Eectroweak and Inflationary Baryogenesis

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reviews

K.F., Prog. Theor. Phys. **96** ('96) 475

V.A. Rubakov and M.E. Shaposhnikov, hep-ph/9603208

A.Cohen, et al., Ann.Rev.Nucl.Part.Sci. **33** ('93) 27

Introduction

Baryon Asymmetry of the Universe

$$\frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} = (1.7 - 8.1) \times 10^{-11}$$

We attempt to explain this quantity, assuming that the Universe was B-symmetric at the beginning.

3 requirements for generation of BAU [Sakharov, '67]

(1) baryon number violation

(2) C and CP violation

(3) departure from equilibrium

candidates:

(1) GUTs, SUSY ($\langle \tilde{q} \rangle \neq 0$, R-viol.) — constraint by p-decay
axial anomaly of $(B + L)$ current in the SM

(2) Yukawa int.($\sim$ chiral gauge int.), scalar self-int., θ-term

1st example satisfying these conditions:

GUTs — decay of heavy X bosons [Yoshimura, '78]

$$\Gamma_X \simeq H(T_{\rm GUT})$$

Sphaleron process

"sphaleron" : classical static solution with one negative mode

— $SU(2)$ gauge-Higgs system

[Manton, '83; Klinkhammer & Manton, '84]

⇕

top of the energy barrier dividing two classical vacua

vacua of the gauge sector ⇔ $A = iU^{-1}dU$ with $U \in SU(2)$

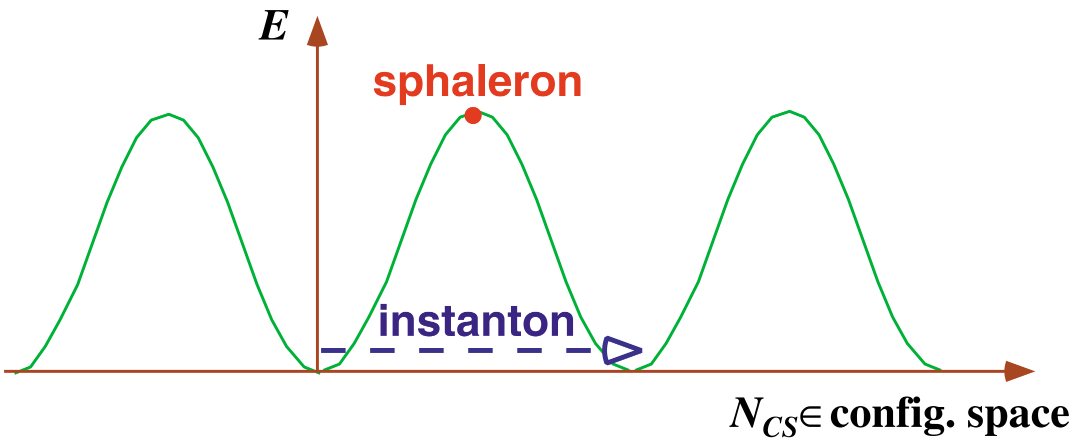
$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2} g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}), \qquad \partial_\mu j^\mu_{B-L} = 0$$

$$\begin{aligned} B(t_f) - B(t_i) &= \frac{N_f g^2}{32\pi^2} \int_{t_i}^{t_f} d^4x \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \\ &= N_f \left[N_{CS}(t_f) - N_{CS}(t_i) \right] \end{aligned}$$

where N_{CS} is the Chern-Simons number:

in the $A_0 = 0$ gauge,

$$N_{CS}(t) = \frac{g^2}{32\pi^2} \int d^3x \, \epsilon_{ijk} \, \mathrm{Tr} \left(F_{ij} A_k - \frac{2}{3} g A_i A_j A_k \right) \Bigg|_t$$



$$E_{\rm sph}(T=0) = \frac{2M_W}{\alpha_W} B\left(\frac{\lambda}{g^2}\right) \simeq 10\text{TeV}$$

λ:the Higgs self coupling, $\alpha_W = g^2/(4\pi)$

$1.5 \leq B \leq 2.7$ for $\lambda/g^2 \in [0, \infty)$

⋆ Transition rate

- $T = 0$ — (constrained or valley) instanton

  $\propto e^{-2S_{\rm inst}} \simeq e^{-378} = 10^{-164}$

- $\omega_-/(2\pi) \lesssim T \lesssim T_C$ $\qquad$ ω_-:negative-mode freq.

$$\Gamma^{(b)}_{\rm sph} \simeq k\,\mathcal{N}_{\rm tr}\,\mathcal{N}_{\rm rot}\,\frac{\omega_-}{2\pi}\left(\frac{\alpha_W(T)T}{4\pi}\right)^3 e^{-E_{\rm sph}/T}$$

zero modes: $\mathcal{N}_{\rm tr} = 26$, $\mathcal{N}_{\rm rot} = 5.3 \times 10^3$ for $\lambda = g^2$

$\omega_-^2 \simeq (1.8 \sim 6.6) m_W^2$ for $10^{-2} \leq \lambda/g^2 \leq 10$

$k \simeq O(1)$

- $T \gtrsim T_C$ $\qquad$ symmetric phase — no mass scale

$$\Gamma^{(s)}_{\rm sph} \simeq \kappa(\alpha_W T)^4$$

Monte Carlo simulation $\qquad$ $\langle N_{CS}^2(t)\rangle = e^{-2\Gamma V t}$ as $t \to \infty$

$\kappa > 0.4$ $\qquad$ $SU(2)$ gauge-Higgs system

[Ambjørn, et al. N.P.B353('91)]

$\kappa = 1.09 \pm 0.04$ $\qquad$ $SU(2)$ pure gauge system

[Ambjørn and Krasnitz, P.L.B362('95)]

Washout of $B + L$

if the sphaleron process is in equilibrium,

$$B \text{ and } L \propto (B - L)_{\text{primordial}}$$

To have nonzero BAU,

(i) we must have nonzero $B - L$ before the sphaleron process decouples, or

(ii) $B + L$ must be created at the first-order EWPT, *and* the sphaleron process must decouple immediately after that.

(i) $\Leftarrow$ GUTs, Affleck-Dine, Leptogenesis, ...

(ii) $=$ Electroweak Baryogenesis

Electroweak Baryogenesis

realization of non-equilibrium state

$$\Gamma_{\text{gauge}},\ \Gamma_{\text{Yukawa}},\ \Gamma^{(s)}_{\text{sph}} \gg H(T \simeq 100\text{GeV})$$

$$\Gamma^{(b)}_{\text{sph}} < H(T \simeq 100\text{GeV}),$$

if $\langle \text{Higgs} \rangle$ is large enough in the broken phase

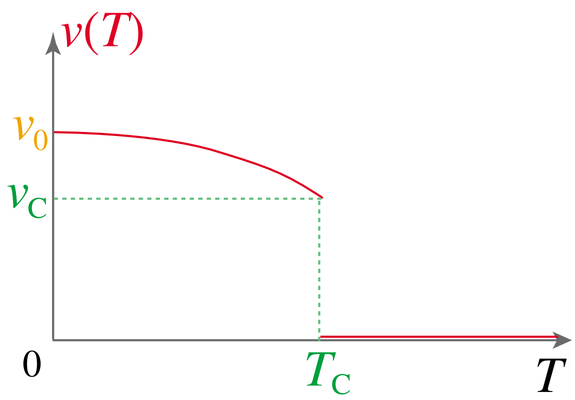
$\Downarrow$

first-order electroweak phase transition (EWPT)

minimal SM

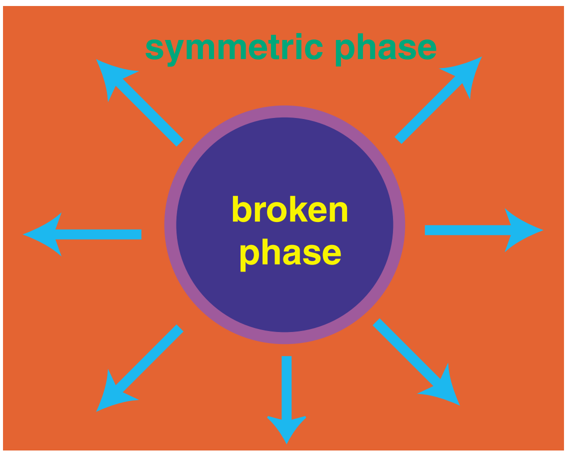
$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$$

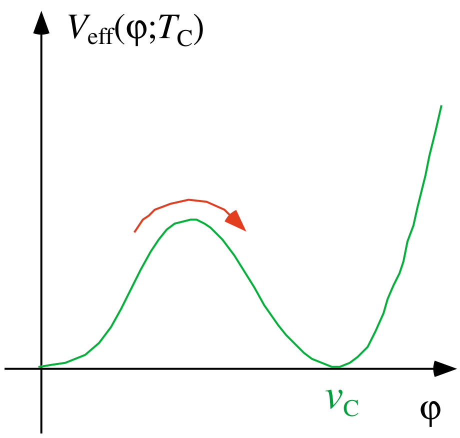
$$v_C \equiv \lim_{T \uparrow T_C} \varphi(T) \neq 0$$



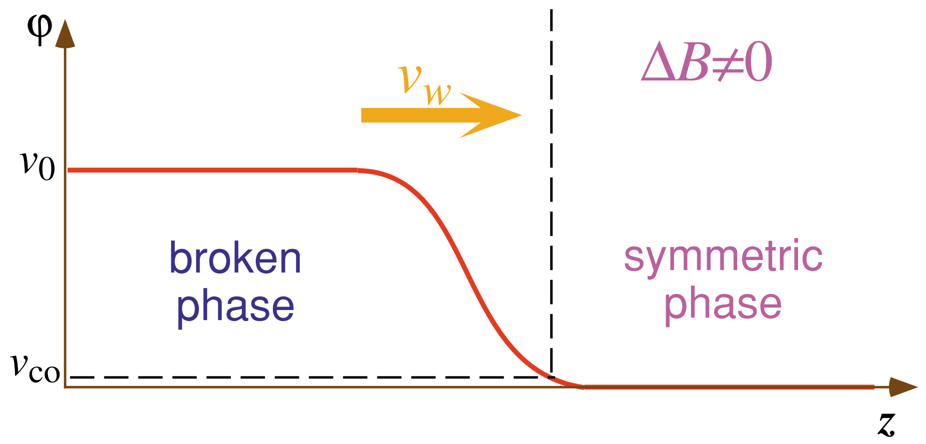
sphaleron decoupling at $T < T_C \Longrightarrow \dfrac{v_C}{T_C} > 1$

with bubble nucleation/growth





⋆ Mechanism



$v_{co} \simeq 0.01 v_0 \Longleftarrow E_{\rm sph}/T_C \simeq 1$

bubble wall $\Leftarrow$ classical config. of the gauge-Higgs system

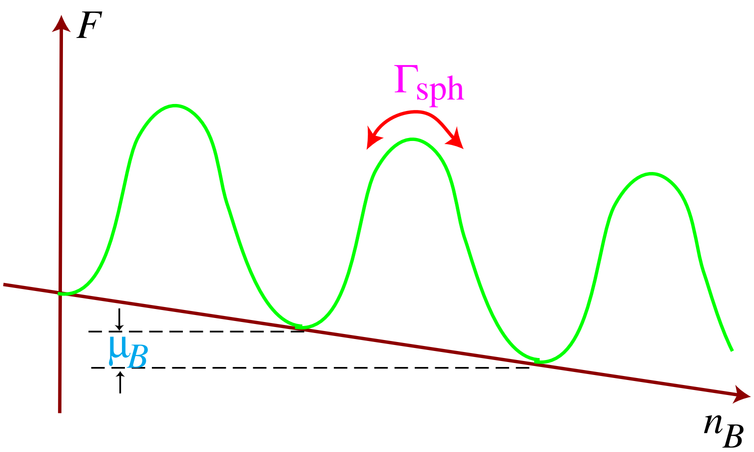
- interactions between the particles and the bubble wall
- accumulation of chiral charge in the symmetric phase

$\Downarrow$

generation of baryon number through sphaleron process

$\Downarrow$

decoupling of sphaleron process in the broken phase



$$\dot{n}_B \simeq -\frac{\mu_B \Gamma^{(s)}_{\rm sph}}{T}$$

$\mu_B \propto$ chiral charge (Y, I_3)

$$\frac{n_B}{s} \simeq \mathcal{N} \frac{100}{\pi^2 g_*} \cdot \kappa \alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T$$

e.g. finite-T perturbation theory (+ high-T expansion)

$$V_{\rm eff}(\varphi;T) \simeq D(T^2 - T_0^2)\varphi^2 - E\,T\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2)$$

$$E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2)$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4}\left[2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right]$$

with $\log\alpha_B = 2\log 4\pi - 2\gamma_E$ and $\log\alpha_F = 2\log\pi - 2\gamma_E$

At T_C, $\quad \varphi_C = \dfrac{2E\,T_C}{\lambda_{T_C}}$

$$\Gamma^{(b)}_{\rm sph}/T_C^3 < H(T_C) \quad \Longleftrightarrow \quad \frac{\varphi_C}{T_C} \gtrsim 1$$

$\Longrightarrow$ upper bound on λ $\qquad [m_h = \sqrt{2}\lambda v_0]$

$$m_h \lesssim 46\text{GeV}$$

$\longleftrightarrow$ inconsistent with the lower bound $m_h > 95.3\text{GeV}$ 107.7GeV(LEP)

⋆ Monte Carlo simulations [MSM]

effective fermion mass : $m_f(T) \sim O(T)$ ← nonzero modes

∴ simulation only with the bosons

QFT on the lattice $\begin{cases} \text{scalar fields:} & \phi(x) \text{ on the sites} \\ \text{gauge fields:} & U_\mu(x) \text{ on the links} \end{cases}$

$$Z = \int [d\phi\, dU_\mu] \exp\{-S_E[\phi, U_\mu]\}$$

- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet
  ⇑
  time-component of the gauge field

  only zero-freq. modes of the bosons survive as $T \to$ large
  matching finite-T Green's functions with 4-dim. theory
  $\Rightarrow$ T-dependent parameters

  [Laine & Rummukainen, hep-lat/9809045]

- 4-dim. $SU(2)$ system with a Higgs doublet

  [Csikor, hep-lat/9910354]

  EWPT is first order for $m_h < 66.5 \pm 1.4$GeV

Both the simulations found end-point of EWPT at

$$m_h = \begin{cases} 72.3 \pm 0.7 \text{ GeV} \\ 72.1 \pm 1.4 \text{ GeV} \end{cases} \Rightarrow \boxed{\text{no PT in the MSM !}}$$

an extension of the SM — MSSM

$$W = \epsilon_{ij}\big(f^{(e)}_{AB} H^i_d L^j_A E_B + f^{(d)}_{AB} H^i_d Q^j_A D_B - f^{(u)}_{AB} H^i_u Q^j_A U_B - \mu H^i_d H^j_u\big)$$

- more scalar fields $\Rightarrow$ { stronger (first-order) PT; 3-dim. order-parameter space }
- many complex parameters $\Rightarrow$ explicit CP violation
  μ, A, B, gaugino masses
- two Higgs doublets $\Rightarrow$ possibility of spontaneous CP viol.

⋆ Sphaleron

- 2-doublet Higgs model [Peccei, Zhang, Kastening, PLB '91]
- squarks vs sphaleron [Moreno, Oakini, Quirós, PLB '97]

⋆ Electroweak phase transition

3 order parametres:

$$\varphi_d = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \varphi_u = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 + i v_3 \end{pmatrix}$$

Higgs potential $\Longleftarrow V_D$ & $\mathcal{L}_{\rm soft}$

$$\begin{aligned} V_0 &= m_1^2\Phi_d^\dagger\Phi_d + m_2^2\Phi_u^\dagger\Phi_u + (m_3^2\Phi_u\Phi_d + \text{h.c.}) \\ &\quad + \frac{g_2^2+g_1^2}{8}(\Phi_d^\dagger\Phi_d - \Phi_u^\dagger\Phi_u)^2 + \frac{g_2^2}{2}(\Phi_d^\dagger\Phi_d)(\Phi_u^\dagger\Phi_u) \end{aligned}$$

$\Longrightarrow$ The tree-level mass relations

$$m_h^2 \le \min\{m_Z^2,\, m_A^2\}, \qquad m_A^2 = \frac{m_3^2}{\sin\beta_0\cos\beta_0}$$

are modified by radiative corrections.[Okada et al. PLB '91]

LEP-Higgs: $m_h \geq 88.3$GeV, $m_H \geq 107.7$GeV
$m_A \geq 88.4$GeV

⋆ EWPT in the MSSM

One-loop effective potential

[K.F., hep-ph/9809517 (PTP '99)]

input:

$v_0 = |\boldsymbol{v}| = 246$GeV, $\tan\beta = \dfrac{\sqrt{v_2^2+v_3^2}}{v_1}$

$\longrightarrow y_t = \sqrt{2}m_t/(v_0\sin\beta)$

M_1, M_2, $m_{\tilde{t}_L}^2$, $m_{\tilde{t}_R}^2$, m_3^2: soft-SUSY-br. parameters

$$m_1^2,\, m_2^2 \quad \Longleftarrow \quad \left.\frac{\partial V_{\rm eff}}{\partial v_1}\right|_{\boldsymbol{v}} = 0, \quad \left.\frac{\partial V_{\rm eff}}{\partial v_2}\right|_{\boldsymbol{v}} = 0$$

N.B.

- stop mass-squared matrix :

$$\begin{pmatrix} m_{\tilde{t}_L}^2 + \left(\frac{g_1^2}{24} - \frac{g_2^2}{8}\right)(v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 & \frac{y_t}{\sqrt{2}}(\mu v_d + A(v_2 - i v_3)) \\ * & m_{\tilde{t}_R}^2 - \frac{g_1^2}{6}(v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 \end{pmatrix}$$

$m_{\tilde{t}_L}^2 = 0$ or $m_{\tilde{t}_R}^2 = 0 \Longrightarrow$ smaller eigenvalue: $m_{\tilde{t}_1}^2 \sim O(v^2)$

$\therefore$ high-T expansion

$$\bar{V}_{\tilde{t}}(\boldsymbol{v}; T) \Rightarrow -\frac{T}{6\pi}(m_-^2)^{3/2}$$

$\longrightarrow$ stronger 1st order PT

- bounds on CP viol. by nEDM [Kizukuri & Oshimo, PRD '92]

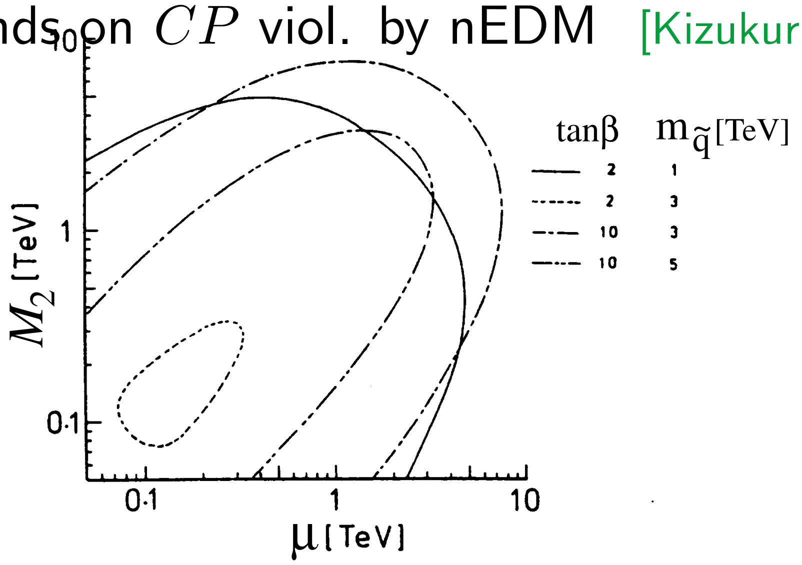


FIG. 4. The parameter regions compatible with the present experimental upper bound on the neutron EDM.

$\theta + \delta_\mu + \delta_2 = \pi/4$

$\mathrm{Arg} A = \pi/4$

inside is excluded

- $\theta + \delta_\mu + \delta_2 = O(1) \Longrightarrow m_{\tilde{q}}, m_{\tilde{l}} \gtrsim 10$TeV
- $m_{\tilde{q}}, m_{\tilde{l}} \lesssim 1$TeV $\Longrightarrow \theta + \delta_\mu + \delta_2 \lesssim 10^{-3}$

output:

masses of the neutral Higgs scalars

$\Longleftarrow$ eigenvalues of $\left.\dfrac{\partial^2 V_{\text{eff}}(\boldsymbol{v}; T=0)}{\partial v_i \partial v_j}\right|_{\boldsymbol{v}}$

$m_{\tilde{t}_{1,2}}$, $m_{\chi^{\pm}_{1,2}}$, $m_{\chi^0_{1-4}}$

$m_{\tilde{t}_1} > 86.4$GeV,
$m_{\chi^0_1} > 32.5$GeV, $m_{\chi^{\pm}_1} > 67.7$GeV for $\tan\beta > 0.7$

when $^{\exists}$explicit CP violation $(\mu, M_2, M_1, A_t \in \mathbf{C})$

$\theta =$ relative phase of the 2 Higgs $= \mathrm{Arg}(v_2 + i v_3)$

$T \neq 0$

$v(T) = |\boldsymbol{v}(T)|$, $\tan\beta(T)$, $\theta(T)$

T_C : transition temperature

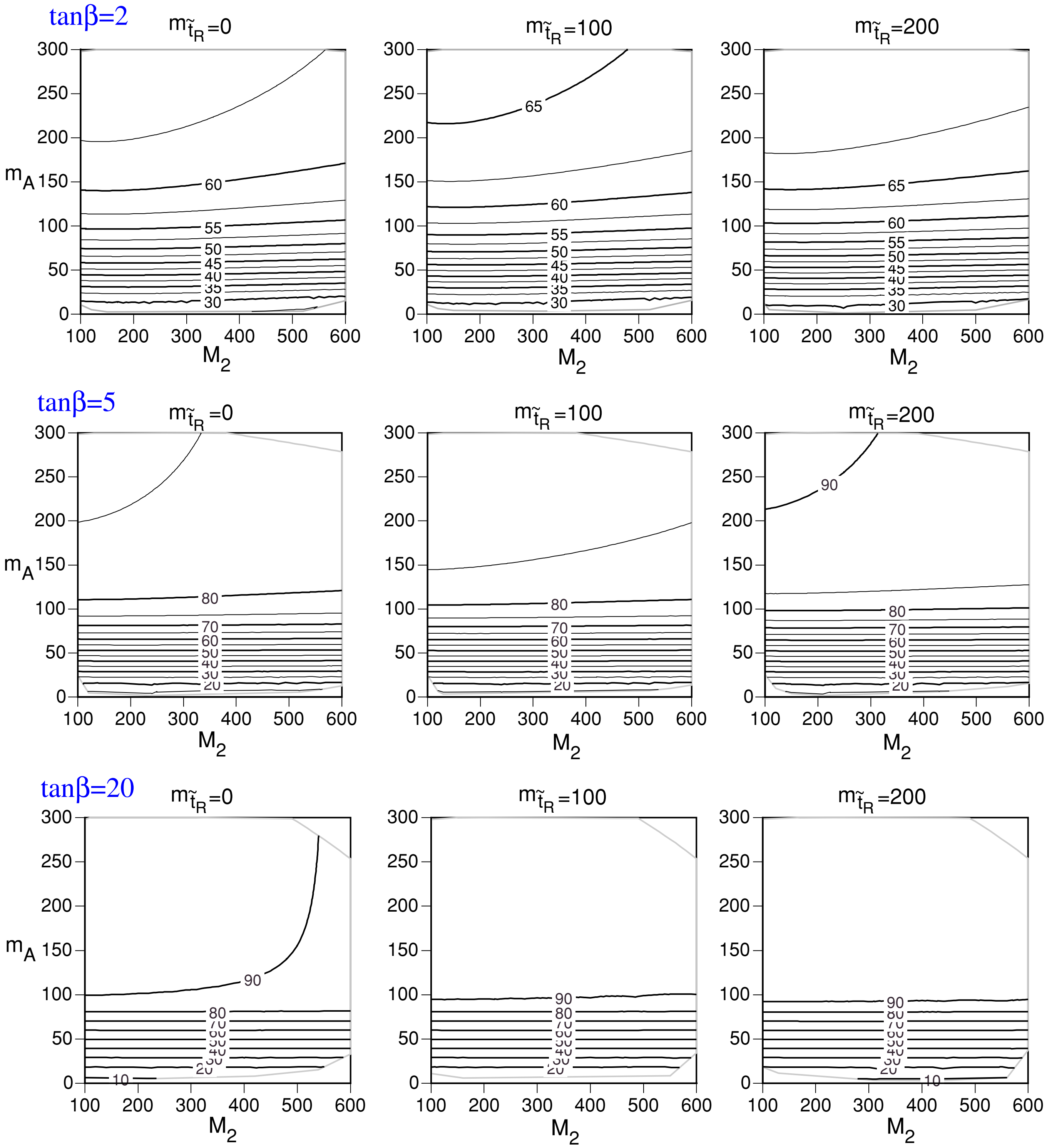
by numerically searching for the min. of V_{eff}

numerical results $M_2 = M_1$

$m_t = 175$ GeV $m_{\tilde{t}_L} = 400$ GeV $\mu = -300$ GeV $A_t = 10$ GeV

without CP violation

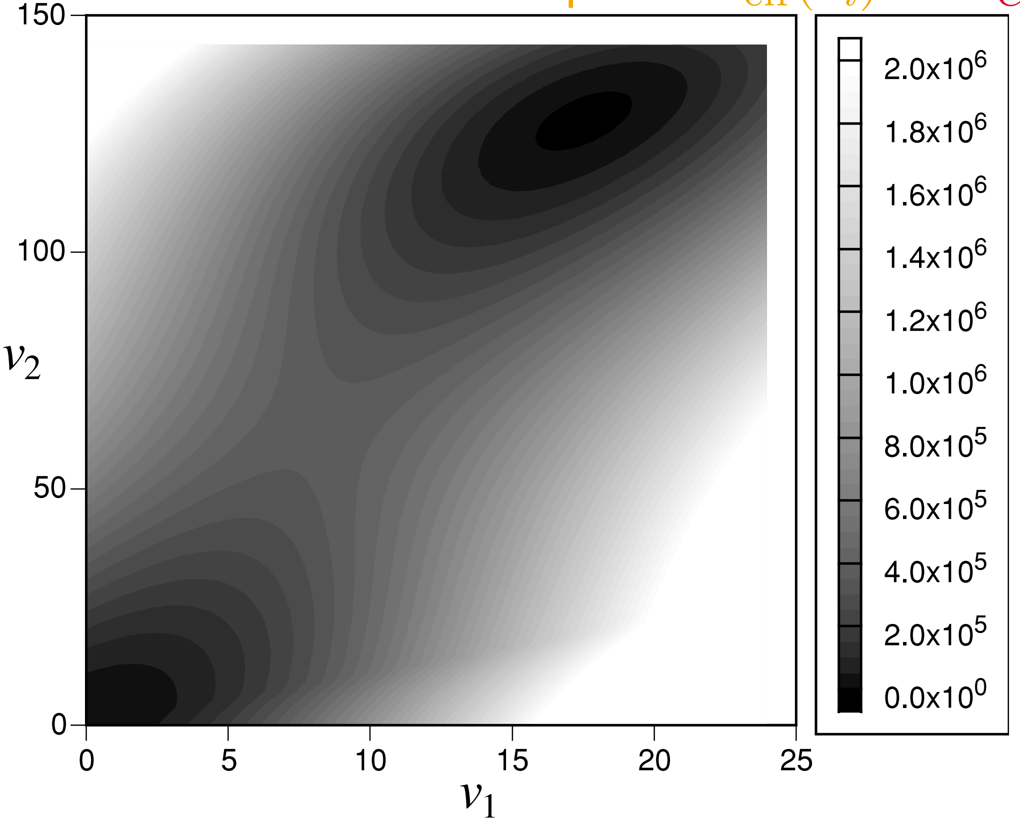
the lighter Higgs scalar mass : m_h (GeV)



at $T \neq 0$

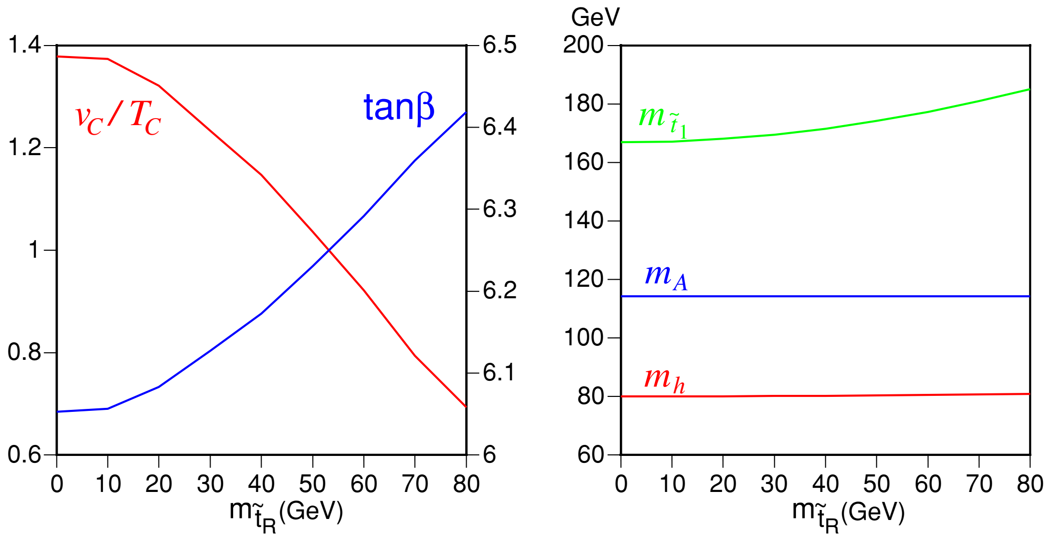
$$\left.\begin{array}{r} m_{\tilde{t}_1} \lesssim m_t \\ m_h \lesssim 100\text{GeV} \end{array}\right\} \Longrightarrow \frac{v_C}{T_C} > 1$$

an example of $V_{\text{eff}}(v_i)$ at T_C



$\tan\beta = 6$, $m_h = 82.3\text{GeV}$, $m_A = 118\text{GeV}$, $m_{\tilde{t}_1} = 168\text{GeV}$

$T_C = 93.4\text{GeV}$, $v_C = 129\text{GeV}$



$\tan\beta = 5$, $m_3^2 = 4326\text{GeV}$

⋆ Lattice MC studies

- 3d reduced model [Laine et al. hep-lat/9809045]

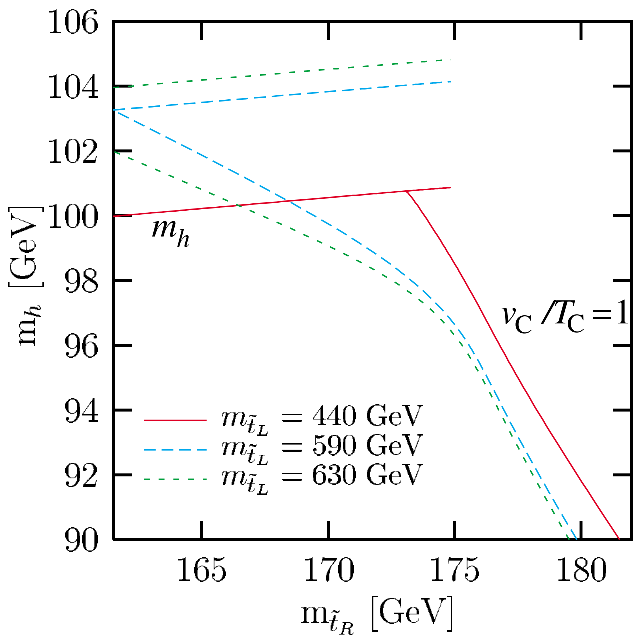
  strong 1st order for $m_{\tilde{t}_1} \lesssim m_t$ and $m_h \leq 110\text{GeV}$

- 4d model [Csikor, et al. hep-lat/0001087]

  with $SU(3)$, $SU(2)$ gauge bosons, 2 Higgs doublets,
  L & R-stops, sbottoms

  no scalar trilinear (A) terms, $\tan\beta \simeq 6$

⟶ agreement with the perturbation theory within the errors

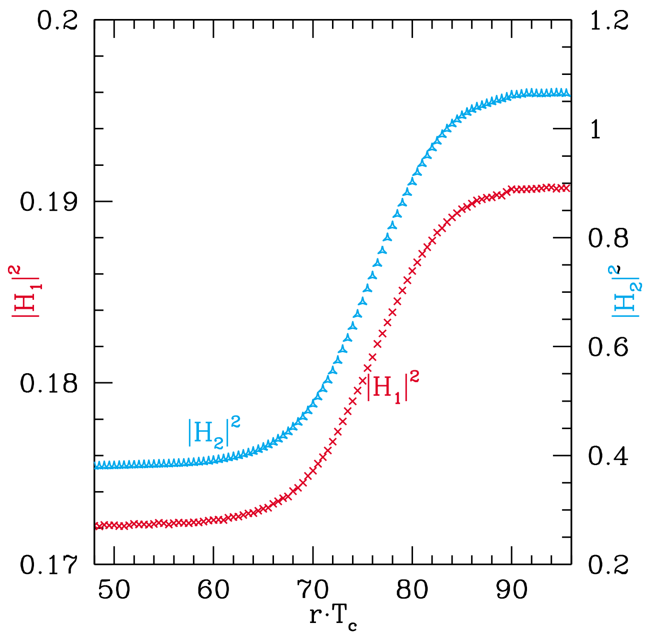


$m_A = 500$ GeV

$v_C/T_C > 1$
below the steeper lines

⇓

max. $m_h = 103 \pm 4$ GeV
for $m_{\tilde{t}_L} \simeq 560$ GeV



bubble-wall profile

$\Delta\beta = 0.0061 \pm 0.0003$

$\Rightarrow \beta \simeq$ const.

wall width $\simeq \dfrac{11}{T_C}$

CP violation relevant to Baryogenesis

— $\theta(x)$ in the bubble wall

Eqs. of motion for $(\rho_i(x), \theta(x))$ with $V_{\text{eff}}(\rho_i, \theta; T_C)$

with B.C. determined by the min. of $V_{\text{eff}}(T_C)$

$\rho(x) \sim 1 + \tanh(ax)$: 0 (sym. phase) $\longrightarrow v_C$ (br. phase)

bubble wall $\sim$ macroscopic, static $\rightarrow$ 1d system

$$\begin{aligned} \frac{d^2\rho_i(z)}{dz^2} - \rho_i(z)\left(\frac{d\theta_i(z)}{dz}\right)^2 - \frac{\partial V_{\text{eff}}}{\partial \rho_i} &= 0, \\ \frac{d}{dz}\left(\rho_i^2(z)\frac{d\theta_i(z)}{dz}\right) - \frac{\partial V_{\text{eff}}}{\partial \theta_i} &= 0 \end{aligned}$$

with gauge-fixing condition

$$\rho_1^2(z)\frac{d\theta_1(z)}{dz} + \rho_2^2(z)\frac{d\theta_2(z)}{dz} = 0$$

Assume that $\tan\beta(z)$ be constant.

Suppose that at $T \simeq T_C$, without explicit CP violation,

$$
\begin{aligned}
& V_{\text{eff}}(\rho_i, \theta = \theta_1 - \theta_2) \\
&= \frac{1}{2}\bar{m}_1^2\rho_1^2 + \frac{1}{2}\bar{m}_2^2\rho_2^2 - \bar{m}_3^2\rho_1\rho_2\cos\theta + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\
&+ \frac{\lambda_3 + \lambda_4}{4}\rho_1^2\rho_2^2 + \frac{\lambda_5}{4}\rho_1^2\rho_2^2\cos 2\theta - \frac{1}{2}(\lambda_6\rho_1^2 + \lambda_7\rho_2^2)\rho_1\rho_2\cos\theta \\
&- [A\rho_1^3 + \rho_1^2\rho_2(B_0 + B_1\cos\theta + B_2\cos 2\theta) \\
&\qquad + \rho_1\rho_2^2(C_0 + C_1\cos\theta + C_2\cos 2\theta) + D\rho_2^3] \\
&= \left[\frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2)\right] \\
&\qquad \times \left[\cos\theta - \frac{2\bar{m}_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)}\right]^2 \\
&\quad + \theta\text{-independent terms}
\end{aligned}
$$

where all the parameters are real

conditions for spontaneous CP violation for a given (ρ_1, ρ_2)

$$
F(\rho_1, \rho_2) \equiv \frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2) > 0,
$$

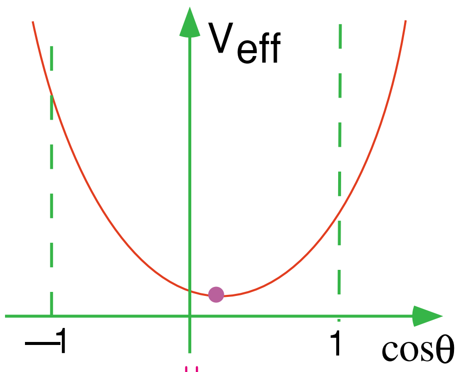
$$
-1 < G(\rho_1, \rho_2) \equiv \frac{2\bar{m}_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)} < 1
$$

At $T \simeq T_C$, around the EW bubble wall

$$(\rho_1, \rho_2) \ : \ (0,0) \longrightarrow (v_C \cos\beta_C, v_C \sin\beta_C)$$

There may be a chance to satisfy the conditions in the transient region.

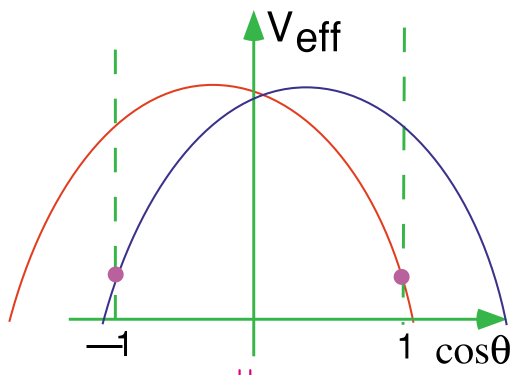
$F(\rho_1, \rho_2) > 0$



$\Downarrow$

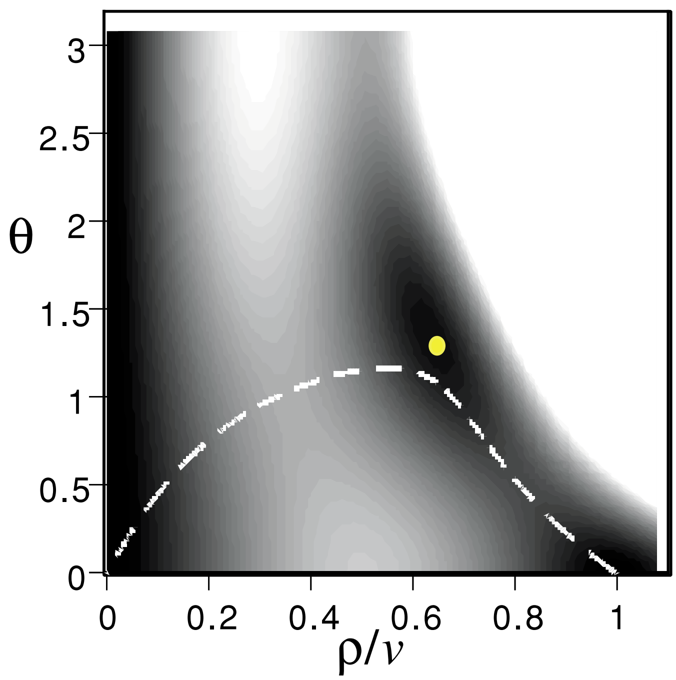
CP-violating
local minimum

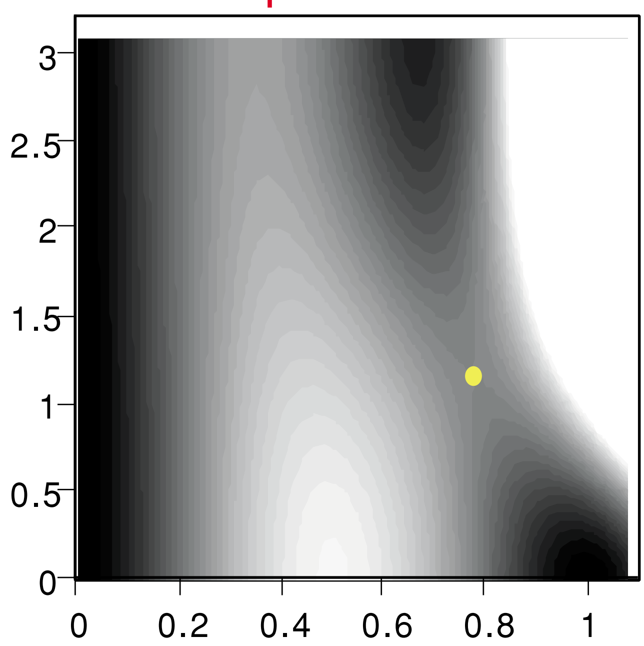
$F(\rho_1, \rho_2) < 0$



$\Downarrow$

CP-violating
saddle point





Transitional CP Violation

N.B. no explicit CP violation $\Rightarrow$ no net BAU

[FKOT, PTP96 ('96)]

spontaneous CP violation in the transient region

$+$ small explicit CP violation

to resolve degeneracy between CP conjugate bubbles

net BAU

$$\frac{n_B}{s} = \frac{\sum_j \left(\frac{n_B}{s}\right)_j N_j}{\sum_j N_j}$$

with

$$\begin{aligned} N_j &= \exp(-4\pi R_C^2 \mathcal{E}_j / T_C) \quad \text{nucleation rate} \\ \mathcal{E}_j &= \text{energy density of the type-}j\text{ bubble} \end{aligned}$$

Example

[K.F., Otsuki & Toyoda, PTP '99]

input parameters

$\tan\beta_0$	m_3^2	μ	A_t	$M_2 = M_1$	$m_{\tilde{t}_L}$	$m_{\tilde{t}_R}$
6	8110 GeV^2	-500 GeV	60 GeV	500 GeV	400 GeV	0

mass spectrum

m_h	m_A	m_H	$m_{\tilde{t}_1}$	$m_{\chi_1^\pm}$	$m_{\chi_1^0}$
82.28 GeV	117.9 GeV	124.0 GeV	167.8 GeV	457.6 GeV	449.8 GeV

at the EWPT

$$T_C = 93.4 \text{ GeV}, \quad v_C = 129.17 \text{ GeV}, \quad \tan\beta = 7.292,$$

inverse wall thickness: $a = \dfrac{\sqrt{8V_{\rm max}}}{v} = 13.23 \text{ GeV} \sim \dfrac{T_C}{7}$

thinner than the MC result

Introduce an explicit CP violation by

$$\bar{m}_3^2 \cos\theta \rightarrow \frac{1}{2}\left(\bar{m}_3^2 e^{i(\theta+\delta)} + \text{h.c.}\right)$$

Net chiral charge flux

$$F_Q^{net} = \frac{N^+ F_Q^+ - N^- F_Q^-}{N^+ + N^-}.$$

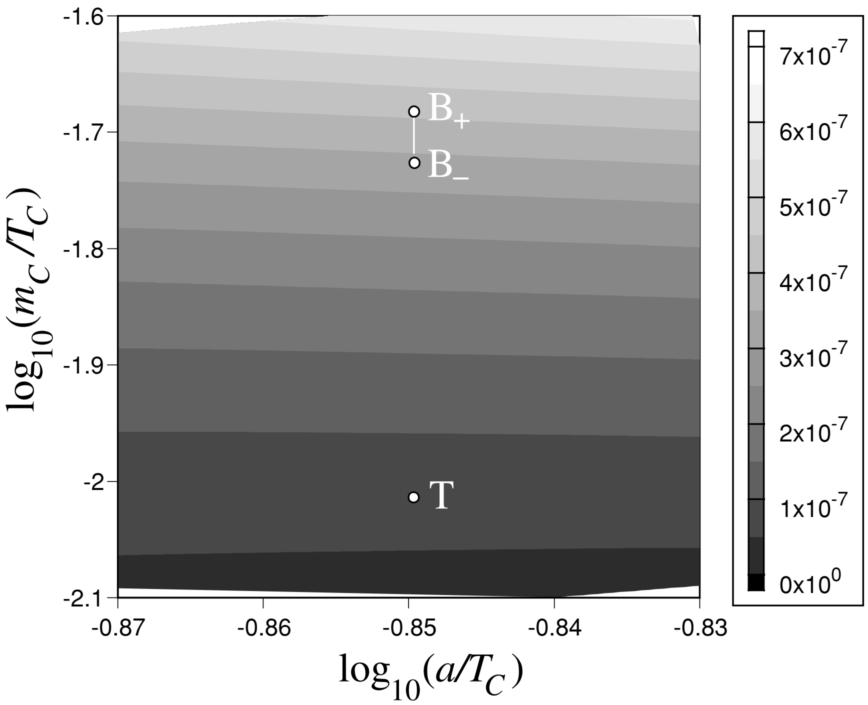
where

$$\frac{N^-}{N^+} = \begin{cases} 0.361 & \text{for } \delta = 0.001 \\ 0.601 & \text{for } \delta = 0.002 \end{cases}$$

by charge transport mechanism

$$\frac{n_B}{s} \sim 10^{-7} \times \frac{F_Q^{net}}{u} \times \frac{\tau}{T_C^2},$$

$$u = 0.1,\ \delta = 10^{-3} \Rightarrow \begin{cases} n_B/s < 10^{-12} & \text{for } b \text{ quark} \\ n_B/s \sim 10^{-(10-12)} & \text{for } \tau \text{ lepton} \end{cases}$$



$$\frac{F_Q^{net}}{(Q^L - Q^R) T_C^3 u}$$

$$|\theta_2| \propto \cos^2\beta \ll 1$$

$$m_C = m\,\frac{v_C \cos\beta_C}{v_0 \cos\beta_0}$$

♠ Enhancement of an explicit CP violation

$$\alpha = \mathrm{Arg}(\mu M_2) = \mathrm{Arg}(\mu M_1), \qquad \beta = \mathrm{Arg}(\mu A_t^*),$$

then

$$\begin{aligned}
\bar{m}_3^2 &= m_3^2 + \Delta_{\phi^\pm}^{(0)} m_3^2 + e^{i\alpha} \Delta_{\chi}^{(0)} m_3^2 + e^{i\beta} \Delta_{\tilde{t}}^{(0)} m_3^2, \\
\lambda_5 &= \Delta_{\phi^\pm}^{(0)} \lambda_5 + e^{i2\alpha} \Delta_{\chi}^{(0)} \lambda_5 + e^{i2\beta} \Delta_{\tilde{t}}^{(0)} \lambda_5, \\
\lambda_{6,7} &= \Delta_{\phi^\pm}^{(0)} \lambda_{6,7} + e^{i\alpha} \Delta_{\chi}^{(0)} \lambda_{6,7} + e^{i\beta} \Delta_{\tilde{t}}^{(0)} \lambda_{6,7}
\end{aligned}$$

$\Delta^{(0)} \equiv$ correction without explicit CP violation

If $\Delta_{\chi}^{(0)} \gg \Delta_{\tilde{t}}^{(0)}, \Delta_{\phi^\pm}^{(0)}$, by rephasing, $\lambda_{5,6,7} \in \mathbf{R}$ and

$$e^{-i\alpha} \bar{m}_3^2 = e^{-i\alpha} m_3^2 + \Delta_{\chi}^{(0)} m_3^2 \equiv e^{-i\delta} \left| \bar{m}_3^2 \right|$$

with $$\tan\delta = -\frac{m_3^2 \sin\alpha}{m_3^2 \cos\alpha + \Delta_{\chi}^{(0)} m_3^2}.$$

N.B $\left| m_3^2 + \Delta_{\chi}^{(0)} m_3^2 \right| \ll m_3^2$ for transitional CP violation

for some parameter set, we have at $T \simeq T_C$

$$\Delta_{\tilde{t}}^{(0)} m_3^2 = 140.69, \qquad \Delta_{\chi}^{(0)} m_3^2 = -2356.73,$$

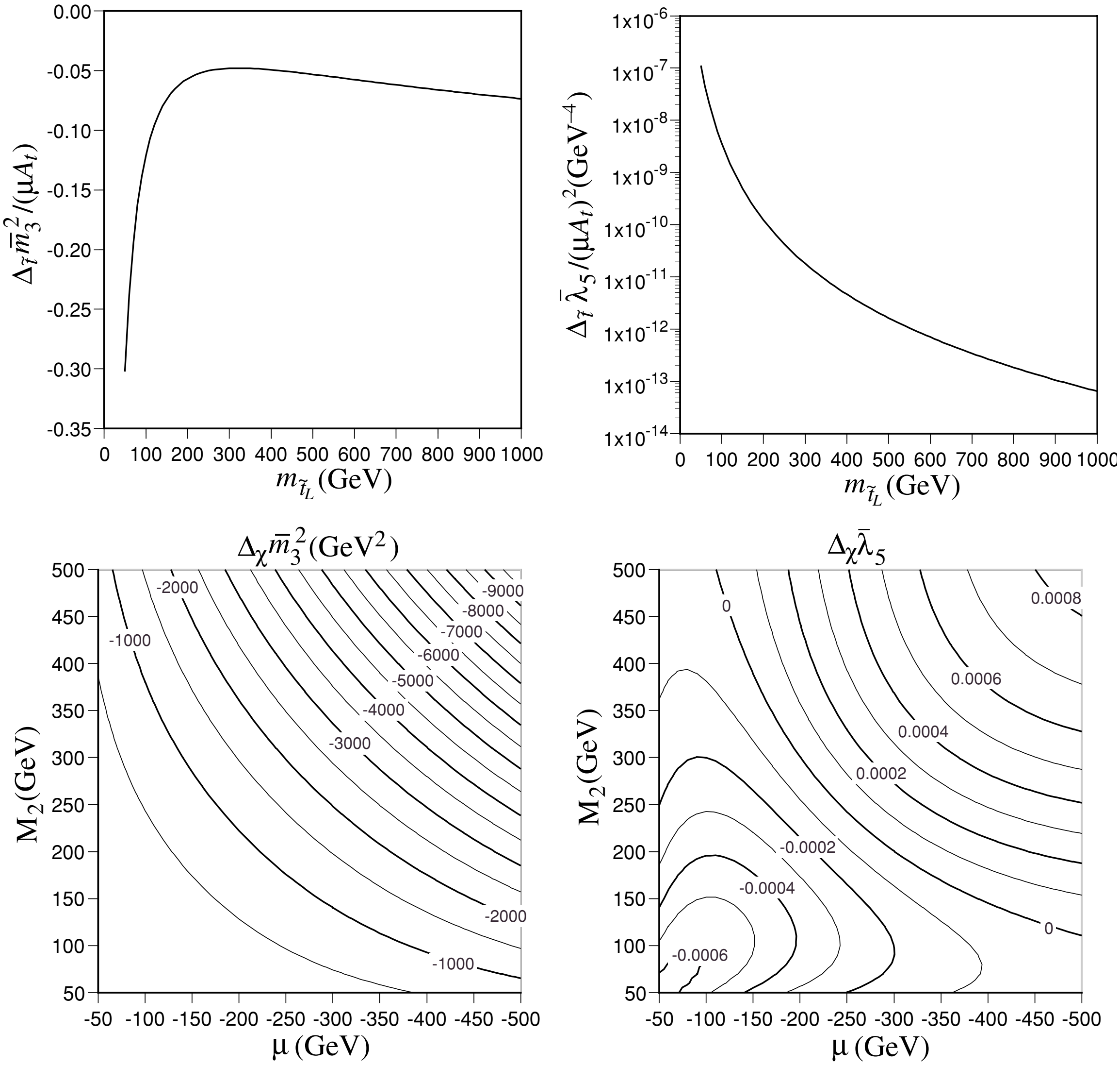
so that even for $\alpha = 10^{-3}$,

$$\begin{aligned}
\tan\delta &= -\frac{(m_3^2 + \Delta_{\tilde{t}}^{(0)} m_3^2) \sin\alpha}{(m_3^2 + \Delta_{\tilde{t}}^{(0)} m_3^2) \cos\alpha + \Delta_{\chi}^{(0)} m_3^2} \\
&\simeq \frac{10^{-3}}{6.8 \times 10^{-3}} = 0.147
\end{aligned}$$

$\Longrightarrow$ only the lowest-energy bubble survives

♠ Possibility of $F < 0$ [$\leftrightarrow \lambda_5 < 0$]

for $m_{\tilde{t}_R} = 0$, at $T = 95$ GeV,



⋆ $\lambda_5 < 0 \Longleftarrow \Delta_\chi\lambda_5 < 0$
$\longrightarrow \Delta_\chi\bar{m}_3^2 < -1500$ GeV^2

⋆ μA_t is restricted to have $\lambda_5 = \Delta_\chi\lambda_5 + \Delta_{\tilde{t}}\lambda_5 < 0$
$\longrightarrow \Delta_{\tilde{t}}\bar{m}_3^2$ is negative and bounded from below.

⋆ to have small $\bar{m}_3^2$, the tree-level $m_3^2 \lesssim 2500$ GeV^2
$\longrightarrow$ too small m_h and m_A (< 67.5 GeV)

$\therefore$ difficult to realize transitional CP violation with $F < 0$ in an acceptable MSSM

Studies after our works:

- Improved perturbation theory [John, hep-ph/0010277]

  no transitional CP violation for parameters with $v_C/T_C > 1$ and acceptable m_A

- 3d Lattice MC [Laine & Rummukainen, hep-lat/0009025]

  no transitional CP violation for $m_h = 105$GeV, $m_A = 105$GeV, $\tan\beta = 12$, $m_{\tilde{q}_L} = 1$TeV

- NMSSM (perturbation)
  [Huber & Schmidt, hep-ph/000312-rev.; hep-ph/0101249]

$$W = \lambda S H_u H_d + \frac{k}{3} S^3 + \mu H_u H_d + r S$$

  S : gauge singlet
  $\langle S \rangle \in \mathbf{C}$ can also violate CP. Transitional CP violation does occur for $\mu = 212.6$GeV, $m_{\tilde{q}_L} = 278.1$GeV, $\tan\beta = -5$, $A_t = -219.8$GeV.
  Both θ and $\theta_S = \mathrm{Arg} S$ has nontrivial values in the transient region.

Inflationary Baryogenesis

Motivation

in an inflationary scenario,

all the particles in the universe

$\Uparrow$

reheating after the de Sitter phase

$\Updownarrow$

decay of coherently oscillating classical scalar field: $\phi(t)$

inflaton

It is natural that B was also generated in this era of particle creation.

process of reheating

- perturbative decay of $\phi(t)$
  classical ϕ-field $=$ zero-momentum ϕ-quanta

- ⊙ preheating by parametric resonance
  particle creation in an oscillating background
  [Kofman, Linde, Starobinsky, PRL73 ('94); PRD56 ('97)]

$\chi(x)$: scalar field interacting with $\phi(t)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}g^2\phi^2\chi^2 + \cdots$$

just after the de Sitter phase,

$$H(t)^2 = \frac{8\pi}{3m_P}(\rho_\phi + \rho_\chi + \cdots) \simeq \frac{8\pi}{3m_P}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + \frac{\partial V(\phi)}{\partial\phi(t)} = 0$$

$$\Longrightarrow \phi(t) = \Phi(t)\sin(mt) \propto \frac{1}{t}\sin(mt)$$

where $V(\phi) \simeq \frac{1}{2}m^2\phi^2$

wave equation for a mode $\chi_k(t)$:

$$\ddot{\chi}_k(t) + 3H(t)\dot{\chi}_k(t) + \left(\frac{k^2}{a^2} + g^2\Phi^2(t)\sin^2(mt)\right)\chi_k(t) = 0$$

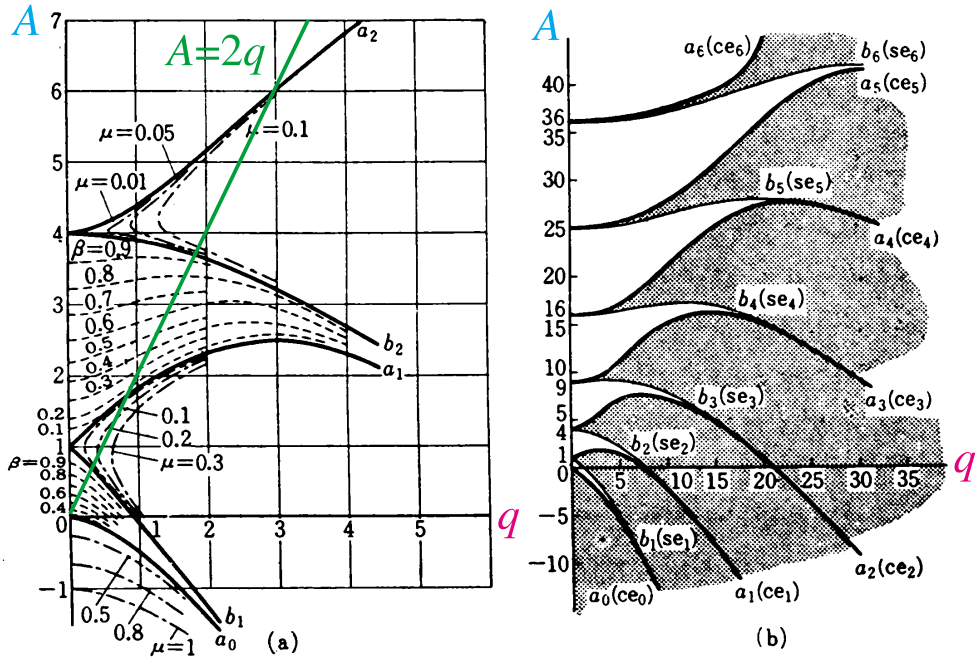
Putting $a(t) = 1$, $z = mt$,

$$\chi_k''(z) + (A_k - 2q\cos 2z)\chi_k(z) = 0$$

where

$$A_k \equiv \frac{k^2}{m^2} + \frac{g^2\Phi^2}{2m^2} = \frac{k^2}{m^2} + 2q, \qquad q \equiv \frac{g^2\Phi^2}{4m^2}$$

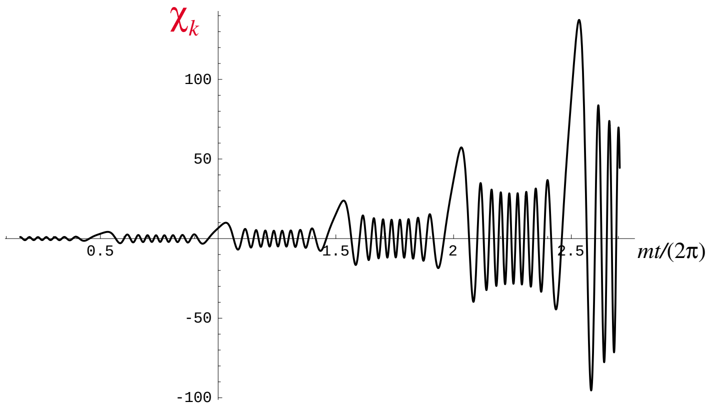
Mathieu equation

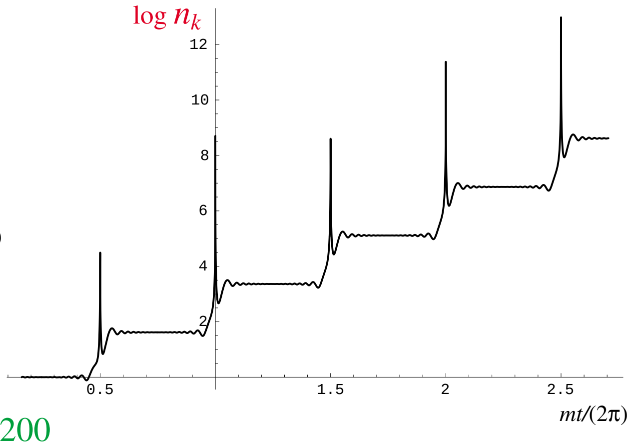


Mathieu の微分方程式の解の安定域

$q \gg 1 \Longrightarrow$ broad resonance

a solution in a resonance band





q=200

n_k changes only at t where $\Phi(t) = 0$

$$\Leftrightarrow \dot{\omega}(t) \gtrsim \omega^2(t) \text{ with } \omega(t) = \sqrt{k^2 + g^2\Phi^2(t)\sin^2(mt)}$$

$$\left.\begin{array}{c} |\chi_k(t)| \\ n_k(t) \end{array}\right\}$$ exponentially increase with t stepwise.

$\Longrightarrow$ seccessive scatterings by a periodic potential

$\Longrightarrow$ descent equation for n_k

We must take into account

- expansion of the universe $a(t) \longrightarrow$ stochastic resonance
- back reaction of the generated χ-particles

$$\begin{cases} \rho \simeq \rho_\phi \rightarrow \rho_\chi & \text{:damping the oscillation} \\ m_\phi^2 \simeq m^2 + g^2\langle\chi^2\rangle & \text{:increase } \phi\text{-frequency} \end{cases}$$

- rescattering of χ-particle by ϕ-particle

  $\Delta m_\chi^2(k) = g^2\langle\delta\phi^2\rangle_k >$ resonance width

  $\Longrightarrow$ terminates the resonance

example:

$m = 10^{-6} m_P$, $\Phi_0 = m_P/5$

the resonance ends after about 10 ϕ-oscillations for $g = 10^{-3\sim-1}$.

state after the preheating

- large occupation number of χ with small k
- large quantum fluctuation of χ

  $\sqrt{\langle\chi^2\rangle} \simeq 3 \times 10^{16}$GeV for $g = 3 \times 10^{-14}$

  $\longleftrightarrow$ thermal fluctuation at $T = 10^{17}$GeV

  $\Downarrow$

> nonthermal symmetry restoration
> nonthermal heavy particle production

⋆Charge Generation

[K.F., Otsuki, Kakuto, Toyoda, hep-ph/0010266]

$$\mathcal{L} = \partial_\mu \chi_a^* \partial^\mu \chi_a - g_a^2 \phi^2(t) \chi_a^* \chi_a - \chi_a^* V_{ab}(t) \chi_b - \frac{1}{2} \left(\chi_a W_{ab}(t) \chi_b + \text{c.c.} \right),$$

We treat CP-violating V_{ab} and W_{ab} as perturbations.
— picture of successive scattering at zero of $\phi(t)$: t_j

for $t \in (t_{j-1}, t_j)$

$$\chi_a(x) = \int d^3\boldsymbol{k} \left(a^j_{a\boldsymbol{k}} f^j_{ak}(t) e^{i\boldsymbol{kx}} + b^{j\dagger}_{a\boldsymbol{k}} f^{j*}_{ak}(t) e^{-i\boldsymbol{kx}} \right)$$

relation between $f^0_{ak}(t)$ and $f^j_{ak}(t)$

$$\Longrightarrow \begin{cases} a^j_{a\boldsymbol{k}} = a^0_{b\boldsymbol{k}} \alpha^j_{ba} + b^{0\dagger}_{b\boldsymbol{k}} \tilde{\beta}^j_{ba} \\ b^{j\dagger}_{a\boldsymbol{k}} = a^0_{b\boldsymbol{k}} \beta^j_{ba} + b^{0\dagger}_{b\boldsymbol{k}} \tilde{\alpha}^j_{ba} \end{cases} \quad : \text{Bogoliubov trf.}$$

CP violation $\Longrightarrow$ $\alpha^j_{ab} \neq \tilde{\alpha}^j_{ab}$ and $\beta^j_{ab} \neq \tilde{\beta}^j_{ab}$

relations among the Bogoliubov coefficients:

$$\alpha^{j\dagger}\alpha^j - \tilde{\beta}^{j\dagger}\tilde{\beta}^j = \tilde{\alpha}^{j\dagger}\tilde{\alpha}^j - \beta^{j\dagger}\beta^j = 1, \quad \beta^{j\dagger}\alpha^j - \tilde{\alpha}^{j\dagger}\tilde{\beta}_j = 0$$

generated particle number and charge densities after j zeros;

$$\begin{aligned} n^j_k &= \frac{1}{V} \langle 0^0 | \sum_{a=1}^{n} \left(a^{j\dagger}_{a\boldsymbol{k}} a^j_{a\boldsymbol{k}} + b^{j\dagger}_{a\boldsymbol{k}} b^j_{a\boldsymbol{k}} \right) | 0^0 \rangle \\ &= \mathrm{Tr} \left(\tilde{\beta}^{j\dagger}\tilde{\beta}^j + \beta^{j\dagger}\beta^j \right) \end{aligned}$$

$$
\begin{aligned}
j^j_{Qk} &= \frac{1}{V}\langle 0^0| \sum_{a=1}^{n} Q_a \left(a^{j\dagger}_{a\boldsymbol{k}} a^j_{a\boldsymbol{k}} - b^{j\dagger}_{a\boldsymbol{k}} b^j_{a\boldsymbol{k}} \right) |0^0\rangle \\
&= \mathrm{Tr}\left[Q \left(\tilde{\beta}^{j\dagger}\tilde{\beta}^j - \beta^{j\dagger}\beta^j \right) \right]
\end{aligned}
$$

N.B. If χ_a is one-component, $\left|\beta^j\right|^2 = \left|\tilde{\beta}^j\right|^2$. $\quad \therefore\ j^j_k = 0$

---

example

2-component χ, $m_1 = m_2$, $V_{11} = V_{22}$, $W_{ab} = 0$

$U(1)$-sym.: $\chi_a \mapsto e^{i\alpha}\chi_a$ and discrete sym.: $\chi_1 \leftrightarrow \chi_2$

We found descent equations for the Bogoliubov coefficients $\left(\alpha^j, \beta^j, \tilde{\alpha}^j, \tilde{\beta}^j\right)$ and $\left(\alpha^{j+1}, \beta^{j+1}, \tilde{\alpha}^{j+1}, \tilde{\beta}^{j+1}\right)$.

$$
\Longrightarrow \begin{cases}
n^j_k = \sum_{a,b=1}^{2} \left(\left|\beta^j_{ab}\right|^2 + \left|\tilde{\beta}^j_{ab}\right|^2 \right) \\
j^j_{1k} = \left|\tilde{\beta}^j_{11}\right|^2 + \left|\tilde{\beta}^j_{21}\right|^2 - \left|\beta^j_{11}\right|^2 - \left|\beta^j_{21}\right|^2 & \text{charge of } \chi_1 \\
j^j_{2k} = \left|\tilde{\beta}^j_{12}\right|^2 + \left|\tilde{\beta}^j_{22}\right|^2 - \left|\beta^j_{12}\right|^2 - \left|\beta^j_{22}\right|^2 & \text{charge of } \chi_2
\end{cases}
$$

$\kappa \equiv \frac{k}{k_*} = \frac{k}{\sqrt{g\Phi m}} \lesssim \frac{1}{\sqrt{\pi}} \quad \Longrightarrow$ in the resonance band

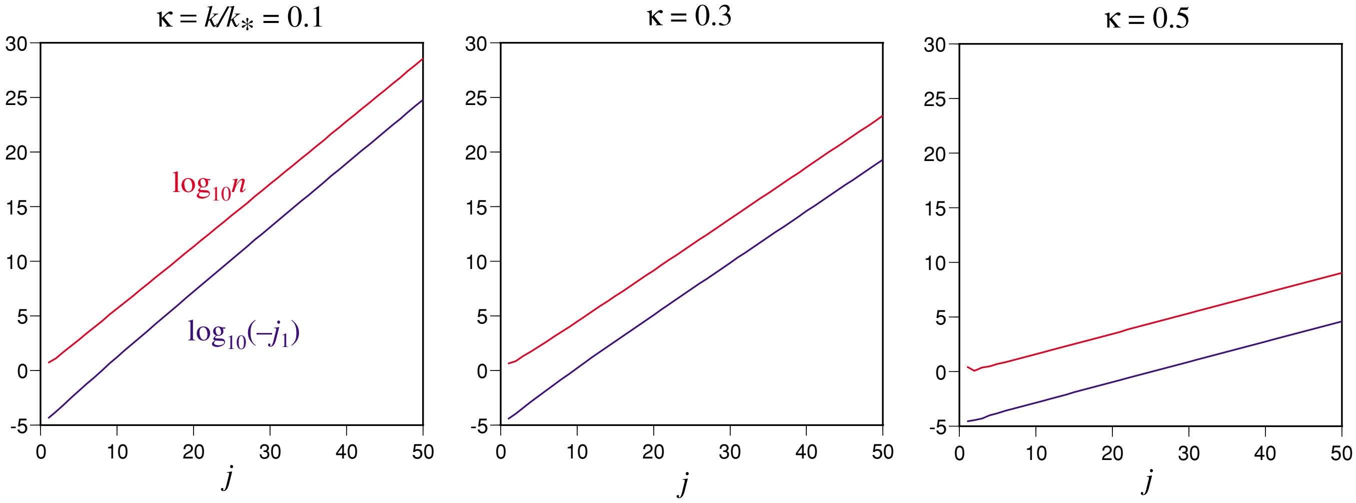
We choose

$$V_{11}(t) = -\frac{2g^2 l_1 \Phi^2}{\cosh^2[m(t-t_j)/\sqrt{2}]}$$

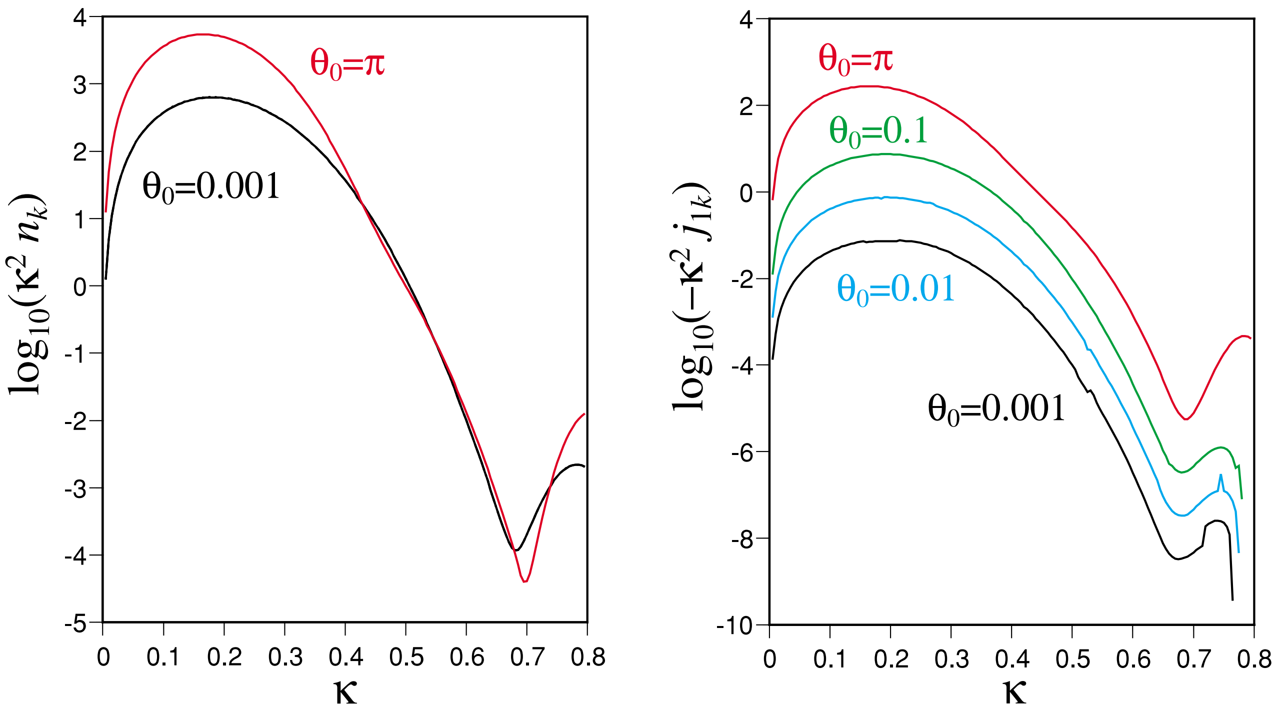
$$V_{12}(t) = -\frac{2g^2 l_2 \Phi^2\, e^{i\theta(t)}}{\cosh^2[m(t-t_j)/\sqrt{2}]}$$

with $\theta(t) = \frac{\theta_0}{1 + e^{m(t-t_j)/\sqrt{2}}}$

time evolution: $q = 200,\ l_1 = 0.01,\ l_2 = 0.02,\ \theta_0 = 10^{-3}$



densities at $j = 10$ for various θ_0



total number and charge densities

$$n = \int d^3\boldsymbol{k}\, n_k = 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, n_k,$$

$$j_1 = 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, j_{1k} = -j_2$$

θ_0	$\int d\kappa\, \kappa^2 n_k$	$\int d\kappa\, \kappa^2 j_{1k}$
10^{-3}	130.5096	-1.609334×10^{-2}
10^{-2}	130.5156	-1.544579×10^{-1}
10^{-1}	131.1163	-1.537716
π	990.7411	-50.84228

Applications — speculation

"charge" $= \begin{cases} B \\ L,\ Y, \cdots \overset{\text{sphaleron}}{\Longrightarrow} B \end{cases}$

oscillating $\phi(t)$ with (at least) 2 complex scalar fields: χ_a

$= \begin{cases} \text{inflaton} & \text{— } T_{\rm rh},\ \delta T/T \\ \text{Affleck-Dine scalar} \end{cases}$

e.g.

- a variant of EW hybrid inflation [Garcia-Bellido et al., PRD60]

  χ_a = 2 Higgs doublet

  "charge" $= Y$, $T_{\rm rh} \gtrsim T_{\rm EW}$ (sphaleron)

  — CP violation in the scalar sector

- Affleck-Dine B- or L-genesis

  initial $\langle \tilde{q} \rangle \propto e^{i\omega t} \longrightarrow j_B$, CP viol.

  Even when $\mathrm{Im}\langle \tilde{q} \rangle \simeq 0$, B can be generated

  χ_a = (quantum) $\tilde{q}$, $\tilde{l}$

Summary

⋆ Electroweak Baryogenesis
— depends on models checked by (future) experiments

▷ minimal SM — rejected: first-order EWPT, CP violation

▷ MSSM:
first-order EWPT for small (allowed) m_h and m_A
explicit CPV is needed

▷ other extended models such as NMSSM

⋆ Other Mechanism — not relying on 1st order EWPT

▷ GUTs

▷ Leptogenesis

▷ Affleck-Dine

▷ Inflationary

▷ ???