Phase Transitions in the NMSSM

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1. Introduction

Baryon Asymmetry of the Universe:

$$\frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} \simeq (0.48 - 0.98) \times 10^{-10}$$

$\longleftarrow$ BBN, consistent with WMAP data

Sakharov's Conditions
for baryogenesis

1. Baryon number violation
2. C and CP violation
3. Out of equilibrium

scenarios to explain the BAU

- GUTs
- Affleck-Dine
- Leptogenesis

$\star$ **EW Baryogenesis** — physics within our reach

⋮

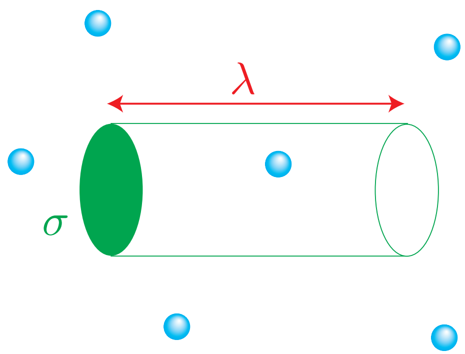
Anomalous $(B+L)$-nonconservation in EW theory

suppresed at $T=0$ by $e^{-2S_{\rm instanton}} \simeq 10^{-164}$

— free from proton decay problem

at $T < T_C$; $\Gamma^{(\rm br)}_{\rm sph} \simeq T e^{-E_{\rm sph}/T}$

at $T > T_C$; $\Gamma^{(\rm sym)}_{\rm sph} \simeq \kappa \alpha_W^4 T \qquad (\kappa \simeq 1.1)$



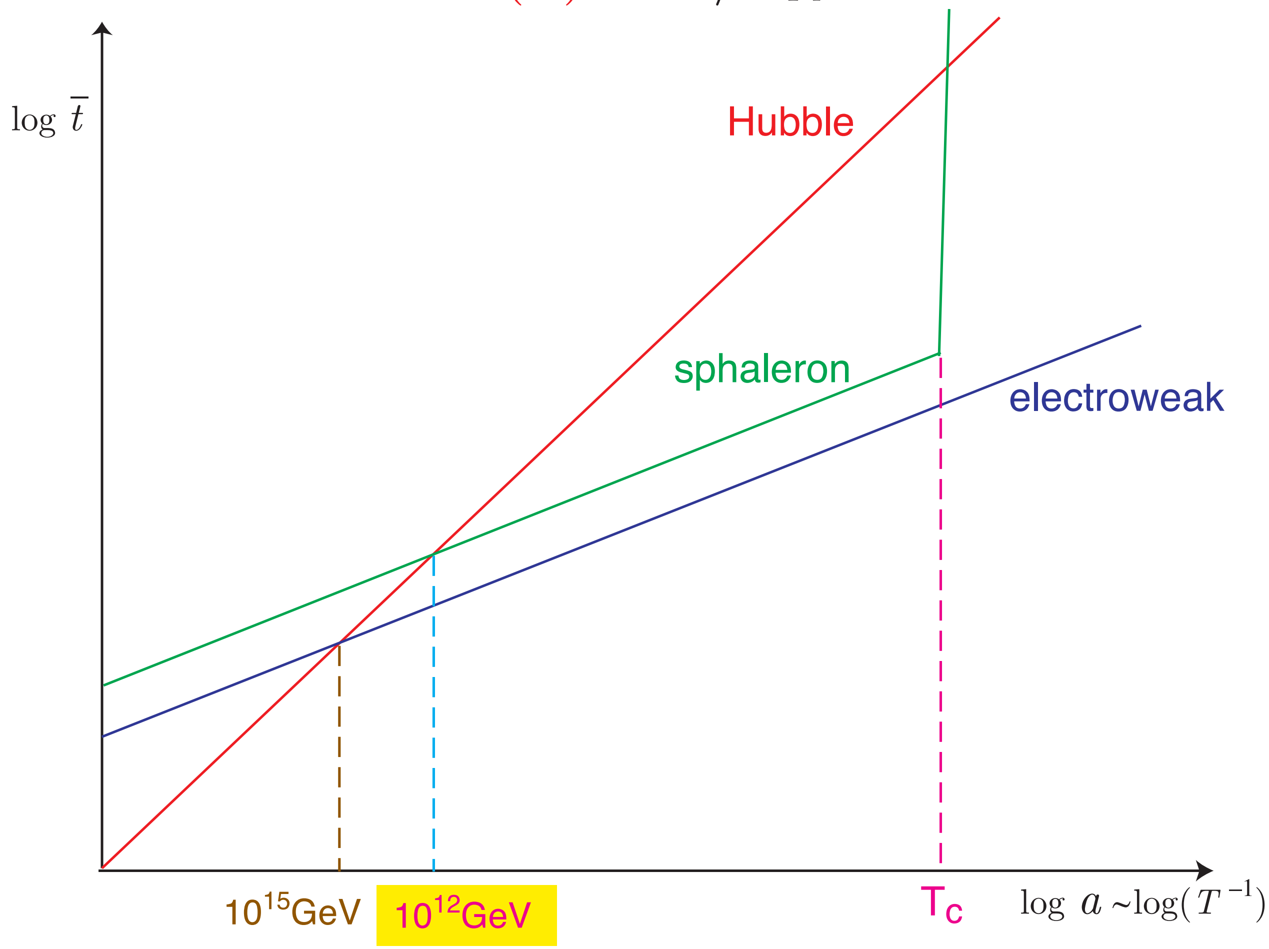
mfp (time scale) of elementary processes: $\lambda \cdot \sigma = \dfrac{1}{n}$

$m \ll T \Rightarrow \lambda \simeq \bar{t} =$ mean free time

For relativistic particles at T,

$$\sigma \simeq \frac{\alpha^2}{s} \simeq \frac{\alpha^2}{T^2} \Longrightarrow \boxed{\lambda \simeq \frac{10}{gT^3}\left(\frac{\alpha^2}{T^2}\right)^{-1} = \frac{10}{g\alpha^2 T}}$$

$H(T) \propto T^2/m_{\rm Pl}$



If $v(T_C) \ll 200\text{GeV}$ (eg. 2nd order EWPT), $^{\exists}T_{\rm dec}$, $s.t.$

$$T_{\rm dec} < T < T_C \implies \Gamma^{(b)}_{\rm sph}(T) > H(T)$$

wash-out of $B + L$ even in the broken phase

To have nonzero BAU,

(i) we must have $B-L$ before the sphaleron process decouples, or

(ii) $B+L$ must be created at the first-order EWPT, *and* the sphaleron process must decouple immediately after that.

N.B.

$\Delta(B+L) \neq 0$ process is in equilibrium, for $T_C \simeq 100\text{GeV} < T < 10^{12}\text{GeV}$.

If $\Delta L \neq 0$ process is in equilibrium in this range of T $\Rightarrow$ $B = L = 0!$

To leave $B \neq 0$, $\Gamma_{\Delta L \neq 0} < H(T)$ for $T \in [T_C, 10^{12}\text{GeV}]$.

$\Longrightarrow$ constraints on models with $\Delta L \neq 0$ processes.

e.g., lower bound on m_N in the seesaw model

$\rightarrow$ upper bound on $m_\nu < 0.8\text{GeV}$

$$T \simeq 100\text{GeV} \Rightarrow \boxed{H^{-1}(T) \simeq 10^{14}\text{GeV}^{-1} \ll \bar{t}_{EW} \simeq \frac{1}{\alpha_W T^2} \sim 10\text{GeV}^{-1}}$$

$\therefore$ All the particles of the SM are in *kinetic* equilibrium.

nonequilibrium state $\Longleftarrow$ **1st order EW phase transition**

study of the EWPT

$\star$ static properties $\Leftarrow$ effective potential $=$ free energy density

$$V_{\text{eff}}(v;T) = -\frac{1}{V} T \log Z = -\frac{1}{V} \log \text{Tr}\left[e^{-H/T}\right]_{\langle\phi\rangle = v}$$

$\star$ dynamics — formation and motion of the bubble wall when 1st order PT

$V_{\text{eff}}(v;T)$ $\Leftarrow$ paramters of the model $\Rightarrow$ mass, coupling of the Higgs bosons

contents

2. Higgs mass and the EWPT in the MSM

perturbation at the 1-loop level

$$V_{\rm eff}(\varphi;T) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2Bv_0^2\varphi^2 + B\varphi^4\left[\log\left(\frac{\varphi^2}{v_0^2}\right) - \frac{3}{2}\right] + \bar{V}(\varphi;T)$$

where $$B = \frac{3}{64\pi^2 v_0^4}(2m_W^4 + m_Z^4 - 4m_t^4),$$

$$\bar{V}(\varphi;T) = \frac{T^4}{2\pi^2}\left[6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)\right], \qquad (a_A = m_A(\varphi)/T)$$

$$I_{B,F}(a^2) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right).$$

high-temperature expansion $[m/T \ll 1]$

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right)\frac{a^4}{2} + O(a^6)$$

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6)$$

For $T > m_W, m_Z, m_t$,

$$V_{\rm eff}(\varphi; T) \simeq D(T^2 - T_0^2)\varphi^2 - E\,T\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2), \qquad E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4}\left(2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right)$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2), \qquad \log\alpha_{F(B)} = 2\log(4)\pi - 2\gamma_E$$

At T_C, $^{\exists}$degenerate minima: $\varphi_C = \dfrac{2E\,T_C}{\lambda_{T_C}}$

$\Gamma_{\rm sph}^{(\rm br)} < H(T_C) \iff \dfrac{\varphi_C}{T_C} \gtrsim 1$ $\Longrightarrow$ upper bound on λ $\qquad [m_H = \sqrt{2\lambda}v_0]$

$m_H \lesssim 46\text{GeV}$ $\Longrightarrow$ MSM is excluded

⋆ Monte Carlo simulations

effective fermion mass : $m_f(T) \sim O(T)$ ← nonzero modes

∴ simulation only with the bosons

QFT on the lattice $\left\{ \begin{array}{ll} \text{scalar fields:} & \phi(x) \text{ on the sites} \\ \text{gauge fields:} & U_\mu(x) \text{ on the links} \end{array} \right.$

$$Z = \int [d\phi\, dU_\mu] \exp\{-S_E[\phi, U_\mu]\}$$

- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet   time-component of U_μ
  [Laine & Rummukainen, hep-lat/9809045]

- 4-dim. $SU(2)$ system with a Higgs doublet   [Csikor, hep-lat/9910354]
  EWPT is first order for $m_h < 66.5 \pm 1.4$GeV

Both the simulations found end-point of EWPT at

$$m_h = \left\{ \begin{array}{l} 72.3 \pm 0.7 \text{ GeV} \\ 72.1 \pm 1.4 \text{ GeV} \end{array} \right. \Rightarrow \boxed{\text{no PT (cross-over) in the MSM !}}$$

3. EWPT in the MSSM

superpotential: $$W = y_b Q_L B_R^c H_d - y_t Q_L T_R^c H_u - \mu H_d H_u$$

2 Higgs doublets: $H_d \ni \Phi_d = \begin{pmatrix} \phi_d^0 \\ \phi_d^- \end{pmatrix}, \qquad H_u \ni \Phi_u = \begin{pmatrix} \phi_u^+ \\ \phi_d^0 \end{pmatrix}$

Higgs potential

$$V_0 = m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u - \left(m_3^2 \epsilon_{ij} \Phi_d^i \Phi_u^j + \text{h.c.}\right) + \frac{g_2^2 + g_1^2}{8}\left(\Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u\right)^2 + \frac{g_2^2}{2}\left|\Phi_d^\dagger \Phi_u\right|^2$$

all the parameters are real: no CP violation

$m_{1,2}^2 = m_{\text{soft}}^2 + |\mu|^2 \leftrightarrow v_0$ and $\tan\beta$ by $\frac{\partial V_0}{\partial v_d} = \frac{\partial V_0}{\partial v_u} = 0$

The Higgs mass is not completely a free parameter.

After EWSB $\longrightarrow$ $\phi_d^0 = \frac{1}{\sqrt{2}}(v_d + h_d + ia_d), \quad \phi_u^0 = \frac{1}{\sqrt{2}}e^{i\theta}(v_u + h_u + ia_u)$

vacuum: $v_0 = \sqrt{v_d^2 + v_u^2} = 246\text{GeV}$, $\tan\beta = v_u/v_d$

1 Nambu-Goldstone mode in (a_d, a_u) and 1 in (ϕ_d^+, ϕ_u^-)

$\Longrightarrow$ physical modes: 3 neutral (h, H, A), 1 charged $(H^\pm)$

tree-level masses

$$m_{h,H}^2 = \frac{1}{2}\left[m_Z^2 + m_A^2 \mp \sqrt{(m_Z^2 + m_A^2)^2 - 4m_Z^2 m_Z^2 \cos^2(2\beta)}\right],$$

$$m_A^2 = \frac{\mathrm{Re}(m_3^2 e^{i\theta})}{\sin\beta\cos\beta}, \qquad m_{H^\pm}^2 = m_A^2 + m_W^2$$

$\longrightarrow$ $m_h \leq \min\{m_Z, m_A\}, \quad m_H \geq \max\{m_Z, m_A\}$

radiative collections from loops of the top quarks and squarks

$\longrightarrow$ $m_h \lesssim 135\text{GeV}$ [Okada, et al. PTP85 ('91) 1]

One-loop Effective potential $(T=0)$

$$V_{\rm eff} = V_0 + \frac{N_C}{32\pi^2}\sum_{q=t,b}\left[\sum_{j=1,2}\left(\bar{m}^2_{\tilde{q}_j}\right)^2\left(\log\frac{\bar{m}^2_{\tilde{q}_j}}{M^2}-\frac{3}{2}\right) - 2\left(\bar{m}^2_q\right)^2\left(\log\frac{\bar{m}^2_q}{M^2}-\frac{3}{2}\right)\right]$$

$\bar{m}^2(v_d, v_u, \theta)$: field-dependent mass

mass^2 at the one-loop level

$$\mathcal{M}^2 = \begin{pmatrix} \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d^2} \right\rangle & \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d \partial h_u} \right\rangle & \frac{1}{\cos\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d \partial a_u} \right\rangle \\ \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d \partial h_u} \right\rangle & \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_u^2} \right\rangle & \frac{1}{\sin\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_u \partial a_d} \right\rangle \\ \frac{1}{\cos\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d \partial a_u} \right\rangle & \frac{1}{\sin\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_u \partial a_d} \right\rangle & \frac{1}{\sin\beta\cos\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial a_d \partial a_u} \right\rangle \end{pmatrix}$$

$$m^2_{H^\pm} = \frac{1}{\sin\beta\cos\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial \phi_d^+ \partial \phi_u^-} \right\rangle$$

$\langle\cdots\rangle$ = values at the vacuum

CP-conserving $\Rightarrow \mathcal{M}^2_{33} = m_A^2$

CP violation in the squark sector $\propto {\rm Im}\left(\mu A_q e^{i\theta}\right) \Rightarrow \mathcal{M}_{13}, \mathcal{M}_{23} \neq 0$

⋆ Electroweak phase transition

$$V_{\rm eff}(\boldsymbol{v};T) = V_{\rm eff}(\boldsymbol{v};T) + 6\sum_{q=t,b}\sum_{j=1,2}\frac{T^4}{2\pi^2}I_B\left(\frac{\bar{m}_{\tilde{q}_j}}{T}\right) + \cdots,$$

where $m^2_{\tilde{t}_j}$ is the eigenvalues of

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}_L} + \left(\frac{g_1^2}{24} - \frac{g_2^2}{8}\right)(v_u^2 - v_d^2) + \frac{y_t^2}{2}v_u^2 & \frac{y_t}{\sqrt{2}}\left(\mu v_d + A(v_2 - iv_3)\right) \\ * & m^2_{\tilde{t}_R} - \frac{g_1^2}{6}(v_u^2 - v_d^2) + \frac{y_t^2}{2}v_u^2 \end{pmatrix}$$

light stop scenario [de Carlos & Espinosa, NPB '97]

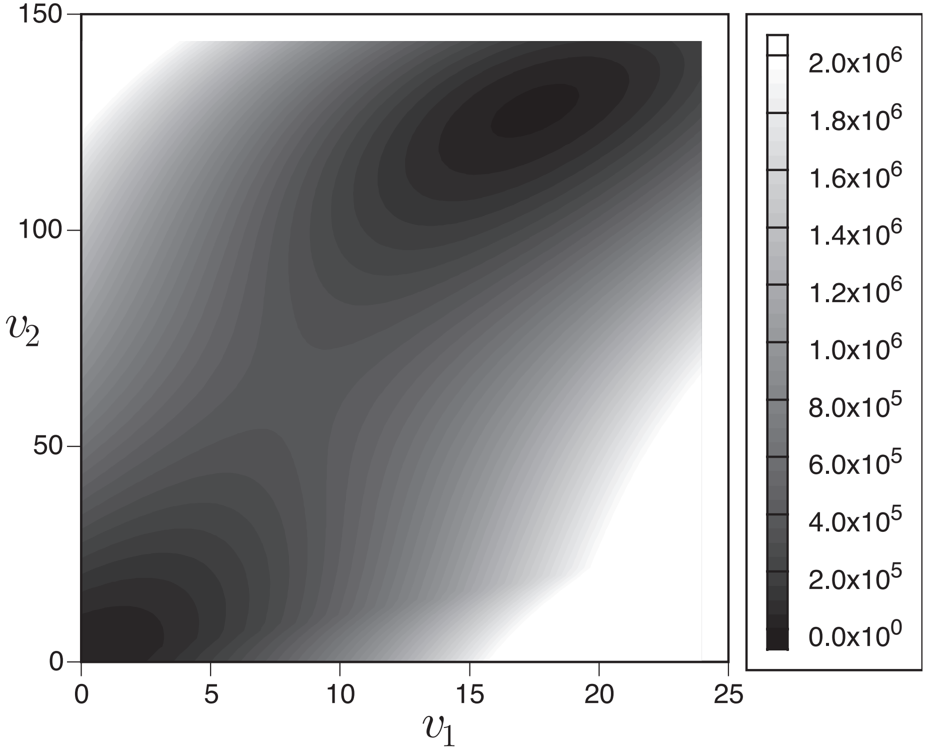
$m^2_{\tilde{t}_L} = 0$ or $m^2_{\tilde{t}_R} = 0 \Longrightarrow$ smaller eigenvalue: $m^2_{\tilde{t}_1} \sim O(v^2)$

$\therefore$ high-T expansion: $\Delta_{\tilde{t}}V_{\rm eff}(\boldsymbol{v};T) \Rightarrow -3\,\frac{T}{6\pi}(m^2_{\tilde{t}_1})^{3/2} \sim -Tv^3$ $\quad\rightarrow$ 1st order PT

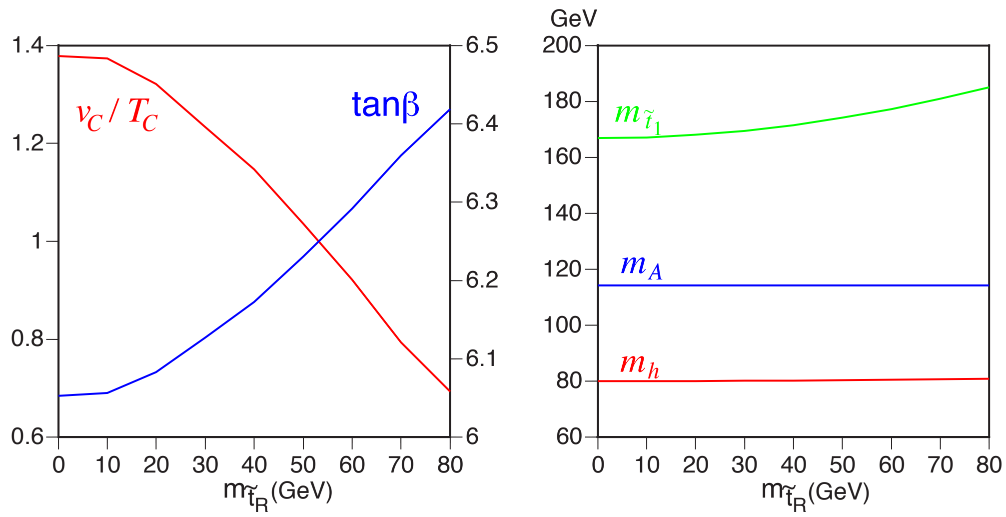
more effective for larger y_t — smaller $\tan\beta$

An example: $\tan\beta = 6$, $m_h = 82.3$GeV, $m_A = 118$GeV, $m_{\tilde{t}_1} = 168$GeV

$T_C = 93.4$GeV, $v_C = 129$GeV [KF, PTP101('99)]



$V_{\rm eff}(v_1, v_2, v_3 = 0; T_C)$



$m_{\tilde{t}_R}$-dependence $(\tan\beta = 6)$

$\star$ Lattice MC studies

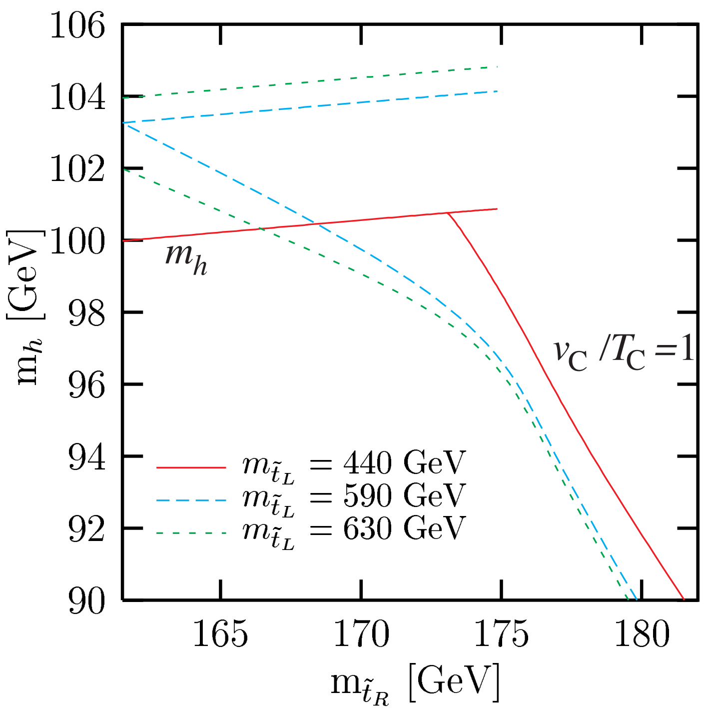
- 3d reduced model [Laine et al. hep-lat/9809045]

  strong 1st order for $m_{\tilde{t}_1} \lesssim m_t$ and $m_h \leq 110\text{GeV}$

- 4d model [Csikor, et al. hep-lat/0001087]

  with $SU(3)$, $SU(2)$ gauge bosons, 2 Higgs doublets, stops, sbottoms

  $A_{t,b} = 0$, $\tan\beta \simeq 6$ $\longrightarrow$ agreement with the perturbation theory within the errors

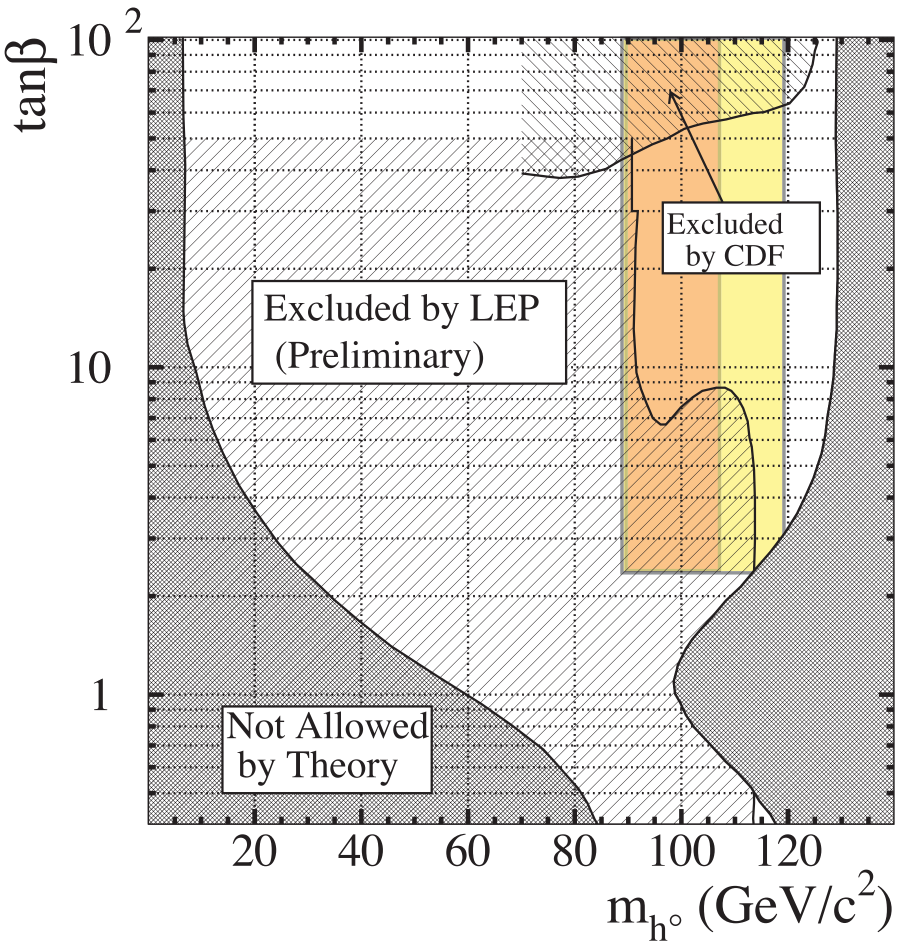


$m_A = 500$ GeV

$v_C/T_C > 1$
below the steeper lines

$\Downarrow$

max. $m_h = 103 \pm 4$ GeV
for $m_{\tilde{t}_L} \simeq 560$ GeV



[PDG, http://ccwww.kek.jp/pdg/]

light stop: $m_{t_R} = 0$

negative soft mass^2: $m^2_{t_R} > -(65\text{GeV})^2$

[Laine & Rummukainen, NPB597]

3. EWPT in the NMSSM

$$W = \epsilon_{ij} \left(y_b H_d^i Q^j B - y_t H_u^i Q^j T + y_l H_d^i L^j E - \lambda N H_d^i H_u^j \right) - \frac{\kappa}{3} N^3$$

$\lambda \langle N \rangle \sim \mu$ in the MSSM

$$\mathcal{L}_{\rm soft} \ni -m_N^2 n^* n + \left[\lambda A_\lambda n H_d H_u + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.} \right].$$

order parameters: $\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{0d} \\ 0 \end{pmatrix}, \quad \langle \Phi_u \rangle = \frac{e^{i\theta_0}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{0u} \end{pmatrix}, \quad \langle n \rangle = \frac{e^{i\varphi_0}}{\sqrt{2}} v_{0n}$

tree-level Higgs potential:

$$\begin{aligned} V_0 \quad = \quad & m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u + m_N^2 n^* n - \left(\lambda A_\lambda \epsilon_{ij} n \Phi_d^i \Phi_u^j + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.} \right) \\ & + \frac{g_2^2 + g_1^2}{8} \left(\Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u \right)^2 + \frac{g_2^2}{2} \left| \Phi_d^\dagger \Phi_u \right|^2 \\ & + |\lambda|^2 n^* n \left(\Phi_d^\dagger \Phi_d + \Phi_u^\dagger \Phi_u \right) + \left| \lambda \epsilon_{ij} \Phi_d^i \Phi_u^j + \kappa n^2 \right|^2 \end{aligned}$$

1. 5 neutral and 1 charged scalars (3 scalar and 2 pseudoscalar when CP cons.)

   The lightest Higgs scalar can escape from the lower bound 114GeV, because of small coupling to Z, W caused by large mixing among 3 scalars. [Miller, et al. NPB 681]

   — "Light Higgs Scenario" —

2. CP violation at the tree level: $\mathrm{Im}\left(\lambda A_\lambda e^{i(\theta+\varphi)}\right)$, $\mathrm{Im}\left(\kappa A_\kappa e^{3i\varphi}\right)$, $\mathrm{Im}\left(\lambda\kappa^* e^{i(\theta-2\varphi)}\right)$

3. $v_n \to \infty$ with λv_n and κv_n fixed $\Longrightarrow$ MSSM [Ellis, et al, PRD 39]

   $\longrightarrow$ new features expected for $v_n = O(100)$GeV

⋆ study of the Higgs spectrum and couplings without/with CP violation [KF and Tao, PTP 113 ('05)]

⋆ study of the EWPT without/with CP violation [KF, Toyoda and Tao, hep-ph/0501052]

⋆ sphaleron solution [KF, Kakuto, Tao, Toyoda, hep-ph/0506156] mass matrix of the neutral Higgs bosons

$$\mathcal{M}^2 \equiv \begin{pmatrix} \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_i \partial h_j} \right\rangle & \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_i \partial a_j} \right\rangle \\ \left\langle \frac{\partial^2 V_{\rm eff}}{\partial a_i \partial h_j} \right\rangle & \left\langle \frac{\partial^2 V_{\rm eff}}{\partial a_i \partial a_j} \right\rangle \end{pmatrix} \xrightarrow{\text{extract NG modes}} \begin{pmatrix} \mathcal{M}_S^2 & \mathcal{M}_{SP}^2 \\ (\mathcal{M}_{SP}^2)^T & \mathcal{M}_P^2 \end{pmatrix}$$

$$\mathcal{M}_S^2 : \ 3 \times 3, \qquad \mathcal{M}_P^2 : \ 2 \times 2. \qquad \mathcal{M}_{SP}^2 : \ 3 \times 2$$

where the basis is (h_d, h_u, h_n, a, a_n),

$$\mathcal{M}_{SP}^2 \propto \begin{cases} \mathrm{Im}\left(\lambda \kappa^* e^{i(\theta_0 - 2\varphi_0)}\right) & \text{at the tree level} \\ \mathrm{Im}\left(\lambda v_n A_{t,b} e^{i(\theta_0 + \varphi_0)}\right) & \text{at the one-loop level} \end{cases}$$

charged Higgs mass

$$m_{H^\pm}^2 = \frac{1}{\sin\beta_0 \cos\beta_0} \left\langle \frac{\partial^2 V_{\rm eff}}{\partial \phi_d^+ \partial \phi_u^-} \right\rangle = \left(m_{H^\pm}^2\right)_{\mu = \lambda v_n e^{i\varphi_0}/\sqrt{2}}^{\rm MSSM}$$

At the tree-level,

$$\mathcal{M}_S^2 = \begin{pmatrix} \left(R_\lambda - \frac{\mathcal{R} v_n}{2}\right) v_n \tan\beta + m_Z^2 \cos^2\beta & -\left(R_\lambda - \frac{\mathcal{R} v_n}{2}\right) v_n - m_Z^2 \sin\beta\cos\beta + |\lambda|^2 v_d v_u & -R_\lambda v_u + \mathcal{R} v_u v_n + |\lambda|^2 v_d v_n \\ \cdots & \left(R_\lambda - \frac{\mathcal{R} v_n}{2}\right) v_n \cot\beta + m_Z^2 \sin^2\beta & -R_\lambda v_d + \mathcal{R} v_d v_n + |\lambda|^2 v_u v_n \\ \cdots & \cdots & R_\lambda \frac{v_d v_u}{v_n} + 3 R_\kappa v_n + 2|\kappa|^2 v_n^2 \end{pmatrix},$$

$$\mathcal{M}_P^2 = \begin{pmatrix} \left(R_\lambda - \frac{1}{2}\mathcal{R} v_n\right) \frac{v_n}{\sin\beta\cos\beta} & (R_\lambda + \mathcal{R} v_n) v_0 \\ (R_\lambda + \mathcal{R} v_n) v_0 & R_\lambda \frac{v_0^2 \sin\beta\cos\beta}{v_n} + 3 R_\kappa v_n - 2\mathcal{R} v_d v_u \end{pmatrix},$$

$$\mathcal{M}_{SP}^2 = \begin{pmatrix} 0 & \frac{3}{2}\sin\beta \\ 0 & \frac{3}{2}\cos\beta \\ -\frac{1}{2} & -2\sin\beta\cos\beta \end{pmatrix} \mathcal{I} v_0 v_n .$$

where we have defined

$$R_\lambda = \frac{1}{\sqrt{2}} \mathrm{Re}\left(\lambda A_\lambda e^{i(\theta_0 + \varphi_0)}\right), \qquad I_\lambda = \frac{1}{\sqrt{2}} \mathrm{Im}\left(\lambda A_\lambda e^{i(\theta_0 + \varphi_0)}\right),$$

$$R_\kappa = \frac{1}{\sqrt{2}} \mathrm{Re}\left(\kappa A_\kappa e^{3i\varphi_0}\right), \qquad I_\kappa = \frac{1}{\sqrt{2}} \mathrm{Im}\left(\kappa A_\kappa e^{3i\varphi_0}\right),$$

$$\mathcal{R} = \mathrm{Re}\left(\lambda\kappa^* e^{i(\theta_0 - 2\varphi_0)}\right), \qquad \mathcal{I} = \mathrm{Im}\left(\lambda\kappa^* e^{i(\theta_0 - 2\varphi_0)}\right)$$

— independent of phase convention

We have used the tadpole conditions: $\left\langle \frac{\partial V_{\rm eff}}{\partial h_i} \right\rangle = \left\langle \frac{\partial V_{\rm eff}}{\partial a_i} \right\rangle = 0$ $\quad (i = d,\, u,\, n)$

$$
\begin{aligned}
m_1^2 &= \left(R_\lambda - \frac{1}{2}\mathcal{R} v_{0n} \right) v_{0n} \tan\beta_0 - \frac{1}{2} m_Z^2 \cos(2\beta_0) - \frac{|\lambda|^2}{2}(v_{0n}^2 + v_{0u}^2) + \cdots \\
m_2^2 &= \left(R_\lambda - \frac{1}{2}\mathcal{R} v_{0n} \right) v_{0n} \cot\beta_0 + \frac{1}{2} m_Z^2 \cos(2\beta_0) - \frac{|\lambda|^2}{2}(v_{0n}^2 + v_{0d}^2) + \cdots \\
m_N^2 &= (R_\lambda - \mathcal{R} v_{0n}) \frac{v_{0d} v_{0u}}{v_{0n}} + R_\kappa v_{0n} - \frac{|\lambda|^2}{2}(v_{0d}^2 + v_{0u}^2) - |\kappa|^2 v_{0n}^2 + \cdots \\
I_\lambda &= \frac{1}{2}\mathcal{I} v_{0n} + \cdots, \qquad I_\kappa = -\frac{3}{2}\mathcal{I}\frac{v_{0d} v_{0u}}{v_{0n}}
\end{aligned}
$$

We shall use $m_{H^\pm}$ instead of R_λ:

$$
m_{H^\pm}^2 = m_W^2 - \frac{1}{2}|\lambda|^2 v^2 + (2R_\lambda - \mathcal{R} v_{0n}) \frac{v_{0n}}{\sin 2\beta_0} + \cdots
$$

Definition of the couplings

gauge vs mass eigenstates: $$\begin{pmatrix} h_d \\ h_u \\ h_n \\ a \\ a_n \end{pmatrix} = \mathcal{O} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{pmatrix}, \qquad \mathcal{O}^T \mathcal{M}^2 \mathcal{O} = \mathrm{diag}(m_{H_1}^2, \cdots, m_{H_5}^2)$$

$$\mathcal{L}_{\rm gauge} \ni g_2 m_W g_{VVH_i} \left(W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu \right) H_i + \frac{g_2}{2\cos\theta_W} g_{ZH_iH_j} Z^\mu (H_i \overleftrightarrow{\partial}_\mu H_j)$$

$$\mathcal{L}_{\rm Yukawa} \ni -\frac{g_2 m_b}{2 m_W} \bar{b}(g_{bbH_i}^S + i\gamma^5 g_{bbH_i}^P) b\, H_i$$

$$\begin{cases} g_{VVH_i} = \mathcal{O}_{1i}\cos\beta + \mathcal{O}_{2i}\sin\beta \\ g_{ZH_iH_j} = \frac{1}{2}\{(\mathcal{O}_{4i}\mathcal{O}_{2j} - \mathcal{O}_{4j}\mathcal{O}_{2i})\cos\beta - (\mathcal{O}_{4i}\mathcal{O}_{1j} - \mathcal{O}_{4j}\mathcal{O}_{1i})\sin\beta\} \\ g_{bbH_i}^S = \mathcal{O}_{1i}\frac{1}{\cos\beta}, \qquad g_{bbH_i}^P = -\mathcal{O}_{4i}\tan\beta \end{cases}$$

$$g_{bbH_i}^2 \equiv \left(g_{bbH_i}^S\right)^2 + \left(g_{bbH_i}^P\right)^2$$

⋆ MSSM vs NMSSM

tree-level mass relation (CP-conserving)

MSSM	NMSSM
$m_h \leq \min\{m_A, m_Z\}$ $m_H \geq \max\{m_A, m_Z\}$ $m_{H^\pm}^2 = m_A^2 + m_W^2$	$m_{A_1} < \hat{m} < m_{A_2}$ For $\hat{m} \gg v_0, v_n$, $m_{S_1} < m_{S_2} < \hat{m} < m_{S_3}$ $\hat{m}^2 = m_{H^\pm}^2 - m_W^2 + \lvert\lambda\rvert^2 v_0^2/2$

tree-level vacuum

MSSM	NMSSM
The tadpole condition $\left\langle \frac{\partial V_0}{\partial \varphi_i} \right\rangle = 0$ is sufficient for the EW vacumm (v_{0d}, v_{0u}) to be the global minimum of the potential.	Even if the tadpole conditions are satisfied, the prescribe vacuum (v_{0d}, v_{0u}, v_{0n}) is *not* always the global minimum. $\leftarrow$ cubic terms in V_0

Although the NMSSM has more parameters than the MSSM,
it must satisfy more constraints than the MSSM.

$$\lambda, \kappa, A_\lambda, A_\kappa, m_N^2$$

$\star$ Constraints on the parameters

1. **vacuum condition**

   The vacuum $(v_0, v_{0n}, \tan\beta_0, \theta_0, \varphi_0)$ be the global minimum of $V_{\rm eff}$.

2. **spectrum condition**

   The neutral Higgs boson with $|g_{VVH}| > 0.1$ be heavier than 114GeV.

We scanned the parameter space for (CP-conserving case)

$$\tan\beta_0 = 2 - 10,\ v_{0n} = 100 - 1000\text{GeV},\ m_{H^\pm} = 100 - 5000\text{GeV},$$

$$-1000\text{GeV} \le A_\kappa \le 0,\ 0 \le \lambda \le 1,\ -1 \le \kappa \le 1$$

$$(m_{\tilde{q}},\ m_{\tilde{t}_R} = m_{\tilde{b}_R}) = \begin{cases} (1000\text{GeV}, 800\text{GeV}) & \text{heavy-squark} \\ (1000\text{GeV}, 10\text{GeV}) & \text{light-squark-1} \\ (500\text{GeV}, 10\text{GeV}) & \text{light-squark-2} \end{cases}$$

$$A_t = A_b = 20\text{GeV}$$

A necessary condition for the vacuum condition: $V_{\text{eff}}(\boldsymbol{v}_0) < V_{\text{eff}}(\mathbf{0})$

$\Downarrow$

$$m_{H^\pm}^2 \quad < \quad \frac{2|\lambda|^2 v_{0n}^2}{\sin^2 2\beta_0} + \frac{2|\kappa|^2 v_{0n}^4}{v_0^2 \sin^2 2\beta_0} + \frac{\mathcal{R} v_{0n}^2}{\sin 2\beta} - \frac{4 R_\kappa v_{0n}^3}{3 v_0^2 \sin^2 2\beta_0}$$
$$+ m_Z^2 \cot^2 2\beta_0 + m_W^2$$

for fixed (λ, κ), upper bound on $m_{H^\pm}$

this bound becomes irrelevant in the MSSM-limit

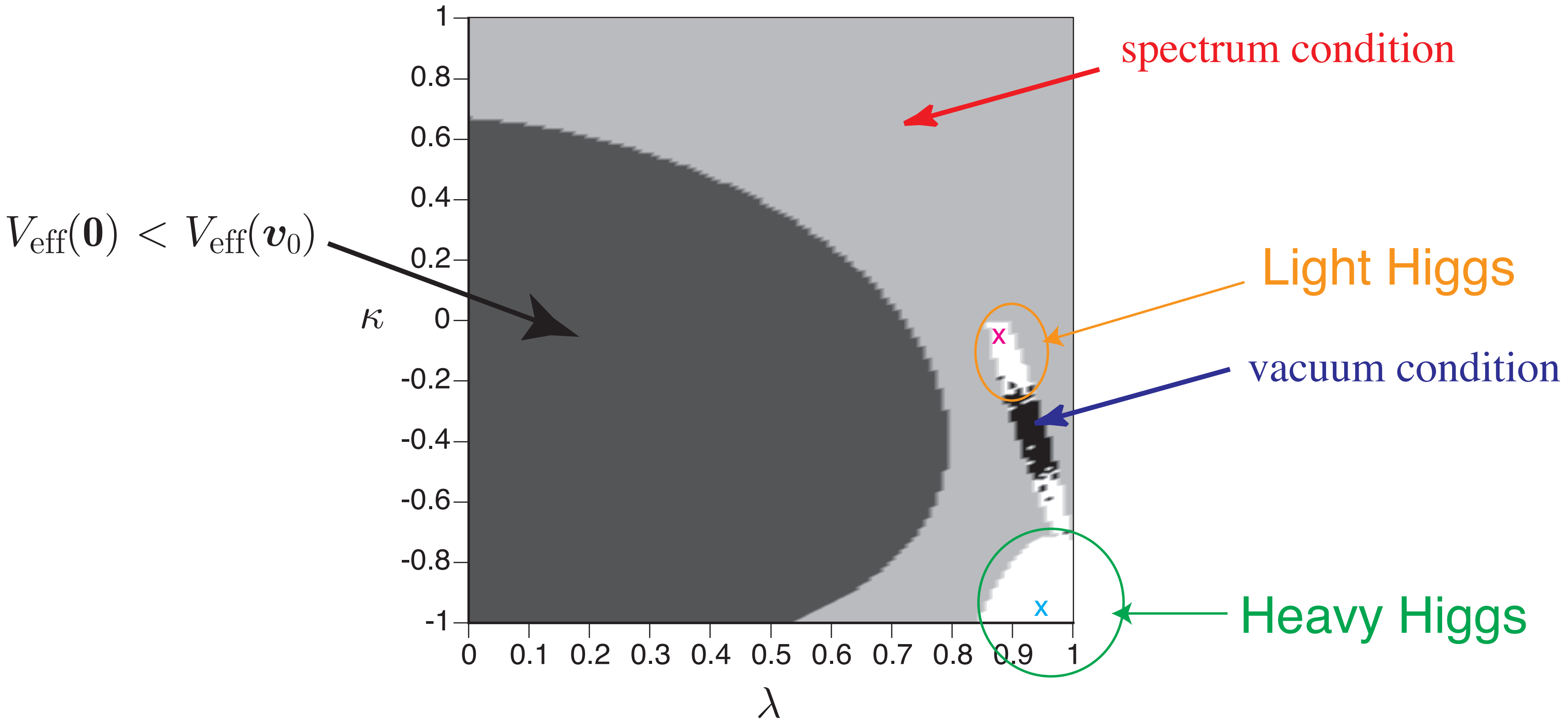
$v_{0n} \to \infty$ with λv_{0n} and κv_{0n} fixed

for fixed $m_{H^\pm}$, an elliptic region in (λ, κ)-plane is excluded

the region shrinks to a point for $v_{0n} \to \infty$

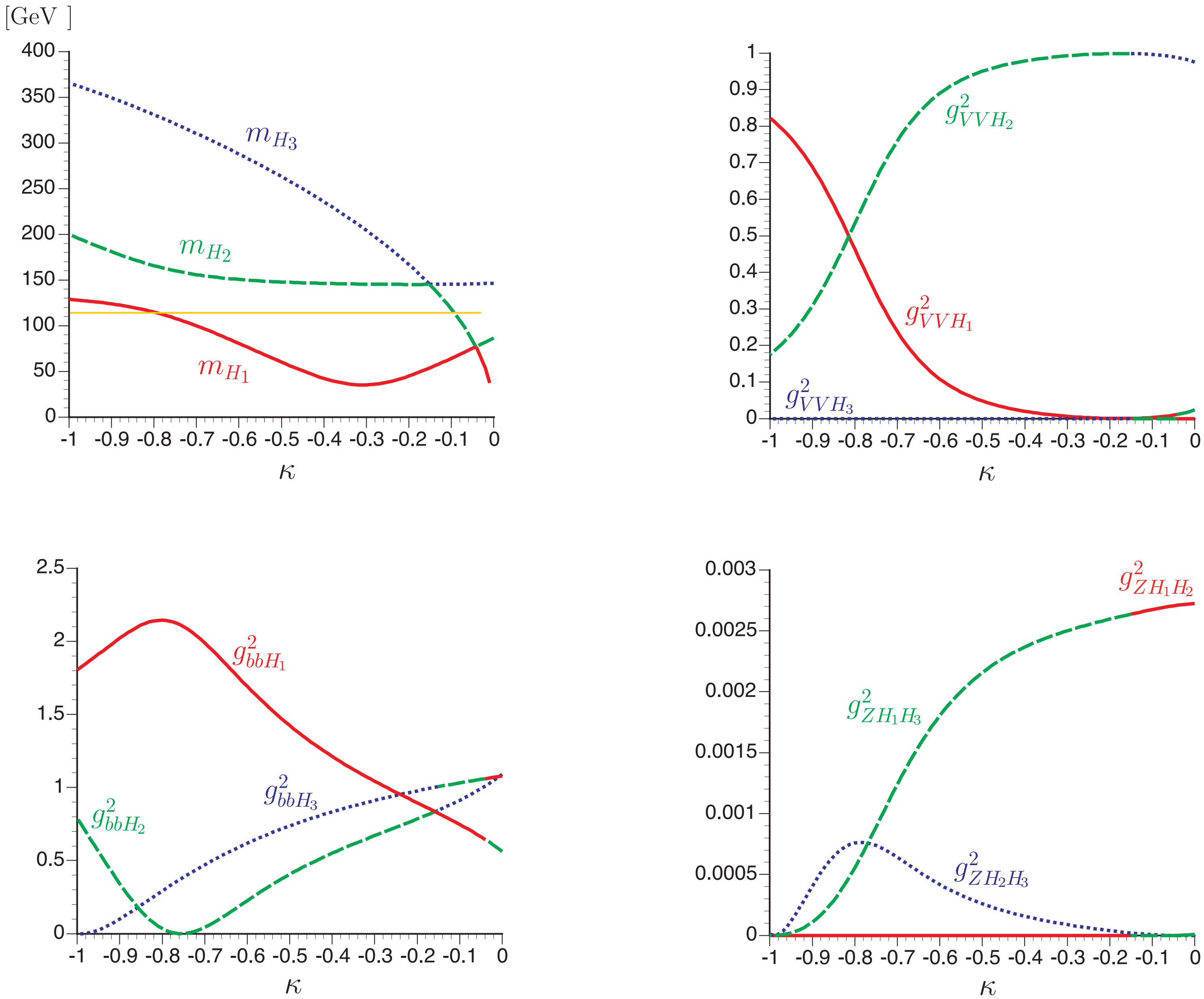
We excuted *numerical search* for the global minimum of V_{eff} to ensure the vacuum condition.

$e.g.$, $\tan\beta_0 = 3$, $v_{0n} = 200$GeV, $m_{H^\pm} = 400$GeV, $A_\kappa = -200$GeV, heavy squark



λ	κ	m_{H_1}	m_{H_2}	m_{H_3}	m_{H_4}	m_{H_5}	$g^2_{VVH_1}$	$g^2_{VVH_2}$	$g^2_{VVH_3}$	$g^2_{VVH_4}$	$g^2_{VVH_5}$
0.9	-0.05	75.0	83.5	145.7	436.0	448.2	0.0094	0	0.9897	0.0009	0
0.95	-0.95	139.6	180.9	360.3	425.4	456.4	0.828	0.170	0	0	0.0028

along the line of $\lambda = 0.9$,



⋆ Phase transitions in the NMSSM

There has been a belief that the EWPT in the NMSSM is strongly first-order because of the cubic terms in the Higgs potential.

naive (?) argument [Pietroni, NPB402('93)27]

order parameters :
$$\begin{cases} v_d = v\cos\beta(T) = y\cos\alpha(T)\cos\beta(T), \\ v_u = v\sin\beta(T) = y\cos\alpha(T)\sin\beta(T), \\ v_n = y\sin\alpha(T) \end{cases}$$

$$V_0 = \frac{1}{2}\left((m_1^2\cos^2\beta + m_2^2\sin^2\beta)\cos^2\alpha + m_N^2\sin^2\alpha\right)y^2 - \left(R_\lambda\cos^2\alpha\sin\alpha\cos\beta\sin\beta + \frac{1}{3}R_\kappa\sin^3\alpha\right)y^3 + \cdots.$$

strongly 1st order PT by the tree-level cubic term ?

Is such a parametrization valid ?

No!

$\because$ **no symmetry** between the doublets and the singlet

$$\boxed{\begin{array}{c}\text{order of phase transitions}\\ (\textit{universality class})\end{array}} \Longleftarrow \left\{\begin{array}{l}\text{dimension of spacetime}\\ \text{symmetry of the system}\end{array}\right.$$

Indeed, we have found various phases and transitions among them.

possible phases and transitions

phase	order parameters	symmetries
EW	$v \neq 0,\ v_n \neq 0$	fully broken
I, I′	$v = 0,\ v_n \neq 0$	local $SU(2)_L \times U(1)_Y$
II	$v \neq 0,\ v_n = 0$	global $U(1)$
SYM	$v = v_n = 0$	$SU(2)_L \times U(1)_Y$, global $U(1)$

global $U(1)$: $v_u e^{i\theta} = v_2 + iv_3 \mapsto e^{i\alpha}(v_2 + iv_3)$ in the subspace of $v_n = 0$

phase-I : heavy Higgs   phase-I′: light Higgs

4 types of phase transitions

A: SYM $\rightarrow$ I $\Rightarrow$ EW   B: SYM $\rightarrow$ I′ $\Rightarrow$ EW

C: SYM $\Rightarrow$ II $\rightarrow$ EW   D: SYM $\Rightarrow$ EW

"$\Rightarrow$" : EWPT

expamles of the phase transitions in the CP-conserving case

common parameters: $\tan\beta_0 = 5$, $v_{0n} = 200\text{GeV}$, $A_\kappa = -100\text{GeV}$

A	$m_{H^\pm} = 600\text{GeV}$	$(\lambda, \kappa) = (0.9, -0.9)$	light-squark-1
B	$m_{H^\pm} = 600\text{GeV}$	$(\lambda, \kappa) = (0.85, -0.1)$	heavy-squark
C	$m_{H^\pm} = 600\text{GeV}$	$(\lambda, \kappa) = (0.82, -0.05)$	light-squark-1
D	$m_{H^\pm} = 700\text{GeV}$	$(\lambda, \kappa) = (0.96, -0.02)$	light-squark-2

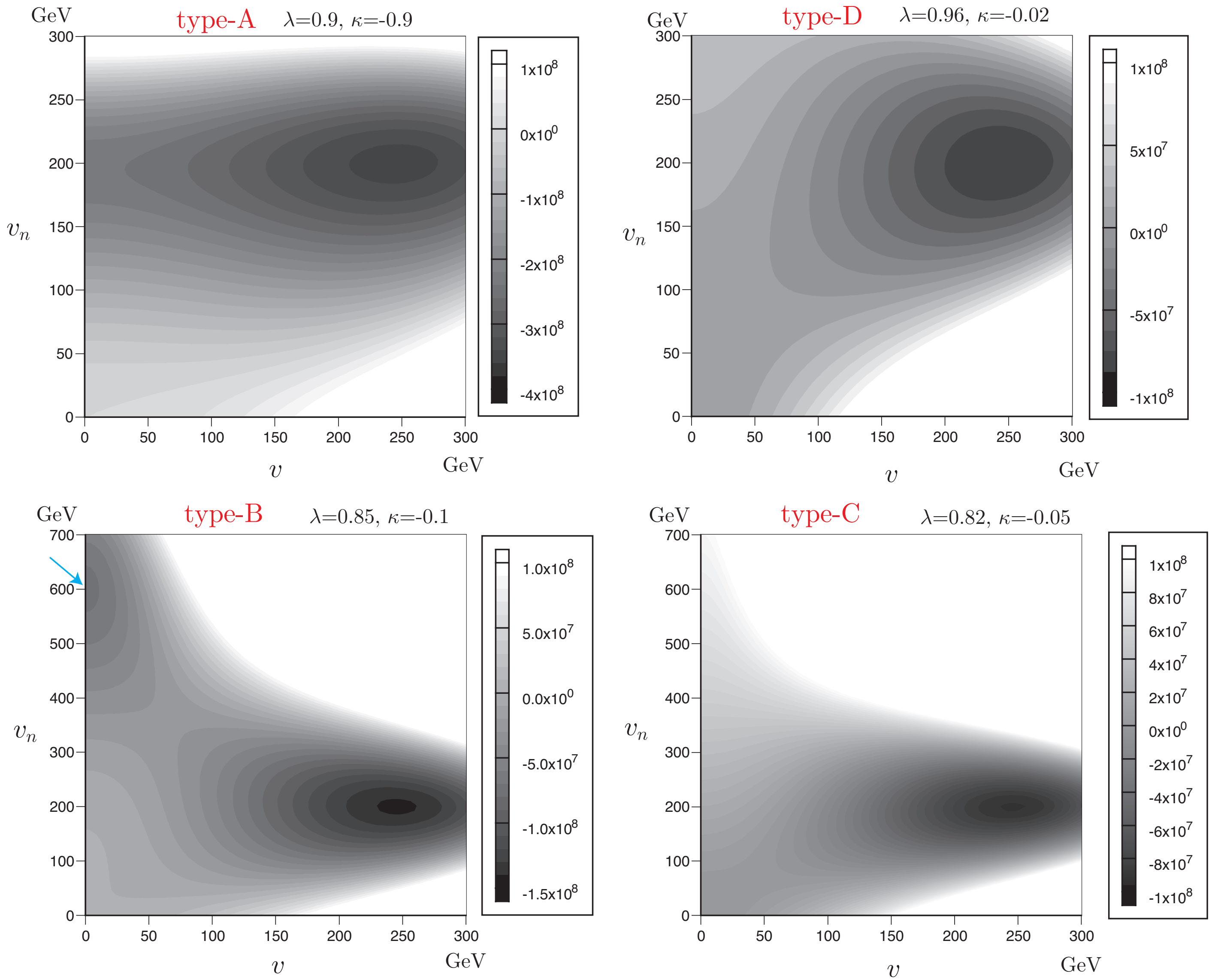
Higgs spectrum and VVH-couplings

		H_1	H_2	H_3	H_4	H_5
A	m_{H_i}(GeV)	119.53	203.59	265.74	617.24	637.47
	$g^2_{VVH_i}$	0.9992	5.926×10^{-4}	0	0	1.884×10^{-4}
B	m_{H_i}(GeV)	38.89	75.31	131.11	625.61	627.95
	$g^2_{VVH_i}$	6.213×10^{-8}	0	0.9999	6.816×10^{-5}	0
C	m_{H_i}(GeV)	42.24	63.49	117.25	625.09	627.44
	$g^2_{VVH_i}$	0.00188	0	0.9980	9.541×10^{-5}	0
D	m_{H_i}(GeV)	41.88	58.62.08	115.15	730.51	734.58
	$g^2_{VVH_i}$	0	1.015×10^{-4}	0.9997	1.632×10^{-4}	0

A: heavy Higgs (MSSM-like), B, C, D: light Higgs

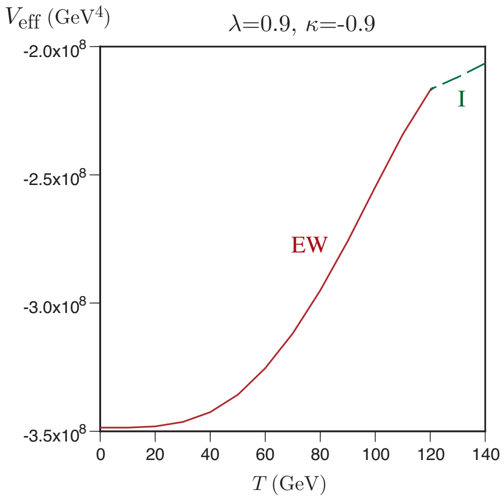
reduced effective potential:

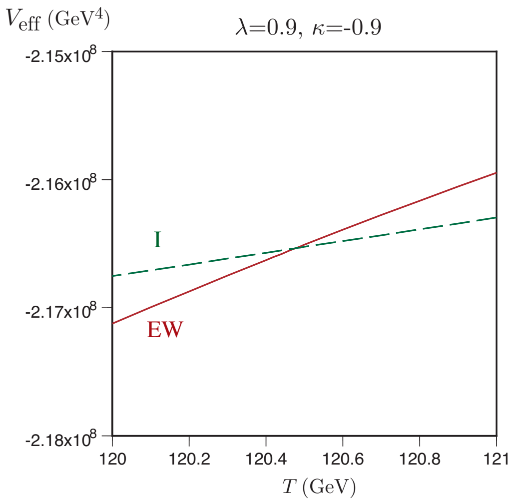
$$\tilde{V}_{\rm eff}(v, v_n; T) = V_{\rm eff}(v\cos\beta(T), v\sin\beta(T), 0, v_n, 0; T) - V_{\rm eff}(0,0,0,0,0;T)$$



⋆ How the phase transitions proceed

type-A



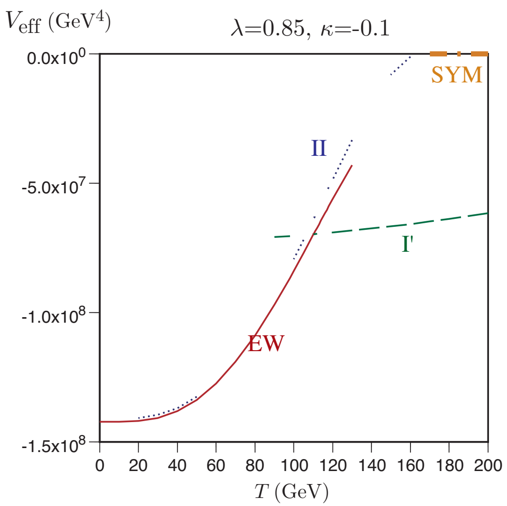


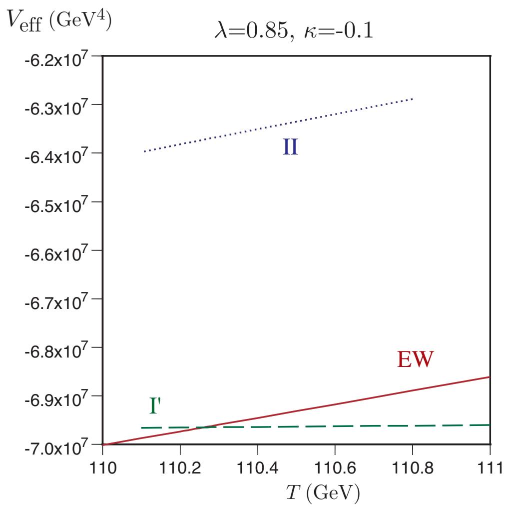
$$(v, v_n) = (106.92, 194.23)(\text{GeV})$$

$$\downarrow \; T_C = 120.47\text{GeV}$$

$$(0, 192.75)(\text{GeV})$$

type-B



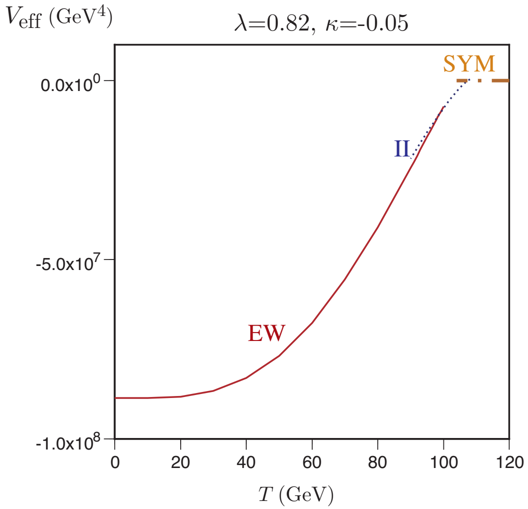


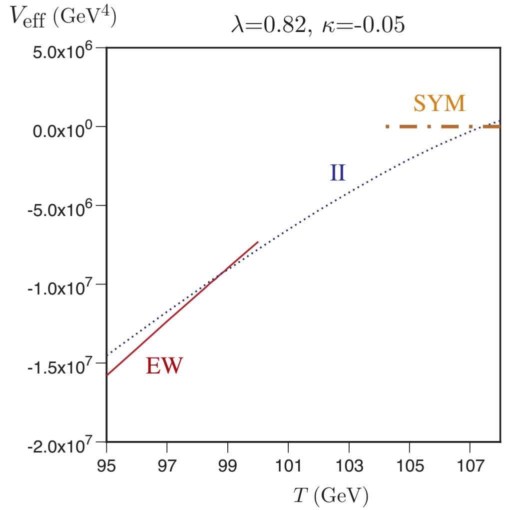
$$(v, v_n) = (208.13, 248.85)(\text{GeV})$$

$$\downarrow \; T_C = 110.26\text{GeV}$$

$$(0, 599.93)(\text{GeV})$$

type-C





$$(v, v_n) = (194.27, 173.75)(\text{GeV})$$

$$\downarrow T_N = 98.76\text{GeV}$$

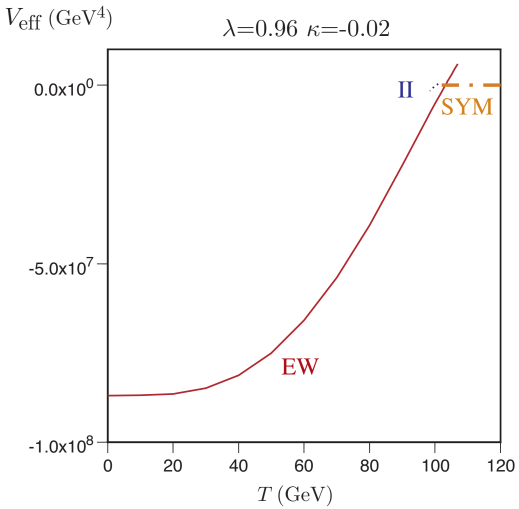
$$(165.97, 0)(\text{GeV})$$

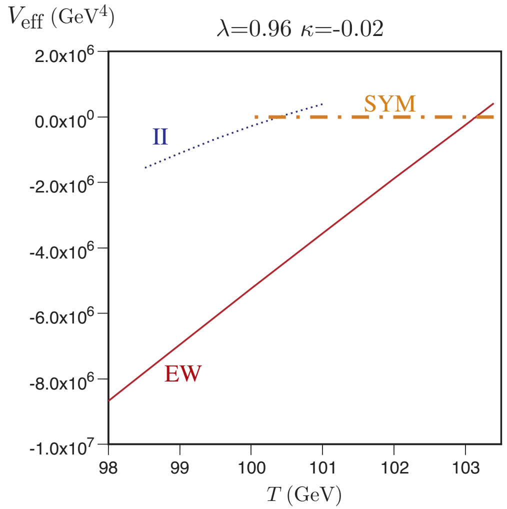
$$(109.54, 0)(\text{GeV})$$

$$\downarrow T_C = 107.44\text{GeV}$$

$$(0, 0)$$

type-D





$$(v, v_n) = (182.49, 192.26)(\text{GeV})$$

$$\downarrow T_C = 103.14\text{GeV}$$

$$(0, 0)$$

type-A MSSM-like EWPT — proceeds along almost constant $v_n \neq 0$

a light stop is needed for it to be strongly first order

type-B new type of 2-stage PT

leap from $(v(T_{C-}), v_n(T_{C-}))$ to $(0, v_n(T_{C+}))$

strongly first order EWPT (no light stop is needed)

type-C new type of 2-stage PT

EWPT proceeds along $v_n = 0$

a light stop is needed for it to be strongly first order

type-D 1-stage PT (so far mainly considered in the NMSSM)

a light stop is needed for the EWPT to be strongly first order

type-B,C,D — light-Higgs scenario — peculiar to NMSSM

4. Summary

EW Baryogenesis

⋆ based on a testable model

⋆ free from the proton decay problem

needs extensions of the MSM for

⋆ CP violation

new sources of CP violation   EDM, precise measurements of CP-viol. BR

μ, A_q, gaugino masses, θ, $\cdots$ in SUSY models

⋆ strongly 1st-order EWPT   extra scalars: 2HDM, MSSM, NMSSM, $\cdots$

$\Longrightarrow$ Higgs spectrum and couplings   LHC, ILC, ...

- $m_H > 120\text{GeV} \Longrightarrow$ 1st-order EWPT in the MSSM **X**

- $m_H > 135\text{GeV} \Longrightarrow$ MSSM **X**

  NMSSM (light Higgs for 1st-order EWPT)

  2HDM, etc.

We studied the phase transitions in the NMSSM to find

- there are 4 phases EW, SYM, I, I′, II

- 4 types of PT, 3 of which have 2-stage nature

  heavy Higgs $\Longrightarrow$ MSSM-like EWPT

  light Higgs $\Longrightarrow$ strongly 1st order EWTP

In particular,

NMSSM in the light Higgs scenario with heavy charged Higgs $(m_{H^\pm} > 300\text{GeV})$

$$m_{H_1} < m_{H_2} < 114\text{GeV} < m_{H_3} < m_{H^\pm} < m_{H_4} < m_{H_5}$$

$$g^2_{VVH_1},\ g^2_{VVH_2} \ll 1$$

$\simeq$ Minimal SM with 1st order EWPT (type-B), extra CP violation

How can we distinguish it from the MSM?

— may be by precision measurements of the couplings

If $\left\{ \begin{array}{c} \text{SUSY particle} \\ \text{Higgs boson } (m_H > 135\text{GeV}) \end{array} \right\}$ are found, the **NMSSM** will be promising.