Electroweak Baryogenesis

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content

I. Introduction

II. Sphaleron Process — $B+L$ violation

III. Electroweak Phase Transition (EWPT)

IV. Mechanism of Electroweak Baryogenesis

V. Discussions

K.F., Prog.Theor.Phys.**96** ('96) 475.
V.A. Rubakov and M.E. Shaposhnikov, hep-ph/9603208.
A.Cohen, et al., Ann.Rev.Nucl.Part.Sci. **33** ('93) 27.

I. Introduction

Baryon Asymmetry of the Universe (BAU)

$$\Longleftrightarrow \frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} = (0.21 - 0.90) \times 10^{-10}$$

$\Longleftarrow$ big-bang nucleosynthesis

$\cdots$constant after decoupling of B-violating processes

evidence of BAU [Steigman, Ann.Rev.Astron.Astrop.**14**('76)]

1. no anti-matter in cosmic rays from our galaxy
   some anti-matter consistent as secondary products
2. nearby clusters of galaxies are stable
   a cluster: $(1 \sim 100) M_{\mathrm{galaxy}} \simeq 10^{12\sim 14} M_\odot$

Starting from a B-symmetric universe ...

$$\frac{n_b}{s} \simeq \frac{n_{\bar{b}}}{s} \quad \sim \quad 8 \times 10^{-11} \quad \text{at } T = 38\text{MeV}$$

$$\sim \quad 7 \times 10^{-20} \quad \text{at } T = 20\text{MeV}$$

$N\bar{N}$-annihilation decouple

At $T = 38$MeV,
mass within a causal region $= 10^{-7} M_\odot \ll 10^{12} M_\odot$.

We must have the BAU $n_B/s = (0.21 - 0.90) \times 10^{-10}$
before the universe was cooled down to $T \simeq 38$MeV.

3 conditions for generation of BAU [Sakharov, '67]

(1) baryon number violation

(2) C and CP violation

(3) departure from equilibrium

if B-violion is in equil. $\Longrightarrow n_b = n_{\bar{b}}$

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(2)
If C *or* CP is conserved, no B is generated:
This is because B is odd under C and CP.

indeed . . .

ρ_0 : baryon-symmetric initial state of the universe $s.t.$

$$\langle n_B \rangle_0 = \mathrm{Tr}[\rho_0 n_B] = 0$$

time evolution of $\rho \Leftrightarrow$ Liouville eq.: $i\hbar \frac{\partial \rho}{\partial t} + [\rho, H] = 0$

If H is C- *or* CP-invariant, $[\rho, \mathcal{C}] = 0$ *or* $[\rho, \mathcal{CP}] = 0$

(spontaneous CP viol. $\Longrightarrow [\rho, \mathcal{CP}] \neq 0$)

Since $\mathcal{C} B \mathcal{C}^{-1} = -B$ and $\mathcal{CP} B \mathcal{CP}^{-1} = -B$

$$\begin{aligned} \langle n_B \rangle &= \mathrm{Tr}[\rho n_B] = \mathrm{Tr}[\rho \mathcal{C} n_B \mathcal{C}^{-1}] = -\mathrm{Tr}[\rho n_B] \\ & or \\ \langle n_B \rangle &= \mathrm{Tr}[\rho \mathcal{CP} n_B (\mathcal{CP})^{-1}] = -\mathrm{Tr}[\rho n_B] \end{aligned}$$

Both C and CP must be violated to have $\langle n_B \rangle \neq 0$.

example — GUTs

[Kolb and Turner, The Early Universe]

out-of-equil. decay of X bosons $m_X \gtrsim 10^{15}$GeV

$X =$ gauge boson or Higgs boson

Consider 2 channels:

$$\begin{cases} X \longrightarrow qq & \Delta B = 2/3 & \text{with branching} \quad r \\ X \longrightarrow \bar{q}\bar{l} & \Delta B = -1/3 & \text{with branching} \quad 1-r \end{cases}$$

in the decay of X-$\bar{X}$ pairs

$$\langle \Delta B \rangle = \frac{2}{3}r - \frac{1}{3}(1-r) - \frac{2}{3}\bar{r} + \frac{1}{3}(1-\bar{r}) = r - \bar{r}$$

$\therefore$ C *or* CP is conserved $(r = \bar{r}) \Longrightarrow \Delta B = 0$

At $T \simeq m_X$, decay rate of $X = \Gamma_D \simeq \alpha m_X$

$\alpha \sim 1/40$ for gauge boson, $\alpha \sim 10^{-6\sim-3}$ for Higgs boson

Hubble parameter : $H \sim 1.7\sqrt{g_*}T^2/m_{Pl}$

$g_* \simeq 10^{2\sim3}$: massless degrees of freedom

$\therefore$ $\Gamma_D \simeq H$ at $T \simeq m_X$

$\Longrightarrow$ decay and production of $X\bar{X}$ are out of equil.

As we shall see, $B + L$ were washed out before EWPT.

$\therefore$ $B - L$-conserving GUT (e.g. minimal $SU(5)$ model) will be useless to generate the BAU.

other candidates for generating BAU

- $^\exists$Majorana neutrino $\Rightarrow$ L-violating interaction
  decoupling of L-violating interaction
  $\Longrightarrow$ constraints on the neutrino mass

- Affleck-Dine mechanism in a supersymmetric model
  [A-D, N.P. B174('86) 45]
  $\langle$squark$\rangle \neq 0$ or $\langle$slepton$\rangle \neq 0$ along (nearly) flat directions, at high temperature
  $\Longrightarrow$ B- and/or L-violation

- Electroweak Baryogenesis

(1) anomaly in $B + L$-current
(2) C-violation (chiral gauge)
    CP-violation in KM matrix or extension of SM
(3) first-order EWPT with expanding bubble walls

- topological defects
  EW string, domain wall $\sim$ EW baryogenesis
  effective volume is too small, mass density of the universe

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N.B.
The BAU may be generated by some combination of these mechanisms. Any way,

$$\boxed{\text{EWPT will be the last chance to obtain the BAU.}}$$

II. Sphaleron Process

II-1. Anomalous fermion number nonconservation

axial anomaly in the standard model

$$\begin{aligned}\partial_\mu j^\mu_{B+L} &= \frac{N_f}{16\pi^2}[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}], \\ \partial_\mu j^\mu_{B-L} &= 0,\end{aligned}$$

$N_f =$ number of the generations, $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

integrating these equations,

$$\begin{aligned}B(t_f) - B(t_i) &= \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x \left[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right] \\ &= N_f\,[N_{CS}(t_f) - N_{CS}(t_i)]\end{aligned}$$

where N_{CS} is the Chern-Simons number defined, in the $A_0 = 0$ gauge, by

$$N_{CS} = \frac{1}{32\pi^2}\int d^3x\, \epsilon_{ijk}\left[g^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g'^2 B_{ij}B_k\right]_t$$

: gauge-noninvariant

For classical vacua of the gauge sector, $N_{CS} \in \mathbf{Z}$

$\Longleftrightarrow F_{ij} = B_{ij} = 0$

$\Longleftrightarrow A = iU^{-1}dU$ and $B = dv$ with $U \in SU(2)$

$U : S^3 \longrightarrow SU(2) \simeq S^3$ is characterized by its winding number, $\pi_3(SU(2)) \simeq \mathbf{Z}$. $\leftrightarrow N_{CS}$

background of gauge fields with $\Delta N_{CS} = 1$

$\Rightarrow \Delta B = 1$ for each generation

($\because$ level-crossing phenomenon) $\longleftrightarrow$ index theorem transition rate between configurations with $\Delta N_{CS} = 1$

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WKB approximation:

$T = 0$

(valley or constrained) instanton $=$ *finite euclidean action*

tunneling probability $\sim \mathrm{e}^{-2S_{\mathrm{inst}}} = \mathrm{e}^{-8\pi^2/g^2}$

for EW theory, $\mathrm{e}^{-2S_{\mathrm{inst}}} \simeq \mathrm{e}^{-378} = 10^{-164}$

$T \neq 0$ [Affleck, P.R.L.46('81)]

$^{\exists}$classical static saddle-point solution with *finite energy*

$\Updownarrow$

top of the energy barrier dividing two classical vacua

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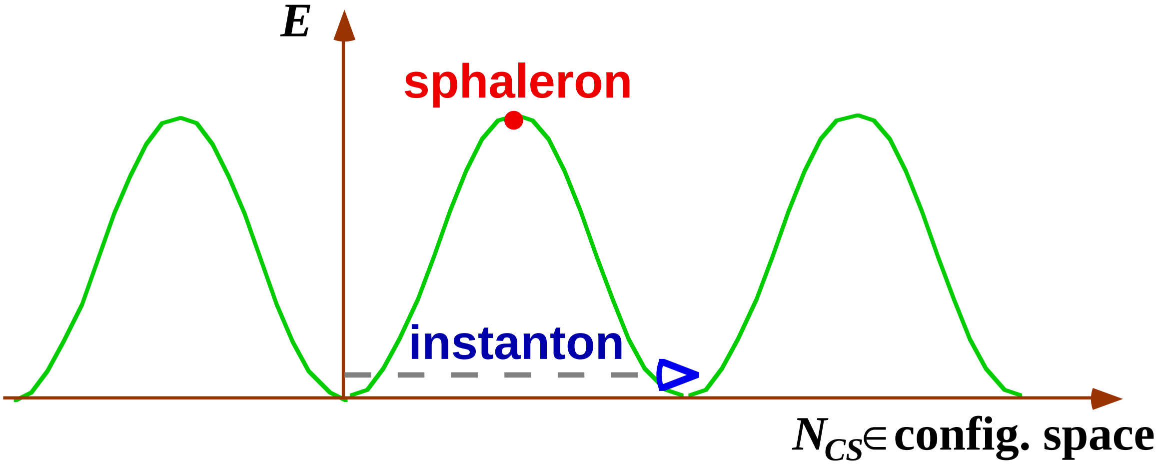
sphaleron solution [Manton, P.R.D28('83)]



$$E_{\mathrm{sph}} = \frac{2M_W}{\alpha_W} B\left(\frac{\lambda}{g^2}\right) \simeq 10\mathrm{TeV}$$

λ:the Higgs self coupling, $\alpha_W = g^2/(4\pi)$

$1.5 \leq B \leq 2.7$ for $\lambda/g^2 \in [0, \infty)$

$E_{\mathrm{sph}} =$ finite $\Longrightarrow$ $^{\exists}$transition over the barrier

sphaleron for $\theta_W \neq 0$	[Brihaye and Kunz, P.R.D50('94)]
2-doublet Higgs model	[Peccei, Zhang and Kastening, P.L.B266('91)]
squark vs sphaleron	[Moreno, Oaknin and Quirós, hep-ph/9612212]

Transition Rate

[Arnold and McLerran, P.R.D36('87)]

♣ $\omega_-/(2\pi) \lesssim T \lesssim T_C$

$$\boxed{\Gamma^{(b)}_{\rm sph} \simeq k \mathcal{N}_{\rm tr} \mathcal{N}_{\rm rot} \frac{\omega_-}{2\pi} \left(\frac{\alpha_W(T)T}{4\pi} \right)^3 {\rm e}^{-E_{\rm sph}/T}}$$

zero modes $\rightarrow$ $\left\{ \begin{array}{l} \mathcal{N}_{\rm tr} = 26 \\ \mathcal{N}_{\rm rot} = 5.3 \times 10^3 \end{array} \right.$ for $\lambda = g^2$

$$\omega_-^2 \simeq (1.8 \sim 6.6) m_W^2 \quad \text{for } 10^{-2} \le \lambda/g^2 \le 10$$

$$k \simeq O(1)$$

♣ $T \gtrsim T_C$ symmetric phase — no mass scale

dimensional analysis :

$$\boxed{\Gamma^{(s)}_{\rm sph} \simeq \kappa (\alpha_W T)^4}$$

check by Monte Carlo simulation $\langle N_{CS}^2(t) \rangle = 2\Gamma V t$ as $t \to \infty$

$\kappa > 0.4$ $SU(2)$ gauge-Higgs system

[Ambjørn, et al. N.P.B353('91)]

$\kappa = 1.09 \pm 0.04$ $SU(2)$ pure gauge system

[Ambjørn and Krasnitz, P.L.B362('95)]

We use 'sphaleron transition' even in the symmetric phase.

II-2. Washout of $B+L$

$B+L$ would be washed out after the EWPT, if
the EWPT is second order or
the sphaleron process does not decouple after it.

decoupling of sphaleron process $\Leftrightarrow$ $\Gamma_{\rm sph} <$ Hubble parameter

at $T = T_C \simeq 100$GeV,

$$H(T_C) = \frac{\dot{R}(t)}{R(t)} \simeq 1.7\sqrt{g_*}\frac{T_C^2}{m_{Pl}} \simeq 10^{-13}\text{GeV}$$

$g_* \sim 100$: effective massless degrees of freedom

At $T > T_C$,

$$\Gamma_{\rm sph} \simeq \Gamma_{\rm sph}^{(s)}/T^3 \simeq \kappa\alpha_W^4 T \sim 10^{-4}\text{GeV} \gg H(T_C)$$

$\Longrightarrow$ $B+L$-changing process in equilibrium

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relic baryon number after the washout
particle number density $[m/T \ll 1$ and $\mu/T \ll 1]$

$$n_+ - n_- = \int \frac{d^3\boldsymbol{k}}{(2\pi)^2}\left[\frac{1}{e^{\beta(\omega_k-\mu)} \mp 1} - \frac{1}{e^{\beta(\omega_k+\mu)} \mp 1}\right]$$

$$\simeq \begin{cases} \dfrac{T^3}{3}\dfrac{\mu}{T} & \text{for bosons} \\ \dfrac{T^3}{6}\dfrac{\mu}{T} & \text{for fermions,} \end{cases}$$

$$\omega_k = \sqrt{\boldsymbol{k}^2 + m^2}$$

chemical equilibrium — all the gauge int., Yukawa, sphaleron

W^-	$u_{L(R)}$	$d_{L(R)}$	$e_{iL(R)}$	ν_{iL}	ϕ^0	ϕ^-
μ_W	$\mu_{u_{L(R)}}$	$\mu_{d_{L(R)}}$	$\mu_{iL(R)}$	μ_i	μ_0	μ_-

gauge int. $\Leftrightarrow$ $\mu_W = \mu_{d_L} - \mu_{u_L} = \mu_{iL} - \mu_i = \mu_- + \mu_0$

$|0\rangle \leftrightarrow u_L d_L d_L \nu_L$ $\Leftrightarrow$ $N_f(\mu_{u_L} + 2\mu_{d_L}) + \sum_i \mu_i = 0$

Quantum number densities [in unit of $T^2/6$]

$$\begin{aligned}
B &= N_f(\mu_{u_L} + \mu_{u_R} + \mu_{d_L} + \mu_{d_R}) = 4N_f\mu_{u_L} + 2N_f\mu_W, \\
L &= \sum_i(\mu_i + \mu_{iL} + \mu_{iR}) = 3\mu + 2N_f\mu_W - N_f\mu_0 \\
Q &= \frac{2}{3}N_f(\mu_{u_L} + \mu_{u_R}) \cdot 3 - \frac{1}{3}N_f(\mu_{d_L} + \mu_{d_R}) \cdot 3 \\
&\quad -\sum_i(\mu_{iL} + \mu_{iR}) - 2 \cdot 2\mu_W - 2m\mu_- \\
&= 2N_f\mu_{u_L} - 2\mu - (4N_f + 4 + 2m)\mu_W + (4N_f + 2m)\mu_0 \\
I_3 &= \frac{1}{2}N_f(\mu_{u_L} - \mu_{d_L}) \cdot 3 + \frac{1}{2}\sum_i(\mu_i - \mu_{iL}) \\
&\quad -2 \cdot 2\mu_W - 2 \cdot \frac{1}{2}m(\mu_0 - \mu_-) \\
&= -(2N_f + m + 4)\mu_W
\end{aligned}$$

$\mu \equiv \sum_i \mu_i$

m : number of Higgs doublets

- symmetric phase

$\Longrightarrow Q = I_3 = 0$

$$B = \frac{8N_f + 4m}{22N_f + 13m}(B - L), \qquad L = -\frac{14N_f + 9m}{22N_f + 13m}(B - L)$$

- broken phase

$\Longrightarrow Q = 0$ and $\mu_0 = 0$

$$\begin{aligned} B &= \frac{8N_f + 4m + 8}{24N_f + 13m + 26}(B - L) \\ L &= -\frac{16N_f + 9m + 18}{24N_f + 13m + 26}(B - L) \end{aligned}$$

$\therefore$ If $(B - L)_{\text{primordial}} = 0$, $B = L = 0$ at present !

To have nonzero BAU,

(i) we must have $B - L$ before the sphaleron process decouples, or

(ii) $B + L$ must be created at the first-order EWPT, and the sphaleron process must decouple immediately after that.

(i) $\Leftarrow$ GUTs, Majorana ν, Affleck-Dine

(ii) $=$ Electroweak Baryogenesis

III. Electroweak Phase Transition (EWPT)

III-1. Static properties of the phase transition

rate of any interacton at $T \sim T_C \ll$ Hubble parameter
$\Longrightarrow$ equil. thermodynamics applicable to static properties

order of the transition, transition temperature,
latent heat and surface tension (if it is first order)

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free energy density $=$ effective potential at $T \neq 0$

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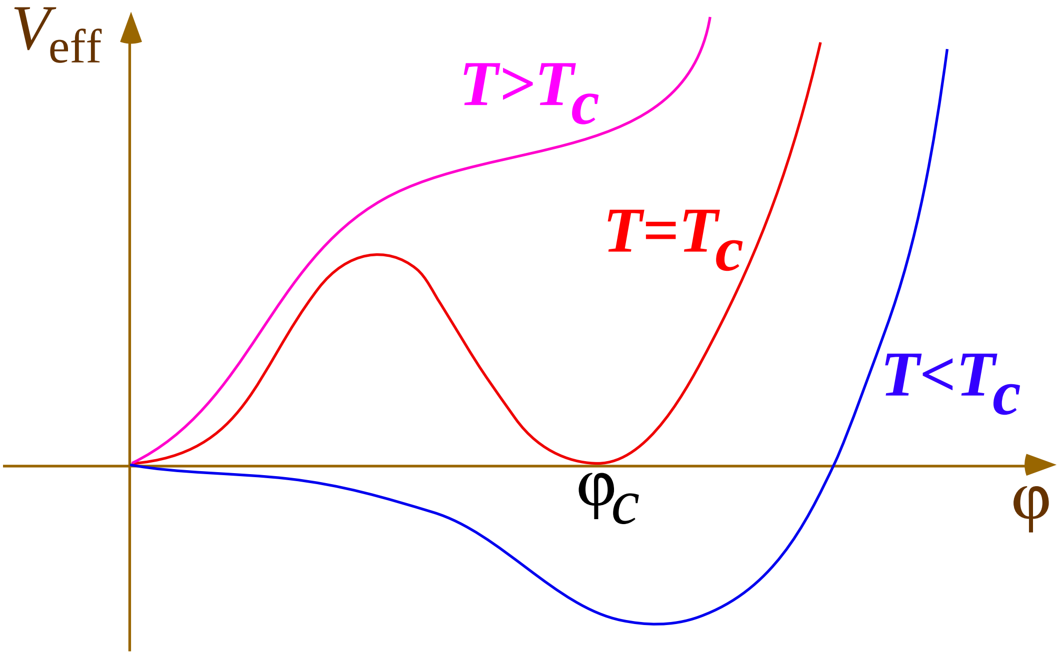
function of the order parameters and T

Example Minimal standard model (MSM)

order parameter: $\langle \Phi \rangle = \dfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$

a first order phase transition

$$\varphi_C \equiv \lim_{T \uparrow T_C} \varphi(T) \neq 0$$



one-loop level,

$$V_{\rm eff}(\varphi;T) = V_{\rm tree}(\varphi) + V^{(1)}(\varphi;T),$$

where

$$\begin{aligned} V_{\rm tree}(\varphi) &= -\frac{1}{2}\mu_0^2\varphi^2 + \frac{\lambda_0}{4}\varphi^4 \\ V^{(1)}(\varphi;T) &= -\frac{i}{2}\sum_A c_A \int_k \log\det\left[i\mathcal{D}_A^{-1}(k;\varphi)\right] \end{aligned}$$

with

μ_0^2, λ_0 : bare parameters $\leftarrow$ renormlized at $T = 0$

A runs over all the partice species

$|c_A|$ counts the degrees of freedom, $\begin{cases} c_A > 0 & \text{for bosons} \\ c_A < 0 & \text{for fermions} \end{cases}$

$$\int_k \equiv iT\sum_{n=-\infty}^{\infty}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}$$

$$\text{with } k^0 = \omega_n = \begin{cases} 2n\pi T & \text{for bosons,} \\ (2n+1)\pi T & \text{for fermions.} \end{cases}$$

W-boson :

$$\begin{aligned} c_W &= 2 \\ i\mathcal{D}^{-1\mu\nu}_W(k;\varphi) &= (-k^2 + m_W^2(\varphi))\eta^{\mu\nu} + (1-\frac{1}{\xi})k^\mu k^\nu \end{aligned}$$

with $m_W(\varphi) = \frac{1}{2}g\varphi$

Dirac fermion :

$$\begin{aligned} c_f &= -2 \\ i\mathcal{D}_f^{-1}(k;\varphi) &= \not{k} - m_f(\varphi) \end{aligned}$$

with $m_f(\varphi) = y_f\varphi/\sqrt{2}$

Higgs boson:

$m_H^2(\varphi) = 3\lambda\varphi^2 - \mu^2$ — negative for small φ

$\Longrightarrow$ complex $V_{\rm eff}$

$\longleftarrow$ sum over *daisy diagrams*, or *improved perturbation*

neglecting the Higgs contribution

$$V_{\rm eff}(\varphi; T) = V_0(\varphi) + \bar{V}(\varphi; T)$$

where

$$V_0(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2Bv_0^2\varphi^2 + B\varphi^4\left[\log\left(\frac{\varphi^2}{v_0^2}\right) - \frac{3}{2}\right]$$

$$\bar{V}(\varphi; T) = \frac{T^4}{2\pi^2}\left[6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)\right]$$

$$B = \frac{3}{64\pi^2 v_0^4}(2m_W^4 + m_Z^4 - 4m_t^4)$$

$$I_{B,F}(a^2) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp \mathrm{e}^{-\sqrt{x^2+a^2}}\right)$$

with

$v_0 = 246\text{GeV}$ is the minimum of $V_0(\varphi)$

$a_A = m_A(\varphi)/T$

high-temperature expansion $[m/T \ll 1]$

$$\begin{aligned} I_B(a^2) \quad &= \quad -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{4\pi} \\ &\qquad -\frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right)\frac{a^4}{2} + O(a^6) \\ I_F(a^2) \quad &= \quad \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6) \end{aligned}$$

For $T > m_W, m_Z, m_t$,

$$V_{\rm eff}(\varphi;T) \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$\begin{aligned}
D &= \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2) \\
E &= \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2} \\
T_0^2 &= \frac{1}{2D}(\mu^2 - 4Bv_0^2) \\
\lambda_T &= \lambda \\
&- \frac{3}{16\pi^2 v_0^4}\left(2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right)
\end{aligned}$$

with $\log\alpha_B = 2\log 4\pi - 2\gamma_E$ and $\log\alpha_F = 2\log\pi - 2\gamma_E$

At T_C, $\varphi_C = \dfrac{2ET_C}{\lambda_{T_C}}$

$$\Gamma_{\rm sph}^{(b)}/T_C^3 < H(T_C) \quad \Longleftrightarrow \quad \frac{\varphi_C}{T_C} \gtrsim 1$$

$\Longrightarrow$ upper bound on λ $[m_H = \sqrt{2}\lambda v_0]$

$$\boxed{m_H \lesssim 46\text{GeV}}$$

$\longleftrightarrow$ inconsistent with the lower bound $m_H > 65$GeV

2-doublet extension of the MSM or MSSM :

more scalars ⟶ more φ^3-temrs

$\Longrightarrow$ stronger first-order EWPT

MSSM with light stop [de Carlos and Espinosa, hep-ph/9703212]

stop mass-squared matrix :

$$\begin{pmatrix} m_{\tilde{q}}^2 + \left(\frac{g_1^2}{24} - \frac{g_2^2}{8} \right) (\rho_u^2 - \rho_d^2) + \frac{1}{2} y_t^2 \rho_u^2 & \frac{y_t}{\sqrt{2}} (\mu \rho_d + A \rho_u e^{-i\theta}) \\ * & m_{\tilde{t}}^2 - \frac{g_1^2}{6} (\rho_u^2 - \rho_d^2) + \frac{1}{2} y_t^2 \rho_u^2 \end{pmatrix}$$

where

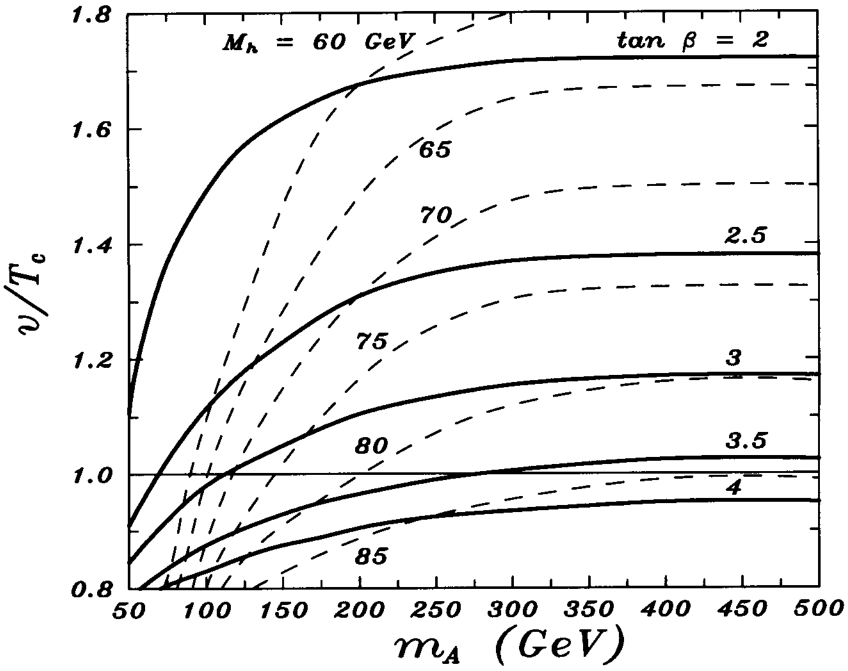
$$\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_d \\ 0 \end{pmatrix} , \qquad \langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho_u \mathrm{e}^{i\theta} \end{pmatrix} ,$$

$m_{\tilde{q}} = 0$ or $m_{\tilde{t}} = 0 \Longrightarrow$ smaller eigenvalue $\equiv m_-^2 \sim O(\rho^2)$

$\therefore$ high-T expansion

$$\bar{V}_{\tilde{t}}(\rho_i, \theta; T) \Rightarrow - \frac{T}{6\pi} (m_-^2)^{3/2}$$

φ_C / T_C vs m_A for various $\tan\beta$ (m_h)



$m_t = 175\text{GeV}$

$m_{\tilde{q}} = 250\text{GeV}$

$m_{\tilde{t}} = 0$

△ Monte Carlo Simulations [MSM][Jansen, N.P.B.Suppl.47('96)]

effective fermion mass : $m_f(T) \sim O(T) \leftarrow$ nonzero modes

- 4-dim. $SU(2)$ system with a Higgs doublet

- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet

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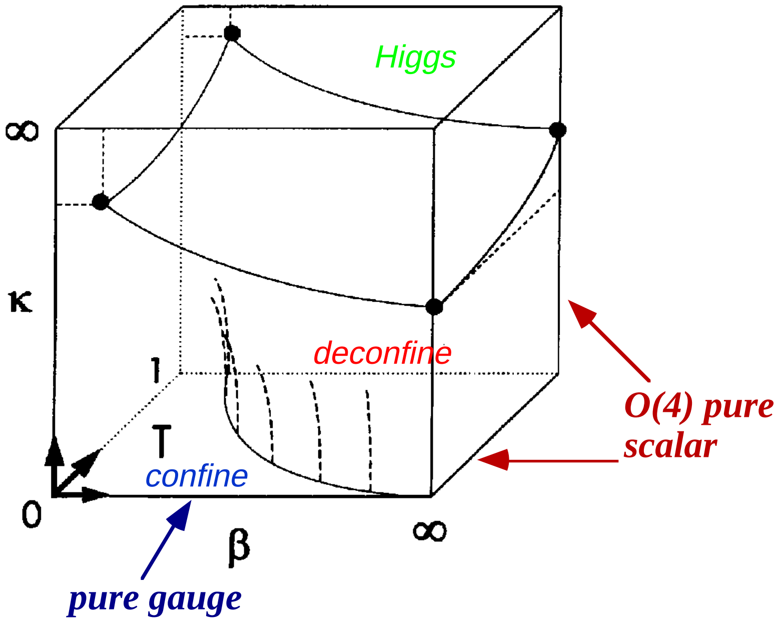
time-component of the gauge field

only zero-freq. modes of the bosons survive as $T \longrightarrow$ large

matching finite-T Green's functions with 4-dim. theory

$\Rightarrow$ T-dependent parameters

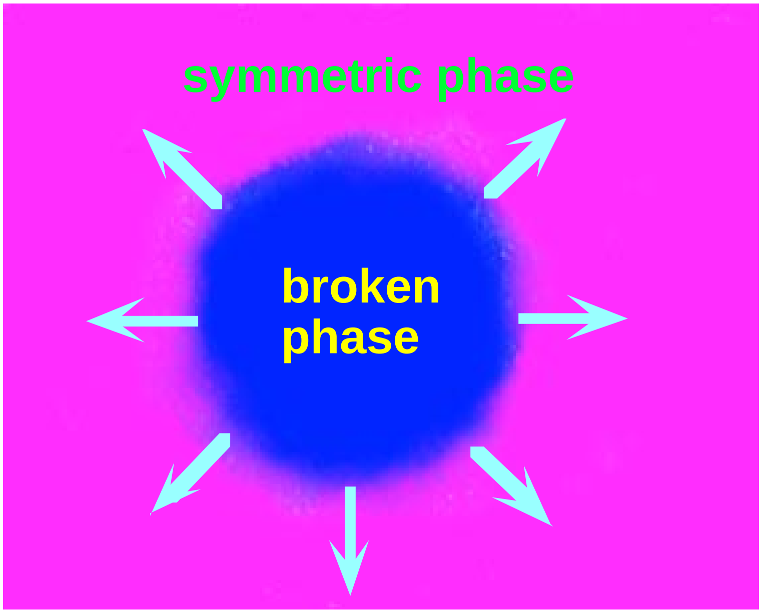
shematic finite-T phase diagram [λ-fixed]

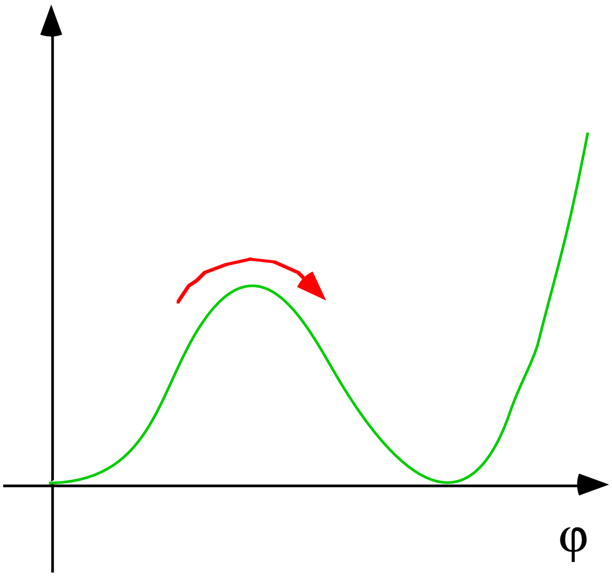


- EWPT is first order for $m_H \lesssim 70\text{GeV}$
  The strength of the transition rapidly decreases as m_H increases.
- $\varphi_C/T_C > 1$ is not satisfied for $m_H \geq 50\text{GeV}$
- The numerical results coincide with those of the continuum two-loop perturbation theory, for $m_H \leq 70\text{GeV}$.

III-2. Dynamics of the phase transition

first-order EWPT accompanying bubble nucleation/growth





Suppose that $V_{\rm eff}(\varphi;T_C)$ is known.

nucleation rate per unit time and unit volume:

$$I(T) = I_0 {\rm e}^{-\Delta F(T)/T}$$

where

$$\Delta F(T) = \frac{4\pi}{3} r^3 [p_s(T) - p_b(T)] + 4\pi r^2 \sigma$$

with

$$p_s(T) = -V_{\rm eff}(0;T), \qquad p_b(T) = -V_{\rm eff}(\varphi(T);T)$$

supercooling $\longrightarrow p_s(T) < p_b(T)$

$\sigma \simeq \int dz (d\varphi/dz)^2$: surface energy density

radius of the critical bubble : $r_*(T) = \dfrac{2\sigma}{p_b(T) - p_s(T)}$

$$\begin{array}{lcll} s & = & \dfrac{\partial V_{\rm eff}}{\partial T} & : \text{ entropy density} \\ \rho & = & V_{\rm eff} - Ts & : \text{ energy density} \end{array}$$

How the EWPT proceeds ?

$f(t)$: fraction of space converted to the broken phase

$$f(t) = \int_{t_C}^{t} dt' \, I(T(t'))[1 - f(t')]V(t', t)$$

where

$V(t', t)$: volume of a bubble at t which was nucleated at t'

$$V(t', t) = \frac{4\pi}{3}\left[r_*(T(t')) + \int_{t'}^{t} dt'' \, v(T(t''))\right]^3$$

$T = T(t) \Leftarrow \rho = (\pi^2/30)g_* T^4 \propto R^{-4}$ for RD universe

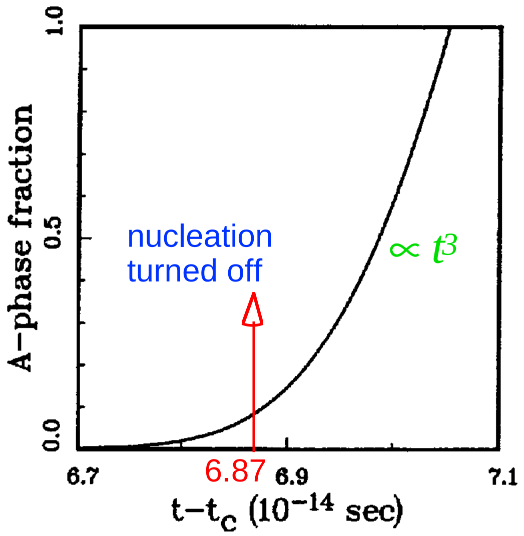
$v(T)$: wall velocity

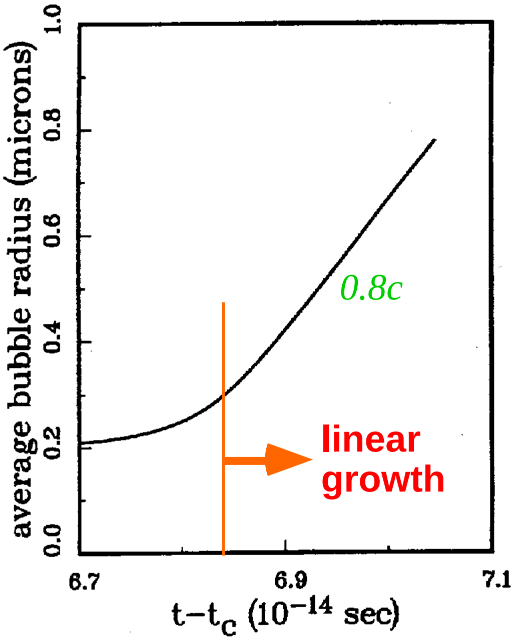
one-loop $V_{\rm eff}$ of MSM with $m_H = 60\text{GeV}$ and $m_t = 120\text{GeV}$

[Carrington and Kapsta, P.R.D47('93)]

At 6.5×10^{-14}sec, bubbles began to nucleate.

[A characteristic time scale of the EW processes is $O(10^{-26})$sec.]





very small supercooling : $\frac{T_C - T_N}{T_C} \simeq 2.5 \times 10^{-4}$

90% of the universe is converted by bubble growth

---

weakly first order $\Longleftrightarrow$ small φ_C and/or lower barrier height

$\Longrightarrow \Bigg\{$ nucleation dominance over growth
large fluctuation between the two phases

IV. Mechanism of Electroweak Baryogenesis

At $T \simeq T_C$, hierarchy of time scales :

EW	$t_{EW} \simeq 1\text{GeV}^{-1}$
Yukawa	$t_Y \simeq (m_W/m_f)^4\text{GeV}^{-1}$
QCD	$t_s \simeq 0.1\text{GeV}^{-1}$
Hubble	$H^{-1} \simeq 10^{13}\text{GeV}^{-1}$
sphaleron	$t_{\text{sph}} = (\kappa\alpha_W^4 T)^{-1} \simeq \kappa^{-1} \cdot 10^4\text{GeV}^{-1}$
wall motion	$t_{\text{wall}} \simeq 0.01 \sim 4\text{GeV}^{-1}$

$$t_{\text{wall}} = \frac{\text{wall width}}{\text{wall velocity}} \simeq \frac{0.01 \sim 0.04\text{GeV}^{-1}}{0.1 \sim 0.8}$$

▷ $t_{EW} \ll H^{-1} \Rightarrow$ all particles are in kinetic equilibrium at the same temperature

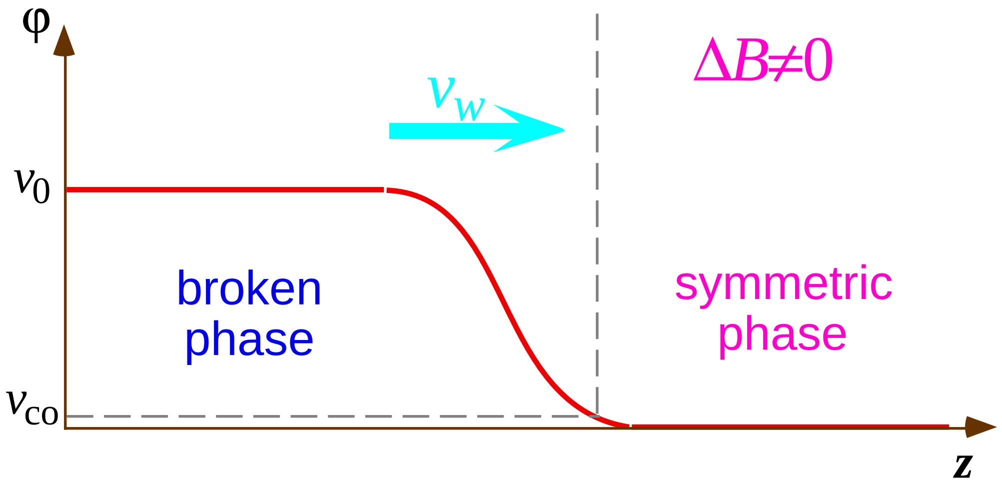
▷ for $m_f \lesssim 0.1\text{GeV}$, $\quad t_Y \sim H^{-1}$
$\therefore$ Yukawa int. of light fermions are out of chemical equil.

▷ some of flavor-changing int. are out of chemical equil.
$\because \quad |V_{ub}|\,, |V_{cb}|\,, |V_{td}|\,, |V_{ts}| \ll 1$

▷ $t_{\text{wall}} \ll t_{\text{sph}}$

$\Rightarrow$ sphaleron process is out of chemical equil. near the bubble wall even in the symmetric phase

$\therefore$ Nonequilibrium state is realized near expanding bubble walls.



$v_{co} \simeq 0.01 v_0 \Longleftarrow E_{\rm sph}/T_C \simeq 1$

bubble wall $\Leftarrow$ classical config. of gauge-Higgs system

Effects of CP violation :

- interactions between the particles and the bubble wall
- propagation of the particles in the plasma

$\Downarrow$

generation of baryon number through sphaleron process

$\Downarrow$

decoupling of sphaleron process in the broken phase

two scenarios to realize EW baryogenesis:

- spontaneous baryogenesis + diffusion
  classical, adiabatic
- charge transport scenario
  quantum mechanical, nonlocal

Both need CP violation other than KM matrix
$\Longleftrightarrow$ extension of the MSM

two-Higgs-doublet model, MSSM, . . .

IV-1. Charge transport mechanism

[Nelson, el al. N.P.B373('92)]

CP violation in the Higgs sector [spacetime-dependent]

⇓

difference in reflections of chiral fermions and antifermions

⇓

net chiral charge flux into the symmetric phase

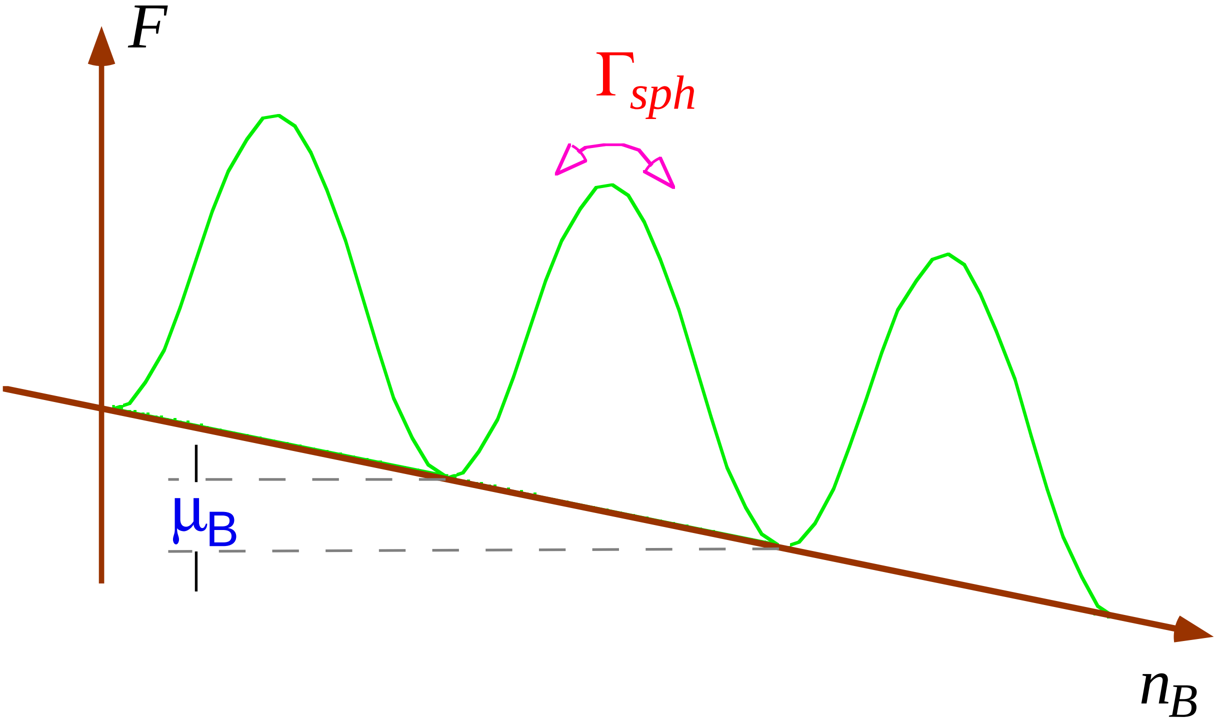
⇓

sphaleron transition converts the charge into B

change of distribution functions by the chiral charge flux
⇐ Boltzmann equations

bubble wall velocity ≃ const. ⇒ constant chiral charge flux

⇒ bias on free energy along B [stationary nonequilibrium]



$$\boxed{\dot{n}_B \simeq -\frac{\mu_B \Gamma_{\rm sph}}{T}}$$

$Q^i_{L(R)}$: charge of a left(right)-handed fermion of species i

${R^s}_{R\to L}$: reflection coeff. for the right-handed fermion incident from the symmetric phase region

${\bar{R}^s}_{R\to L}$: the same as above for the right-handed antifermion

⟨injected charge into symmetric phase⟩ brought by the fermions and antifermions in the symmetric phase :

$$
\begin{aligned}
&{\Delta Q_i}^s \\
= \quad & [({Q_R}^i - {Q_L}^i){R^s}_{L\to R} + (-{Q_L}^i + {Q_R}^i){\bar{R}^s}_{R\to L} \\
& +(-{Q_L}^i)({T^s}_{L\to L} + {T^s}_{L\to R}) - (-{Q_R}^i)({\bar{T}^s}_{R\to L} + {\bar{T}^s}_{R\to R})]{f^s}_{Li} \\
& +[({Q_L}^i - {Q_R}^i){R^s}_{R\to L} + (-{Q_R}^i + {Q_L}^i){\bar{R}^s}_{L\to R} \\
& +(-{Q_R}^i)({T^s}_{R\to L} + {T^s}_{R\to R}) - (-{Q_L}^i)({\bar{T}^s}_{L\to L} + {\bar{T}^s}_{L\to R})]{f^s}_{Ri}
\end{aligned}
$$

the same brought by the transmission from the broken phase :

$$
\begin{aligned}
{\Delta Q_i}^b \quad = \quad & {Q_L}^i({T^b}_{L\to L}{f^b}_{Li} + {T^b}_{R\to L}{f^b}_{Ri}) \\
& +{Q_R}^i({T^b}_{L\to R}{f^b}_{Li} + {T^b}_{R\to R}{f^b}_{Ri}) \\
& +(-{Q_L}^i)({\bar{T}^b}_{R\to L}{f^b}_{Li} + {\bar{T}^b}_{L\to L}{f^b}_{Ri}) \\
& +(-{Q_R}^i)({\bar{T}^b}_{R\to R}{f^b}_{Li} + {\bar{T}^b}_{L\to R}{f^b}_{Ri})
\end{aligned}
$$

by use of

unitarity: ${R^s}_{L\to R} + {T^s}_{L\to L} + {T^s}_{L\to R} = 1,$ etc.

reciprocity: ${T^s}_{R\to L} + {T^s}_{R\to R} = {T^b}_{L\to L} + {T^b}_{R\to L},$ etc.

$$f^{s(b)}_{iL} = f^{s(b)}_{iR} \equiv f^{s(b)}_i$$

we obatin

$$\Delta {Q^s}_i + \Delta {Q^b}_i = ({Q_L}^i - {Q_R}^i)({f^s}_i - {f^b}_i)\Delta R$$

where

$$\Delta R \equiv {R^s}_{R\to L} - {\bar{R}^s}_{R\to L}$$

total flux injected into the *symmetric phase* region

$$F^i_Q = \frac{{Q_L}^i - {Q_R}^i}{4\pi^2\gamma}\int_{m_0}^{\infty} dp_L \int_0^{\infty} dp_T\, p_T \times \left[{f_i}^s(p_L,p_T) - {f_i}^b(-p_L,p_T)\right] \Delta R\left(\frac{m_0}{a},\frac{p_L}{a}\right)$$

where

$$ {f_i}^s(p_L,p_T) = \frac{p_L}{E}\frac{1}{\exp[\gamma(E - v_w p_L)/T]+1}$$

$$ {f_i}^b(-p_L,p_T) = \frac{p_L}{E}\frac{1}{\exp[\gamma(E + v_w\sqrt{p_L^2 - m_0^2})/T]+1}$$

are the fermion flux densities in the symmetric and broken phases.

m_0 : fermion mass in the broken pase

v_w : wall velocity

p_T : transverse momentum

$1/a$: wall width

$$\gamma = 1/\sqrt{1 - v_w^2}$$

$$E = \sqrt{p_L^2 + p_T^2}$$

$\boxed{\Delta R \Longrightarrow \text{effects of CP violation}}$

- MSM — KM matrix
  dispersion relation of the fermion $\sim O(\alpha_W)$
  [Farrar and Shaposhnikov, P.R.D50('94)]
  — decoherence by QCD effects (short range)
- CP violation in mass matrices — Higgs sector & SUSY-br.
  tree-level quantum scattering by the bubble wall

choich of the charge :

$$\left.\begin{array}{c} Q_L - Q_R \neq 0 \\ \text{conserved in the symmetric phase}\end{array}\right\} \Longrightarrow \quad Y, \quad I_3$$

change of the state by the injection of the flux

assume :

- bubble is macroscopic and expand with const. velo.
- deep in the sym. phase, elementary processes are fast enough to realize a new stationary state

$\Longrightarrow$ chemical potential argument

charged-current interection :

$$\mu_W = \mu_0 + \mu_- = -\mu_{t_L} + \mu_{b_L} = -\mu_{\nu_\tau} + \mu_{\tau_L}$$

Yukawa interaction :

$$\mu_0 = -\mu_{t_L} + \mu_{t_R} = -\mu_{b_L} + \mu_{b_R} = -\mu_{\tau_L} + \mu_{\tau_R}$$

no further independent relations

chem. potentials of conserved or almost conserved quantum numbers :

$$\mu_{B-L}, \quad \mu_Y, \quad \mu_{I_3}; \quad \mu_B$$

We assume the sphaleron process is out of equilibrium.

$$\begin{aligned}
\mu_{t_L(b_L)} &= \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} + \frac{1}{6}\mu_Y + (-)\frac{1}{2}\mu_{I_3}, \\
\mu_{t_R} &= \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} + \frac{2}{3}\mu_Y \\
\mu_{b_R} &= \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} - \frac{1}{3}\mu_Y \\
\mu_{\tau_L(\nu_\tau)} &= -\mu_{B-L} - \frac{1}{2}\mu_Y + (-)\frac{1}{2}\mu_{I_3} \\
\mu_{0(-)} &= +(-)\frac{1}{2}\mu_Y - \frac{1}{2}\mu_{I_3} \\
\mu_W &= -\mu_{I_3}
\end{aligned}$$

baryon and lepton number densities:

$$\begin{aligned} n_B &= 3 \cdot \frac{1}{3} \cdot \frac{T^2}{6}(\mu_{t_L} + \mu_{t_R} + \mu_{b_L} + \mu_{b_R}) \\ &= \frac{T^2}{9}(2\mu_B + 2\mu_{B-L} + \mu_Y) \\ n_L &= \frac{T^2}{6}(\mu_{\nu_\tau} + \mu_{\tau_L} + \mu_{\tau_R}) = \frac{T^2}{6}(-3\mu_{B-L} - 2\mu_Y) \end{aligned}$$

If $n_B = n_L = 0$ before the injection of the hypercharge flux,

$$\mu_{B-L} = -\frac{2}{3}\mu_Y, \qquad \mu_B = \frac{1}{6}\mu_Y$$

hypercharge density:

$$\begin{aligned} \frac{Y}{2} &= \frac{T^2}{6}\left\{3\left[\frac{1}{6}(\mu_{t_L} + \mu_{b_L}) + \frac{2}{3}\mu_{t_R} - \frac{1}{3}\mu_{b_R})\right]\right. \\ &\quad \left. - \frac{1}{2}(\mu_{\nu_\tau} + \mu_{\tau_L}) - \mu_{\tau_R}\right\} + \frac{T^2}{3}\frac{1}{2}(\mu_0 - \mu_-)m \\ &= \frac{T^2}{6}(m + \frac{5}{3})\mu_Y \qquad [m = \#(\text{Higgs doublets})] \end{aligned}$$

$$\therefore \quad \mu_B = \frac{Y}{2(m+5/3)T^2}$$

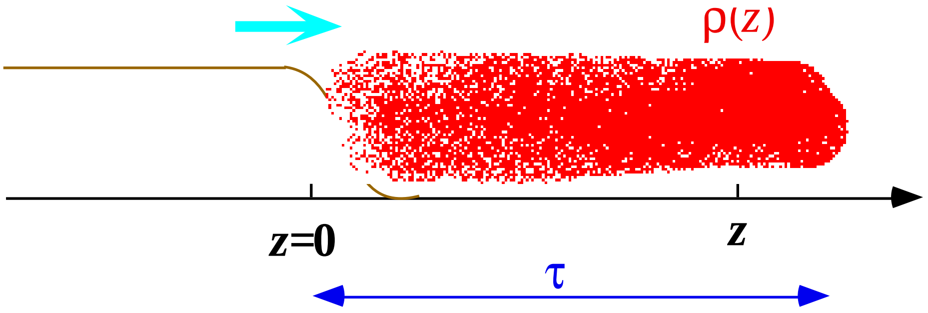
Integrating the equation for $\dot{n}_B$,

$$n_B = -\frac{\Gamma_{\text{sph}}}{T}\int dt\, \mu_B = -\frac{\Gamma_{\text{sph}}}{2(m+5/3)T^3}\int dt\, Y$$

where

$$\int dt\, Y = \int_{-\infty}^{z/v_w} dt\, \rho_Y(z - v_w t) = \frac{1}{v_w}\int_0^\infty dz\, \rho_Y(z).$$

[z = distance from the bubble wall]



The last integral is approximated as

$$\frac{1}{v_w}\int_0^\infty dz\,\rho_Y(z) \simeq \frac{F_Y\tau}{v_w}$$

where

τ = transport time within which the scattered fermions are captured by the wall

generated BAU :

$$\boxed{\frac{n_B}{s} \simeq \mathcal{N}\frac{100}{\pi^2 g_*}\cdot\kappa\alpha_W^4\cdot\frac{F_Y}{v_w T^3}\cdot\tau T}$$

where

$$\mathcal{N} \sim O(1)$$

$$\tau T \simeq \begin{cases} 1 & \text{for quarks} \\ 10^2 \sim 10^3 & \text{for leptons} \end{cases}$$

MC simulation⇒ forward scattering enhanced :

for top quark

$\tau T \simeq 10 \sim 10^3$ max. at $v_w \simeq 1/\sqrt{3}$

for this optimal case [top quark]

$$\boxed{\frac{n_B}{s} \simeq 10^{-3}\cdot\frac{F_Y}{v_w T^3}}$$

$\Longrightarrow$ $F_Y/(v_w T^3) \sim O(10^{-7})$ would be sufficient to explain the BAU.

charge carriers :
$(\tau T)_{\rm quark} \ll \tau T$ for leptons, higgsino, Wino, Bino

---

- Calculation of $\Delta R \longrightarrow$ chiral charge flux

relative phase of $\langle\Phi_1\rangle$ and $\langle\Phi_2\rangle \Rightarrow$ CP violating angle θ
$\Longrightarrow$Dirac equation through Yukawa coupling

$$-f\langle\phi(x)\rangle = m(x) \quad \in \mathbf{C}$$

$$i\partial\!\!\!/\psi(x) - m(x)P_R\psi(x) - m^*(x)P_L\psi(x) = 0$$

(i) perturbative method [FKOTT, P.R.D50('94)]
(ii) numerical method [CKN, N.P.B373('92), FKOT, P.T.P.95('96)]

$$\Downarrow$$

ΔR as a function of (m, a, p_L)

$$\Downarrow$$

chiral charge flux $F_Q(T, m, a, v_w)$

Example [CKN, N.P.B373('92)]

$$m(z) = m_0\frac{1+\tanh(az)}{2}\exp\left(-i\pi\frac{1-\tanh(az)}{2}\right)$$

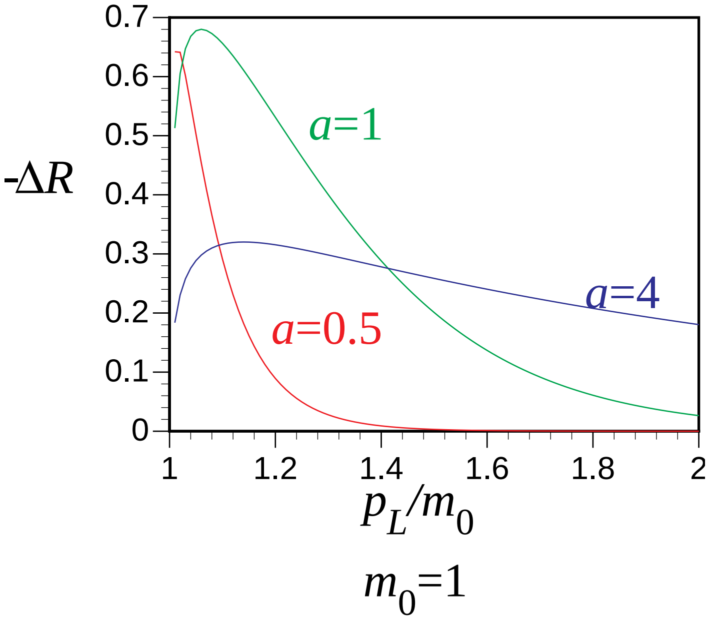
— no CP violation in the broken phase $[z \sim \infty]$

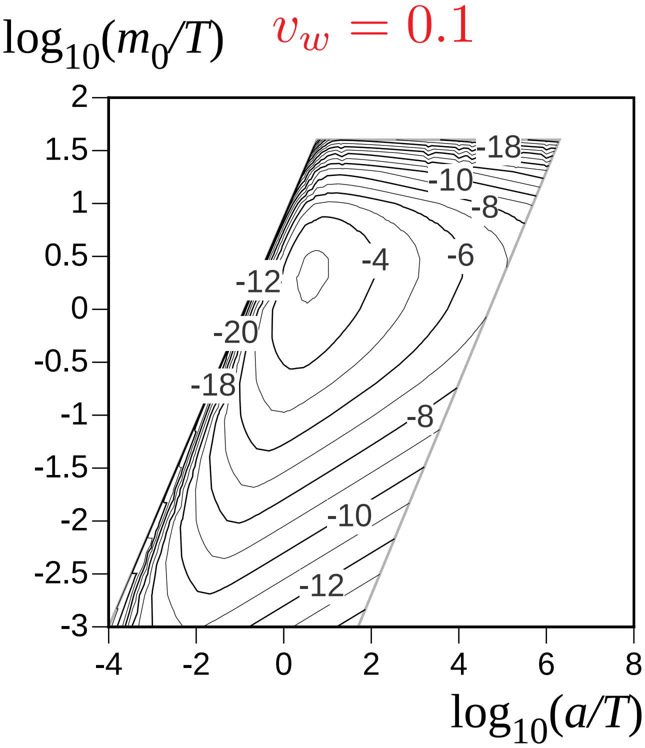
- $\Delta R \equiv {R^s}_{R\to L} - {\bar{R}^s}_{R\to L}$
  wall width $\simeq$ wave length of the carrier $\Rightarrow \Delta R \sim O(1)$
  for larger energy, ΔR decays exponentially
- chiral charge flux
  normalized as

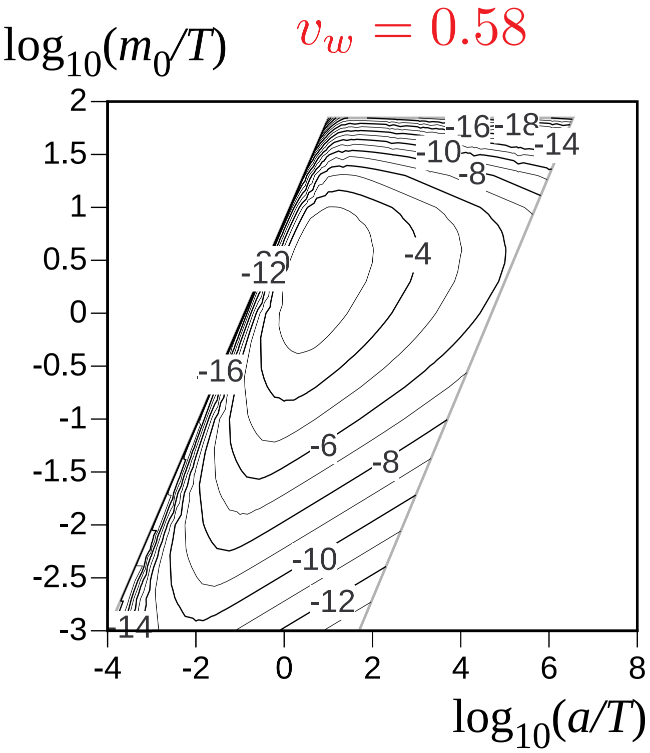
$$\frac{F_Q}{v_w T^3(Q_L - Q_R)} \qquad \text{[dimensionless]}$$

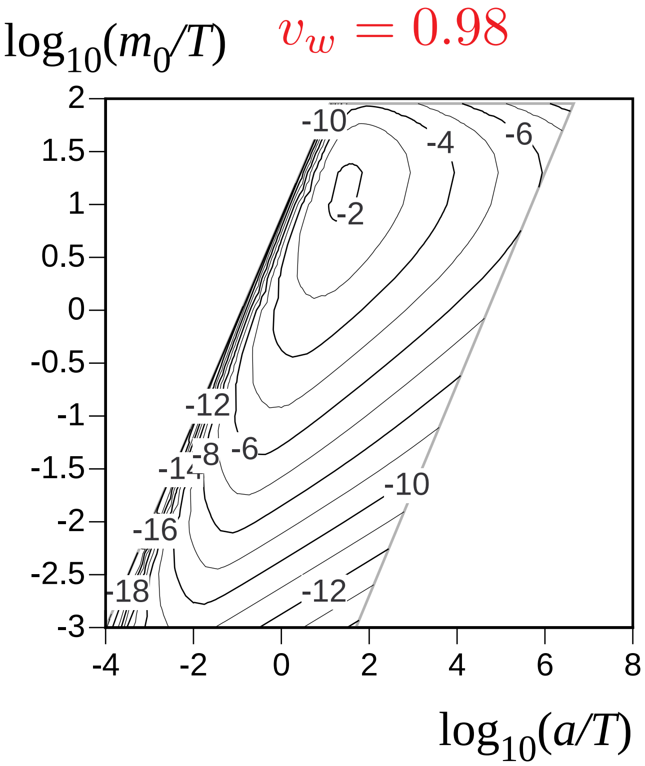
Numerical results

$T = 100$ GeV









$$\frac{n_B}{s} \simeq \mathcal{N}\frac{100}{\pi^2 g_*}\cdot\kappa\alpha_W^4\cdot\frac{F_Y}{v_w T^3}\cdot\tau T$$

$$\simeq 10^{-3}\cdot\frac{F_Y}{v_w T^3}$$

for an optimal case (top quark)

IV-2. Spontaneous baryogenesis

(i) in two-Higgs-doublet model [at $T = 0$]

$$\Delta\mathcal{L}_{\rm eff} = -\frac{g^2 N_f}{24\pi^2}\theta(x) F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

—CP-even $\Leftarrow$ $\theta(x)$, $F\tilde{F}$: CP-odd

$\Longrightarrow$ $\dot{\theta} \sim$ chem.pot. for N_{CS}

At high-T, suppressed by $\left(\frac{m_t}{T}\right)^2$.

(ii) bias for the hypercharge instead of N_{CS} [CKN,P.L.B263('91)]

neutral comp. of 2 Higgs scalars :

$$\phi_j^0(x) = \rho_j(x) \mathrm{e}^{i\theta_j}, \qquad (j = 1, 2)$$

Suppose only ϕ_1 couples to the fermions.

Eliminate θ_1 in Yukawa int. by anomaly-free $U(1)_Y$ trf.

fermion kinetic term induces:

$$2\partial_\mu\theta_1(x)\left[\frac{1}{6}\bar{q}_L(x)\gamma^\mu q_L(x) + \frac{2}{3}\bar{u}_R(x)\gamma^\mu u_R(x)\right.$$
$$\left. -\frac{1}{3}\bar{d}_R(x)\gamma^\mu d_R(x) - \frac{1}{2}\bar{l}_L(x)\gamma^\mu l_L(x) - \bar{e}_R(x)\gamma^\mu e_R(x)\right]$$

$\langle\dot{\theta}_1\rangle \neq 0$ during EWPT $\Rightarrow$ *charge potential*

⋆ criticism by Dine-Thomas [P.L.B328('94)]

▷ The current is not the conserved Y-current, but the fermionic part of it.
Nonconservation of Y in the broken phase leads to

$$\partial_\mu \theta_1 \cdot j^\mu_Y \propto \frac{{m_t}^2}{T^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

▷ The bias for Y exists where $(v/T)^2 > 0$.
The sphaleron process is effective for $v < v_{co}$
$\therefore$ The generated B is suppressed by $(v_{co}/T^2 \sim O(10^{-6})$.

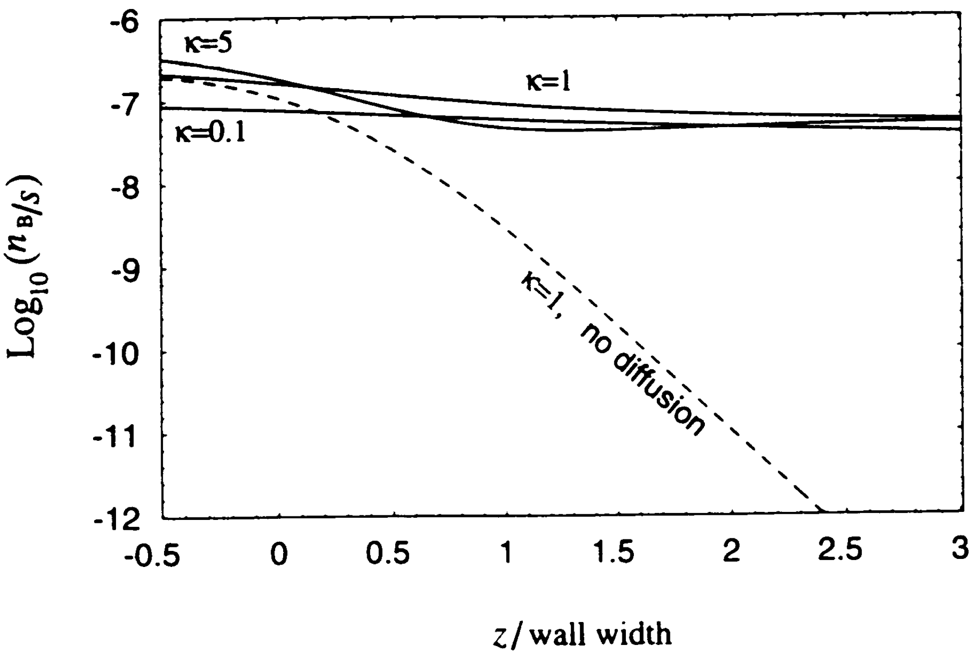
⋆ enhancement by diffusion [CKN, P.L.B336('94)]

Diffusion carries Y into the symmetric phase.
$\longrightarrow$ nonlocal baryogenesis

for the profile

$$\langle \phi(z) \rangle = v \frac{1-\tanh(az)}{2} \exp\left[-i \frac{\pi}{2} \frac{1-\tanh(az)}{2} \right]$$

z_{co} vs $\log_{10}[(n_B/s)(g_*/100)]$ with $v_{co} = \varphi(z_{co})$



— almost independent of z_{co}

V. Discussions

Minimal Standard Model :

difficulties $\left\{\begin{array}{l} \text{first order EWPT} \\ \text{decoupling of sphaleron} \\ \text{sufficient } CP \text{ violation} \end{array}\right.$ $\Big]$ for $m_H > 70\text{GeV}$ (bracket covers first two items)

$\Downarrow$

2-doublet extension of the SM [$\supset$ MSSM]

- first order EWPT
  $\Longrightarrow$ constraints on mass parameters in scalar sector
- two mechanisms work

Problems to be solved

1. EWPT in the extended models
   - effective potential with 3 order parameters
     bubble profile $\Rightarrow$ wall width
     dynamics, CP violation
   - Lattice MC simulation
   - dynamics of EWPT
2. unified treatment of the mechanism
   Huet and Nelson, P.L.**B355** ('95), P.R.**D53** ('96)
   field theory in CP-viol. background
3. relation to the observed CP violation
   - phases of the soft-SUSY-breaking parameters

     chargino mass $= \begin{pmatrix} M_2 & -\frac{i}{\sqrt{2}} g_2 v_u e^{i\theta} \\ -\frac{i}{\sqrt{2}} g_2 v_d & -\mu \end{pmatrix}$ $\Rightarrow$ complex parameters
   - CP-violating bubble wall profile
     $\Longrightarrow$ Higgs sector CP violation