} + g^2\Phi^2(t)\sin^2(mt)\right)\chi_k(t) = 0$$

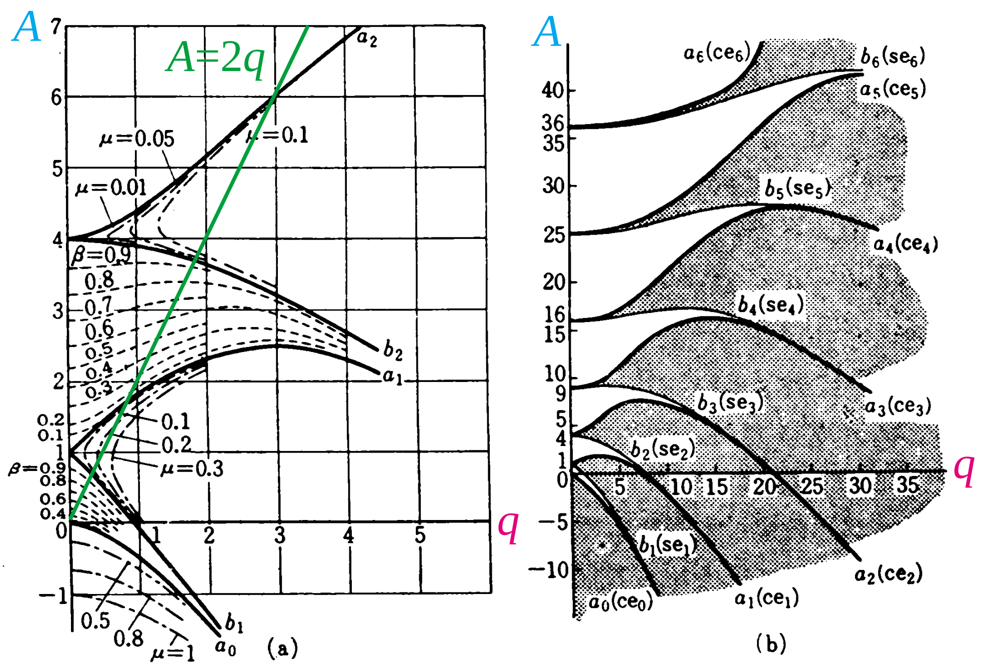
In the Minkowski spacetime $(a(t) \equiv 1,\ \Phi(t) = \text{const.})$

$$\chi_k''(z) + (A_k - 2q\cos 2z)\chi_k(z) = 0$$

where $z = mt$,

$$A_k \equiv \frac{k^2}{m^2} + \frac{g^2\Phi^2}{2m^2} = \frac{k^2}{m^2} + 2q, \qquad q \equiv \frac{g^2\Phi^2}{4m^2}$$

Mathieu equation



Mathieu の微分方程式の解の安定域

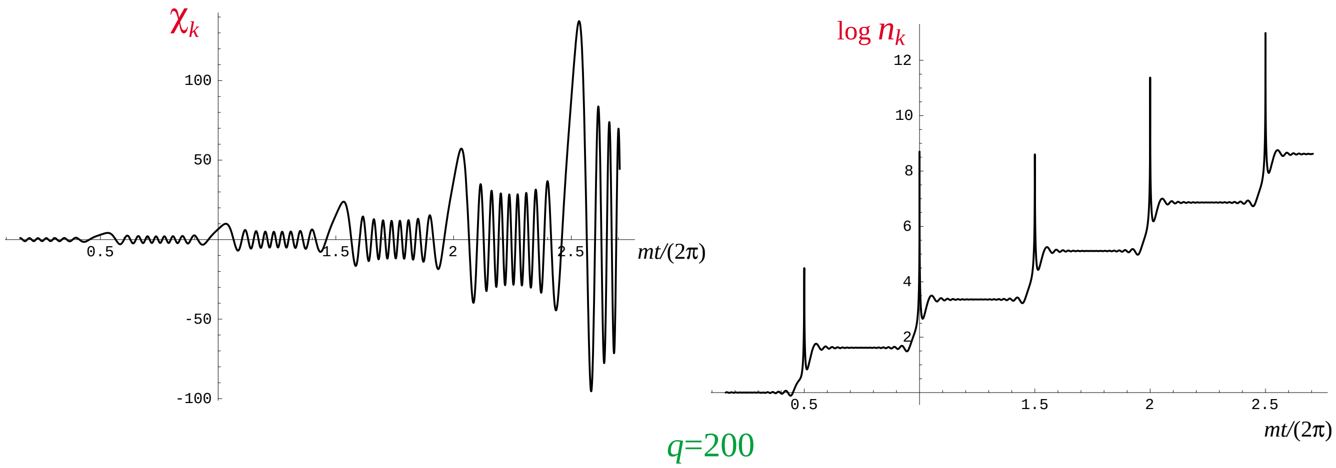
Parametric Resonance

wave function in a periodic potential

$$= \begin{cases} \text{Bloch wave} \\ \text{exponentially growing or damping waves} \end{cases}$$

For $q \gg 1$, the waves are in broad resonance

a solution in a resonance band



q=200

$$n_k \equiv \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2}$$

n_k changes only at t where $\Phi(t) = 0$

$$\Leftrightarrow \dot{\omega}(t) \gtrsim \omega^2(t) \text{ with } \omega(t) = \sqrt{k^2 + g^2\Phi^2(t)\sin^2(mt)}$$

$$\left.\begin{matrix} |\chi_k(t)| \\ n_k(t) \end{matrix}\right\} \text{exponentially increase with } t \text{ stepwise.}$$

$\Longrightarrow$ seccessive scatterings by a periodic potential

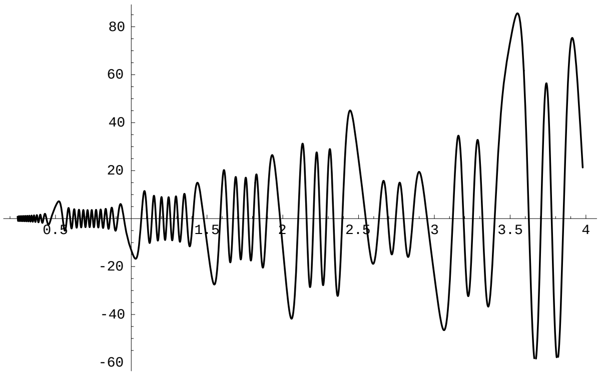
$\Longrightarrow$ descent equation for n_k

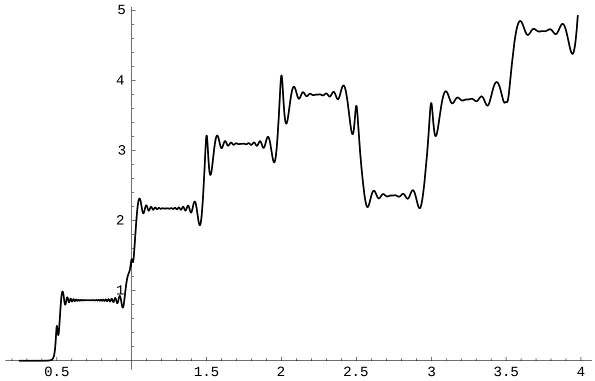
We must take into account ...

▷ **Expantion of the Universe** $a(t)$, $\Phi(t)$

narrow resonance $q \lesssim O(1)$ $\longrightarrow$ resonanceが即終了

broad resonace $q \gg 1$ $\longrightarrow$ stochastic resonance





それでも、successive scatteringの描像は使える

$$n_k^{j+1} \simeq \left(1 + 2e^{-\pi\kappa_j^2} - 2\sin\hat{\theta}\, e^{-\pi\kappa_j^2/2}\sqrt{1 + e^{-\pi\kappa_j^2}}\right) n_k^j$$

ここで$\hat{\theta}$はrandom phase、

$$\kappa_j \equiv \frac{k}{a_j k_{*j}}、\quad k_{*j} \equiv \sqrt{gm\Phi_j} = \sqrt{2}\, m q_j^{1/4}$$

($j \leftrightarrow j$-th zero of $\phi(t)$)

▷ **生成されたχ粒子の** back reaction

$$\begin{cases} \rho \simeq \rho_\phi \rightarrow \rho_\chi & \text{:damping the oscillation} \\ m_\phi^2 \simeq m^2 + g^2\langle\chi^2\rangle & \text{:increase } \phi\text{-frequency} \end{cases}$$

▷ **χ粒子とϕ粒子の** rescattering

$\Delta m_\chi^2(k) = g^2\langle\delta\phi^2\rangle_k >$ resonance width

$\Longrightarrow$ terminates the resonance

state after preheating

- large occupation number of χ with small k

  resonance band $\Leftrightarrow \pi\kappa^2 < 1 \Leftrightarrow \kappa < \frac{1}{\sqrt{\pi}} \simeq 0.56$

- large quantum fluctuation of χ

  e.g.

$$m = 10^{-6} m_P, \quad \Phi_0 = \frac{m_P}{5}, \quad g = 10^{-3 \sim -1}$$

$\Rightarrow$ resonance terminates after about 10 ϕ-oscillations

$\sqrt{\langle \chi^2 \rangle} \simeq 3 \times 10^{16}$GeV for $g = 3 \times 10^{-4}$

$\longleftrightarrow$ thermal fluctuation at $T = 10^{17}$GeV

$\Downarrow$

nonthermal symmetry restoration nonthermal heavy particle production

Evolution of this state;

⋆ decay to light particles — conventional reheating process

⋆ relaxation to thermal distribution

numerical simulation [Felder & Kofman, hep-ph/0011160]

relaxation time $\ll \dfrac{1}{n\sigma_{\text{int}}}$ ($\because$ large occupation no.)

§ 2. Charge Generation

[K.F., Otsuki, Kakuto, Toyoda, hep-ph/0010266]

Extension to the case of n-component complex scalar fields

$$\mathcal{L} = \partial_\mu \chi_a^* \partial^\mu \chi_a - g_a^2 \phi^2(t) \chi_a^* \chi_a - \chi_a^* V_{ab}(t) \chi_b - \frac{1}{2} \left(\chi_a W_{ab}(t) \chi_b + \text{c.c.} \right),$$

$\phi(t)$: oscillating background

"effective potential": $V_{ab}(t) = V_{ba}^*(t)$, $W_{ab}(t)$

induced by couplings to ϕ and/or by radiative and finite-T corrections

$W_{ab}(t) = 0 \Rightarrow$ global $U(1)$

$\mathrm{Im} V_{ab}(t) \neq 0$ or $\mathrm{Im} W_{ab}(t) \neq 0 \Rightarrow$ C and CP violation

We assume that

- charge is generated when $\phi(t) = 0$, as particles are created.
- $V_{ab}(t)$ and $W_{ab}(t)$ can be treated perturbatively.

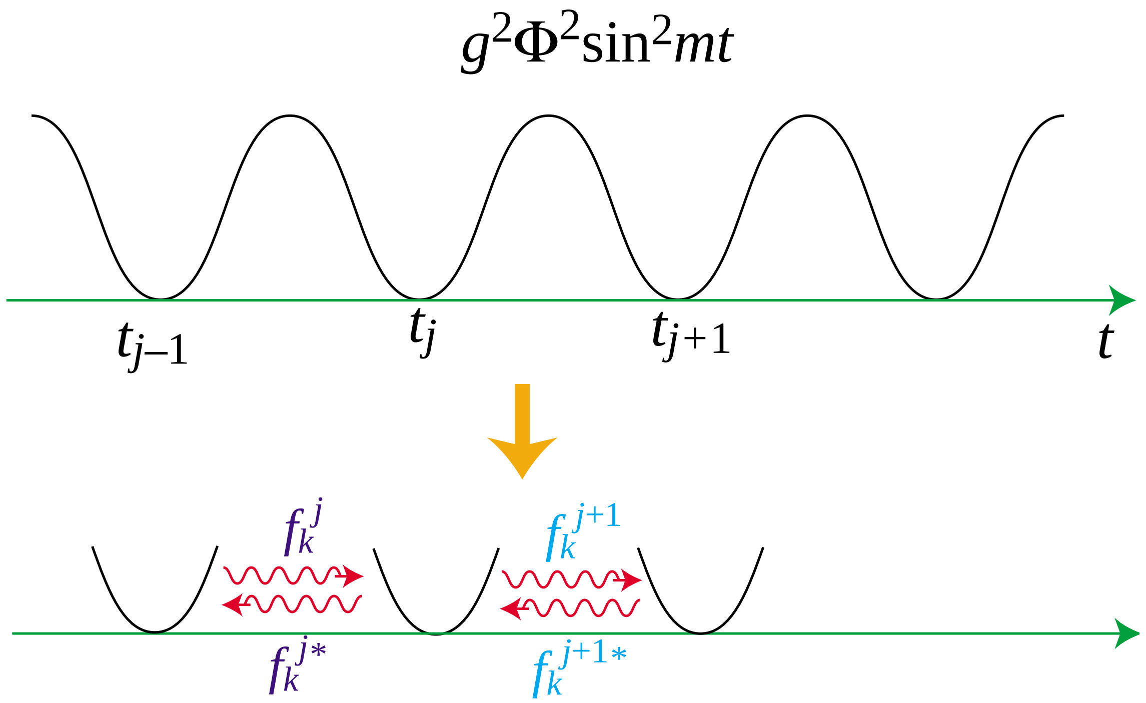
successive scattering approximation (for broad resonance)

for $t_{j-1} \ll t \ll t_j$, $(t_j = \pi j/m)$

$$\chi_a(x) = \int d^3\boldsymbol{k} \left(a^j_{a\boldsymbol{k}} f^j_{ak}(t) e^{i\boldsymbol{kx}} + b^{j\dagger}_{a\boldsymbol{k}} f^{j*}_{ak}(t) e^{-i\boldsymbol{kx}} \right)$$

ここでmode関数 $f^j_k(t)$ は次の方程式の解:

$$\ddot{f}^j_k(t) + \left(k^2 + g_a^2 \Phi^2 \sin^2 mt\right) f^j_k(t) = 0$$



- t_j の近傍以外では**断熱近似**

$$f^j_k(t) \simeq \frac{1}{\sqrt{2\omega_a(t)}} e^{-i\int_0^t dt' \omega_a(t)}$$

を用いる。$\left(\omega_a(t) = \sqrt{k^2 + g_a^2\Phi^2 \sin^2 mt}\right)$

- t_j の近傍では、$\sin^2 mt$ を他の関数で近似して散乱問題を解く。 $\left(\sin^2 mt \simeq 2\tanh^2\left(\frac{m(t-t_j)}{\sqrt{2}}\right)\right)$

各$\phi(t)$のゼロ点毎の散乱により正振動モードと負振動モードが混合する

$$\begin{pmatrix} 0 \\ \vdots \\ f^0_{ak}(t) \\ \vdots \\ 0 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{pmatrix} \alpha^j_{a1} f^j_{1k}(t) + \beta^j_{a1} f^{j*}_{1k}(t) \\ \vdots \\ \alpha^j_{ab} f^j_{bk}(t) + \beta^j_{ab} f^{j*}_{bk}(t) \\ \vdots \\ \alpha^j_{an} f^j_{nk}(t) + \beta^j_{an} f^{j*}_{nk}(t) \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ \vdots \\ f^{0*}_{ak}(t) \\ \vdots \\ 0 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{pmatrix} \tilde\beta^j_{a1} f^j_{1k}(t) + \tilde\alpha^j_{a1} f^{j*}_{1k}(t) \\ \vdots \\ \tilde\beta^j_{ab} f^j_{bk}(t) + \tilde\alpha^j_{ab} f^{j*}_{bk}(t) \\ \vdots \\ \tilde\beta^j_{an} f^j_{nk}(t) + \tilde\alpha^j_{an} f^{j*}_{nk}(t) \end{pmatrix}$$

CP violation $\Longrightarrow \alpha^j_{ab} \neq \tilde\alpha^j_{ab},\ \beta^j_{ab} \neq \tilde\beta^j_{ab}$

Bogoliubov変換

$$a^j_{a\boldsymbol{k}} = a^0_{b\boldsymbol{k}} \alpha^j_{ba} + b^{0\dagger}_{b\boldsymbol{k}} \tilde\beta^j_{ba}$$
$$b^{j\dagger}_{a\boldsymbol{k}} = a^0_{b\boldsymbol{k}} \beta^j_{ba} + b^{0\dagger}_{b\boldsymbol{k}} \tilde\alpha^j_{ba}$$

Bogoliubov係数が満たすべき条件

commutation rel.

($n \times n$行列表記で)

$$\alpha^{j\dagger}\alpha^j - \tilde\beta^{j\dagger}\tilde\beta^j = \tilde\alpha^{j\dagger}\tilde\alpha^j - \beta^{j\dagger}\beta^j = 1, \quad \beta^{j\dagger}\alpha^j - \tilde\alpha^{j\dagger}\tilde\beta_j = 0$$

$|0^0\rangle$ ($a^0_{a\boldsymbol{k}}|0^0\rangle = b^0_{a\boldsymbol{k}}|0^0\rangle$) に対して第 j 区間で生成される**粒子数密度と charge 密度**

$$
\begin{aligned}
n_k^j &\equiv \frac{1}{V}\langle 0^0|\sum_{a=1}^{n}\left(a^{j\dagger}_{a\boldsymbol{k}}a^j_{a\boldsymbol{k}} + b^{j\dagger}_{a\boldsymbol{k}}b^j_{\boldsymbol{k}}\right)|0^0\rangle \\
&= \mathrm{Tr}\left(\tilde{\beta}^{j\dagger}\tilde{\beta}^j + \beta^{j\dagger}\beta^j\right) \\
j_k^j &\equiv \frac{1}{V}\langle 0^0|\sum_{a=1}^{n}Q_a\left(a^{j\dagger}_{a\boldsymbol{k}}a^j_{a\boldsymbol{k}} - b^{j\dagger}_{a\boldsymbol{k}}b^j_{a\boldsymbol{k}}\right)|0^0\rangle \\
&= \mathrm{Tr}\left[Q\left(\tilde{\beta}^{j\dagger}\tilde{\beta}^j - \beta^{j\dagger}\beta^j\right)\right]
\end{aligned}
$$

$$Q = \mathrm{diag}\left(Q_1, Q_2, \cdots, Q_n\right)$$

$n=1$ の場合、Bogoliubov 係数の条件は

$$\left|\alpha^j\right|^2 = \left|\tilde{\beta}^j\right|^2 + 1, \quad \left|\tilde{\alpha}^j\right|^2 = \left|\beta^j\right|^2 + 1$$

$$\left|\alpha^j\right|^2\left|\beta^j\right|^2 = \left|\tilde{\alpha}^j\right|^2\left|\tilde{\beta}^j\right|^2$$

これから

$$\left|\beta^j\right|^2 = \left|\tilde{\beta}^j\right|^2 \quad \Rightarrow \quad \boxed{j_k^j = 0}$$

$\Longleftrightarrow$ heavy particle の崩壊では CP violation は 2 つ以上の channel の干渉として現れる

$$\left|\mathcal{A}_1 + e^{i\theta}\mathcal{A}_2\right|^2$$

Example

$n = 2$: $m_1 = m_2 \equiv m$, $V_{11} = V_{22}$, $W_{ab} = 0$

$U(1)$-sym.: $\chi_a \mapsto e^{i\alpha}\chi_a$ and discrete sym.: $\chi_1 \leftrightarrow \chi_2$

$$\begin{cases} n_k^j = \sum_{a,b=1}^{2} \left(\left|\beta_{ab}^j\right|^2 + \left|\tilde{\beta}_{ab}^j\right|^2 \right) \\ j_{1k}^j = \left|\tilde{\beta}_{11}^j\right|^2 + \left|\tilde{\beta}_{21}^j\right|^2 - \left|\beta_{11}^j\right|^2 - \left|\beta_{21}^j\right|^2 \quad \text{charge of } \chi_1 \\ j_{2k}^j = \left|\tilde{\beta}_{12}^j\right|^2 + \left|\tilde{\beta}_{22}^j\right|^2 - \left|\beta_{12}^j\right|^2 - \left|\beta_{22}^j\right|^2 \quad \text{charge of } \chi_2 \end{cases}$$

$$\kappa \equiv \frac{k}{k_*} = \frac{k}{\sqrt{g\Phi m}} \lesssim \frac{1}{\sqrt{\pi}} \qquad \Longrightarrow \text{ in the resonance band}$$

For definiteness, take

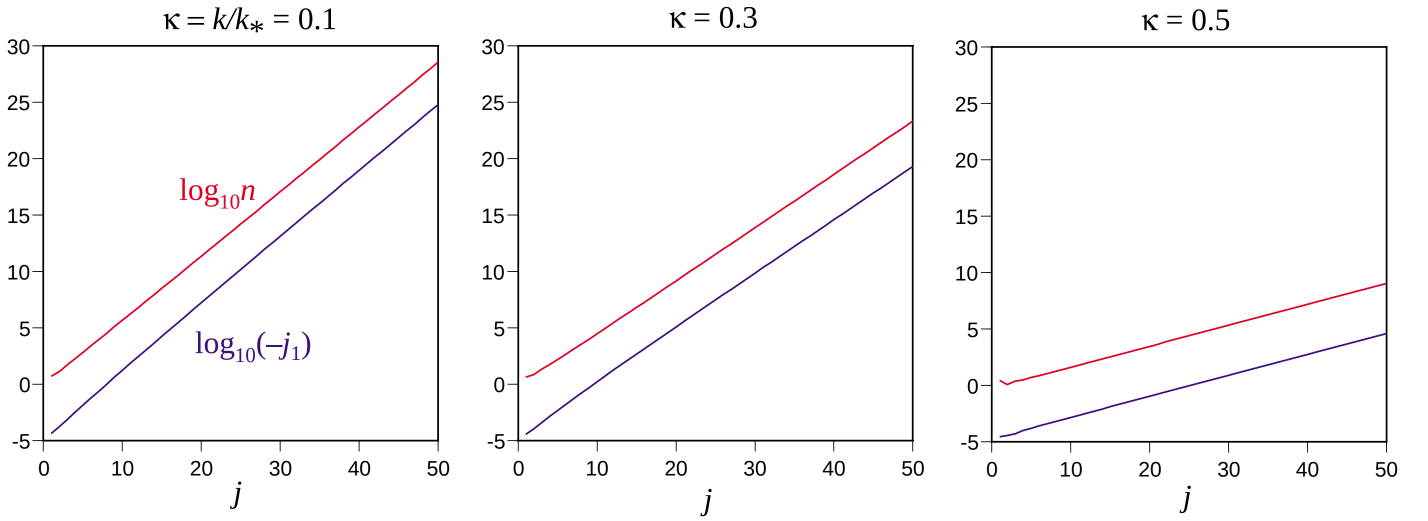
$$V_{11}(t) = -\frac{2g^2 l_1 \Phi^2}{\cosh^2[m(t-t_j)/\sqrt{2}]}$$

$$V_{12}(t) = -\frac{2g^2 l_2 \Phi^2 \, e^{i\theta(t)}}{\cosh^2[m(t-t_j)/\sqrt{2}]}$$

with $$\theta(t) = \frac{\theta_0}{1 + e^{m(t-t_j)/\sqrt{2}}}$$

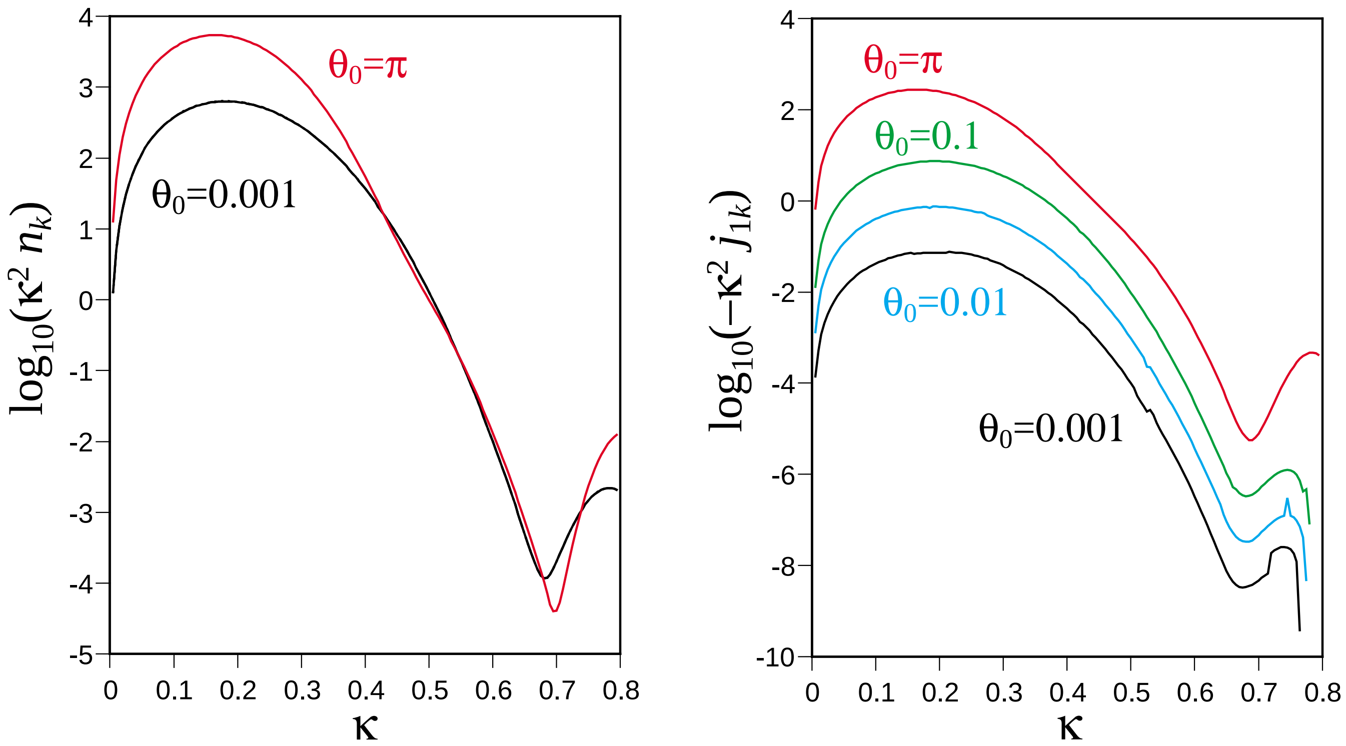
粒子数密度とcharge密度の時間発展

$$q = 200,\ l_1 = 0.01,\ l_2 = 0.02,\ \theta_0 = 10^{-3}$$



resonanceが終わる頃

densities at $j = 10$ for various θ_0



total number and charge densities

$$n = \int d^3\boldsymbol{k}\, n_k = 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, n_k,$$

$$j_1 = 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, j_{1k} = -j_2$$

θ_0	$\int d\kappa\, \kappa^2 n_k$	$\int d\kappa\, \kappa^2 j_{1k}$
10^{-3}	130.5096	-1.609334×10^{-2}
10^{-2}	130.5156	-1.544579×10^{-1}
10^{-1}	131.1163	-1.537716
π	990.7411	-50.84228

$$-j_1 \sim 10^{-1} \times \theta_0 \times n$$

§ 3. Application

Affleck-Dine mechanism

[Dine, et al. NPB458 ('96)]

flat directions in the MSSM [Gherghetta, et al. NPB468 ('96)]

parametrized by a complex no. ϕ s.t. $V_D = V_F = 0$

during inflation

$$V_{AD} = (m^2 - cH_I^2)\,|\phi|^2 + \left(\frac{A\lambda\phi^n}{M^{n-3}} + \text{h.c.}\right) + \frac{\lambda^2\,|\phi|^{2(n-1)}}{M^{2(n-3)}}$$

where c, A come from SUSY breaking coupled to the inflaton

if $c \geq 0$ and $\mathrm{Arg}A \ll 1 \Rightarrow \phi$ oscillates around 0

In general, ϕ couples to more than 2 scalar fields, B and/or L will be generated when $\phi \neq 0$ violates the $U(1)$ symmetry.

e.g. LH_u flat direction

$$L_1^0 = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \qquad H_u^0 = \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \qquad \text{other scalars} = 0$$

fluctuation modes after the gauge fixing

$$\begin{aligned} L_1 &= L_1^0 + \bar{L}_1 = L_1^0 + \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{e}_1 \end{pmatrix}, \\ H_u &= H_u^0 + \bar{H}_u = H_u^0 + \begin{pmatrix} 0 \\ h_u \end{pmatrix} \end{aligned}$$

U(1)- and CP-violating terms in the scalar potential

$$D_2^2 + D_1^2 \quad \rightarrow \quad (\phi^* \tilde{\nu}_1 + \phi \tilde{\nu}_1^*)^2$$

$$\left|F_{H_d}\right|^2 \quad \rightarrow \quad \left[(-\mu \bar{H}_u^0)^\dagger f_{1B}^{(e)} L_1^0 e_B + \text{h.c.}\right]$$

$$= \quad -\mu^* \phi\, f_{1B}^{(e)}\, h_u \tilde{e}_B + \text{h.c.}$$

$$\Downarrow$$

$U(1)_L$-breaking potentials of W-type with

$$\text{“}m\text{”} \longrightarrow \text{soft-SUSY-br. mass or } |\mu|$$

$$\text{“}g\text{”} \longrightarrow g_2,\ g_1,\ \text{Yukawa coupl.}$$

$$\text{“}\Phi\text{”} \longrightarrow \phi_{\text{initial}}$$

For $m \simeq m_W$,

$$q^{1/2} \equiv \frac{g\Phi}{2m} \sim \frac{\phi_{\text{initial}}}{\langle H \rangle} = \frac{\phi_{\text{initial}}}{246\text{GeV}}$$

$\therefore \quad \phi_{\text{initial}} \gtrsim 10\text{TeV} \Longrightarrow$ broad resonance