

Electroweak Baryogenesis

K. Funakubo, Saga Univ.

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K.F., Prog.Theor.Phys.**96** ('96) 475.

V.A. Rubakov and M.E. Shaposhnikov, hep-ph/9603208.

A.Cohen, et al., Ann.Rev.Nucl.Part.Sci. **33** ('93) 27.

I. Introduction

Baryon Asymmetry of the Universe (BAU)

$$\iff \frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} = (0.21 - 0.90) \times 10^{-10}$$

← big-bang nucleosynthesis

... constant after decoupling of B-violating processes

evidence of BAU

[Steigman, Ann.Rev.Astron.Astrop.**14**('76)]

1. no anti-matter in cosmic rays from our galaxy

some anti-matter consistent as secondary products

2. nearby clusters of galaxies are stable

a cluster: $(1 \sim 100) M_{\text{galaxy}} \simeq 10^{12 \sim 14} M_{\odot}$

Starting from a B -symmetric universe ...

$$\frac{n_b}{s} \simeq \frac{n_{\bar{b}}}{s} \sim 8 \times 10^{-11} \quad \text{at } T = 38\text{MeV}$$

$$\sim 7 \times 10^{-20} \quad \text{at } T = 20\text{MeV}$$

$N\bar{N}$ -annihilation decouple

At $T = 38\text{MeV}$,

mass within a causal region = $10^{-7} M_{\odot} \ll 10^{12} M_{\odot}$.

We must have the BAU $n_B/s = (0.21 - 0.90) \times 10^{-10}$ before the universe was cooled down to $T \simeq 38\text{MeV}$.

3 conditions for generation of BAU [Sakharov, '67]

- (1) baryon number violation
- (2) C and CP violation
- (3) departure from equilibrium

if B -violation is in equil. $\implies n_b = n_{\bar{b}}$

(2)

If C or CP is conserved, no B is generated:

This is because B is odd under C and CP .

indeed ...

ρ_0 : baryon-symmetric initial state of the universe *s.t.*

$$\langle n_B \rangle_0 = \text{Tr}[\rho_0 n_B] = 0$$

time evolution of $\rho \Leftrightarrow$ Liouville eq.: $i\hbar \frac{\partial \rho}{\partial t} + [\rho, H] = 0$

If H is C - or CP -invariant, $[\rho, C] = 0$ or $[\rho, CP] = 0$

(spontaneous CP viol. $\implies [\rho, CP] \neq 0$)

Since $CBC^{-1} = -B$ and $CPBCP^{-1} = -B$

$$\langle n_B \rangle = \text{Tr}[\rho n_B] = \text{Tr}[\rho C n_B C^{-1}] = -\text{Tr}[\rho n_B]$$

or

$$\langle n_B \rangle = \text{Tr}[\rho CP n_B (CP)^{-1}] = -\text{Tr}[\rho n_B]$$

Both C and CP must be violated to have $\langle n_B \rangle \neq 0$.

example — GUTs

[Kolb and Turner, The Early Universe]

out-of-equil. decay of X bosons $m_X \gtrsim 10^{15}\text{GeV}$

$X =$ gauge boson or Higgs boson

Consider 2 channels:

$$\begin{cases} X \longrightarrow qq & \Delta B = 2/3 & \text{with branching } r \\ X \longrightarrow \bar{q}\bar{l} & \Delta B = -1/3 & \text{with branching } 1 - r \end{cases}$$

in the decay of $X\text{-}\bar{X}$ pairs

$$\langle \Delta B \rangle = \frac{2}{3}r - \frac{1}{3}(1 - r) - \frac{2}{3}\bar{r} + \frac{1}{3}(1 - \bar{r}) = r - \bar{r}$$

$\therefore C$ or CP is conserved ($r = \bar{r}$) $\implies \Delta B = 0$

At $T \simeq m_X$, decay rate of $X = \Gamma_D \simeq \alpha m_X$

$\alpha \sim 1/40$ for gauge boson, $\alpha \sim 10^{-6 \sim -3}$ for Higgs boson

Hubble parameter : $H \sim 1.7\sqrt{g_*}T^2/m_{Pl}$

$g_* \simeq 10^{2 \sim 3}$: massless degrees of freedom

$\therefore \Gamma_D \simeq H$ at $T \simeq m_X$

\implies decay and production of $X\bar{X}$ are out of equil.

As we shall see, $B + L$ were washed out before EWPT.

$\therefore B - L$ -conserving GUT (e.g. minimal $SU(5)$ model) will be useless to generate the BAU.

other candidates for generating BAU

- \exists Majorana neutrino $\Rightarrow L$ -violating interaction
decoupling of L -violating interaction
 \Rightarrow constraints on the neutrino mass
- Affleck-Dine mechanism in a supersymmetric model
[A-D, N.P. B174('86) 45]
 $\langle \text{squark} \rangle \neq 0$ or $\langle \text{slepton} \rangle \neq 0$ along (nearly) flat directions,
at high temperature
 $\Rightarrow B$ - and/or L -violation
- Electroweak Baryogenesis
 - (1) anomaly in $B + L$ -current
 - (2) C -violation (chiral gauge)
 CP -violation in KM matrix or extension of SM
 - (3) first-order EWPT with expanding bubble walls
- topological defects
EW string, domain wall \sim EW baryogenesis
effective volume is too small, mass density of the universe

N.B.

The BAU may be generated by some combination of these mechanisms. Any way,

EWPT = the last period to affect the BAU.

II. Sphaleron Process

II-1. Anomalous fermion number nonconservation

axial anomaly in the standard model

$$\partial_\mu j_{B+L}^\mu = \frac{N_f}{16\pi^2} [g^2 \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu} \tilde{B}^{\mu\nu}],$$

$$\partial_\mu j_{B-L}^\mu = 0,$$

$N_f =$ number of the generations, $\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$

integrating these equations,

$$\begin{aligned} B(t_f) - B(t_i) &= \frac{N_f}{32\pi^2} \int_{t_i}^{t_f} d^4x \left[g^2 \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu} \tilde{B}^{\mu\nu} \right] \\ &= N_f [N_{CS}(t_f) - N_{CS}(t_i)] \end{aligned}$$

where N_{CS} is the Chern-Simons number defined, in the $A_0 = 0$ gauge, by

$$N_{CS} = \frac{1}{32\pi^2} \int d^3x \epsilon_{ijk} \left[g^2 \text{Tr} \left(F_{ij} A_k - \frac{2}{3} g A_i A_j A_k \right) - g'^2 B_{ij} B_k \right]_t$$

: gauge-noninvariant

For **classical vacua** of the gauge sector, $N_{CS} \in \mathbf{Z}$

$$\iff F_{ij} = B_{ij} = 0$$

$$\iff A = iU^{-1}dU \text{ and } B = dv \text{ with } U \in SU(2)$$

$U : S^3 \longrightarrow SU(2) \simeq S^3$ is characterized by its winding number, $\pi_3(SU(2)) \simeq \mathbf{Z}$. $\leftrightarrow N_{CS}$

background of gauge fields with $\Delta N_{CS} = 1$

$$\Rightarrow \Delta B = 1 \text{ for each generation}$$

(\because level-crossing phenomenon) \longleftrightarrow index theorem

transition rate between configurations with $\Delta N_{CS} = 1$



WKB approximation:

$T = 0$

(valley or constrained) instanton = *finite euclidean action*

tunneling probability $\sim e^{-2S_{inst}} = e^{-8\pi^2/g^2}$

for EW theory, $e^{-2S_{inst}} \simeq e^{-378} = 10^{-164}$

$T \neq 0$

[Affleck, P.R.L.46('81)]

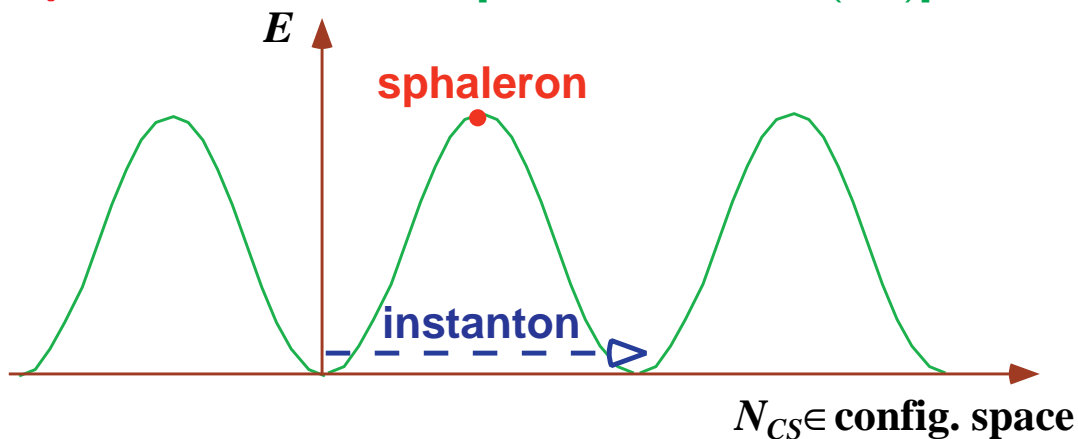
\exists classical static **saddle-point** solution with *finite energy*



top of the energy barrier dividing two classical vacua



sphaleron solution [Manton, P.R.D28('83)]



$$E_{sph}(T = 0) = \frac{2M_W}{\alpha_W} B \left(\frac{\lambda}{g^2} \right) \simeq 10\text{TeV}$$

λ : the Higgs self coupling, $\alpha_W = g^2/(4\pi)$

$1.5 \leq B \leq 2.7$ for $\lambda/g^2 \in [0, \infty)$

$E_{sph} = \text{finite} \implies \exists$ transition over the barrier

sphaleron for $\theta_W \neq 0$	[Brihaye and Kunz, P.R.D50('94)]
2-doublet Higgs model	[Peccei, Zhang and Kastening, P.L.B266('91)]
squark vs sphaleron	[Moreno, Oaknin and Quirós, hep-ph/9612212]

Transition Rate

[Arnold and McLerran, P.R.D36('87)]

♣ $\omega_- / (2\pi) \lesssim T \lesssim T_C$

$$\Gamma_{\text{sph}}^{(b)} \simeq k \mathcal{N}_{\text{tr}} \mathcal{N}_{\text{rot}} \frac{\omega_-}{2\pi} \left(\frac{\alpha_W(T) T}{4\pi} \right)^3 e^{-E_{\text{sph}}/T}$$

zero modes $\rightarrow \begin{cases} \mathcal{N}_{\text{tr}} = 26 \\ \mathcal{N}_{\text{rot}} = 5.3 \times 10^3 \end{cases}$ for $\lambda = g^2$

$\omega_-^2 \simeq (1.8 \sim 6.6) m_W^2$ for $10^{-2} \leq \lambda/g^2 \leq 10$

$k \simeq O(1)$

♣ $T \gtrsim T_C$ symmetric phase — no mass scale

dimensional analysis :

$$\Gamma_{\text{sph}}^{(s)} \simeq \kappa (\alpha_W T)^4$$

check by Monte Carlo simulation $\langle N_{CS}^2(t) \rangle = 2\Gamma V t$ as $t \rightarrow \infty$

$\kappa > 0.4$ $SU(2)$ gauge-Higgs system

[Ambjørn, et al. N.P.B353('91)]

$\kappa = 1.09 \pm 0.04$ $SU(2)$ pure gauge system

[Ambjørn and Krasnitz, P.L.B362('95)]

We use 'sphaleron transition' even in the symmetric phase.

II-2. Washout of $B + L$

$B + L$ would be washed out after the EWPT, if the EWPT is **second order** or the sphaleron process does **not decouple** after it.

decoupling of sphaleron process $\Leftrightarrow \Gamma_{\text{sph}} < \text{Hubble parameter}$

at $T = T_C \simeq 100\text{GeV}$,

$$H(T_C) = \frac{\dot{R}(t)}{R(t)} \simeq 1.7\sqrt{g_*}\frac{T_C^2}{m_{Pl}} \simeq 10^{-13}\text{GeV}$$

$g_* \sim 100$: effective massless degrees of freedom

At $T > T_C$,

$$\Gamma_{\text{sph}} \simeq \Gamma_{\text{sph}}^{(s)}/T^3 \simeq \kappa\alpha_W^4 T \sim 10^{-4}\text{GeV} \gg H(T_C)$$

\Rightarrow $B + L$ -changing process in equilibrium

relic baryon number after the washout

particle number density [$m/T \ll 1$ and $\mu/T \ll 1$]

$$n_+ - n_- = \int \frac{d^3\mathbf{k}}{(2\pi)^2} \left[\frac{1}{e^{\beta(\omega_k - \mu)} \mp 1} - \frac{1}{e^{\beta(\omega_k + \mu)} \mp 1} \right]$$
$$\simeq \begin{cases} \frac{T^3}{3} \frac{\mu}{T} & \text{for bosons} \\ \frac{T^3}{6} \frac{\mu}{T} & \text{for fermions,} \end{cases}$$

chemical equilibrium — all the gauge int., Yukawa, sphaleron

W^-	$u_{L(R)}$	$d_{L(R)}$	$e_{iL(R)}$	ν_{iL}	ϕ^0	ϕ^-
μ_W	$\mu_{u_{L(R)}}$	$\mu_{d_{L(R)}}$	$\mu_{iL(R)}$	μ_i	μ_0	μ_-

$$\text{gauge int.} \Leftrightarrow \mu_W = \mu_{d_L} - \mu_{u_L} = \mu_{iL} - \mu_i = \mu_- + \mu_0$$

$$|0\rangle \Leftrightarrow u_L d_L d_L \nu_L \Leftrightarrow N_f(\mu_{u_L} + 2\mu_{d_L}) + \sum_i \mu_i = 0$$

Quantum number densities [in unit of $T^2/6$]

$$B = N_f(\mu_{u_L} + \mu_{u_R} + \mu_{d_L} + \mu_{d_R}) = 4N_f\mu_{u_L} + 2N_f\mu_W,$$

$$L = \sum_i (\mu_i + \mu_{iL} + \mu_{iR}) = 3\mu + 2N_f\mu_W - N_f\mu_0$$

$$\begin{aligned} Q &= \frac{2}{3}N_f(\mu_{u_L} + \mu_{u_R}) \cdot 3 - \frac{1}{3}N_f(\mu_{d_L} + \mu_{d_R}) \cdot 3 \\ &\quad - \sum_i (\mu_{iL} + \mu_{iR}) - 2 \cdot 2\mu_W - 2m\mu_- \\ &= 2N_f\mu_{u_L} - 2\mu - (4N_f + 4 + 2m)\mu_W + (4N_f + 2m)\mu_0 \end{aligned}$$

$$\begin{aligned} I_3 &= \frac{1}{2}N_f(\mu_{u_L} - \mu_{d_L}) \cdot 3 + \frac{1}{2} \sum_i (\mu_i - \mu_{iL}) \\ &\quad - 2 \cdot 2\mu_W - 2 \cdot \frac{1}{2}m(\mu_0 - \mu_-) \\ &= -(2N_f + m + 4)\mu_W \end{aligned}$$

$$\mu \equiv \sum_i \mu_i$$

m : number of Higgs doublets

- symmetric phase

$$\implies Q = I_3 = 0$$

$$B = \frac{8N_f + 4m}{22N_f + 13m}(B - L), \quad L = -\frac{14N_f + 9m}{22N_f + 13m}(B - L)$$

- broken phase

$$\implies Q = 0 \text{ and } \mu_0 = 0$$

$$B = \frac{8N_f + 4m + 8}{24N_f + 13m + 26}(B - L)$$

$$L = -\frac{16N_f + 9m + 18}{24N_f + 13m + 26}(B - L)$$

\therefore If $(B - L)_{\text{primordial}} = 0$, $B = L = 0$ at present !

To have nonzero BAU,

- (i) we must have $B - L$ before the sphaleron process decouples, or
- (ii) $B + L$ must be created at the first-order EWPT, and the sphaleron process must decouple immediately after that.

(i) \Leftarrow GUTs, Majorana ν , Affleck-Dine

(ii) $=$ Electroweak Baryogenesis

III. Electroweak Phase Transition (EWPT)

III-1. Static properties of the phase transition

rate of any interaction at $T \sim T_C \ll$ Hubble parameter
 \implies equil. thermodynamics applicable to static properties

order of the transition, transition temperature,
latent heat and surface tension (if it is first order)



free energy density = effective potential at $T \neq 0$



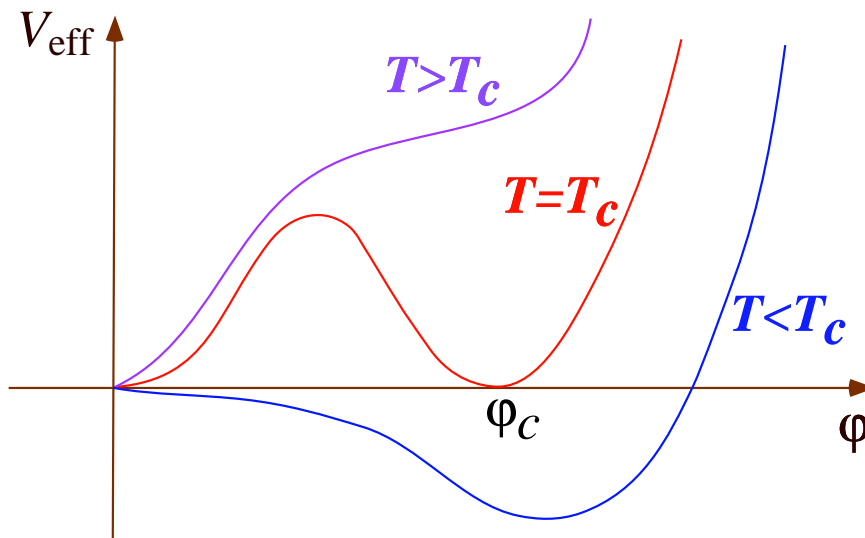
function of the order parameters and T

Example Minimal standard model (MSM)

order parameter: $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$

a first order phase transition

$$\varphi_C \equiv \lim_{T \uparrow T_C} \varphi(T) \neq 0$$



one-loop level,

$$V_{\text{eff}}(\varphi; T) = V_{\text{tree}}(\varphi) + V^{(1)}(\varphi; T),$$

where

$$V_{\text{tree}}(\varphi) = -\frac{1}{2}\mu_0^2\varphi^2 + \frac{\lambda_0}{4}\varphi^4$$

$$V^{(1)}(\varphi; T) = -\frac{i}{2} \sum_A c_A \int_k \log \det [i\mathcal{D}_A^{-1}(k; \varphi)]$$

with

μ_0^2, λ_0 : bare parameters \leftarrow renormlized at $T = 0$

A runs over all the partice species

$|c_A|$ counts the degrees of freedom, $\begin{cases} c_A > 0 & \text{for bosons} \\ c_A < 0 & \text{for fermions} \end{cases}$

$$\int_k \equiv iT \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{k}}{(2\pi)^3}$$

$$\text{with } k^0 = \omega_n = \begin{cases} 2n\pi T & \text{for bosons,} \\ (2n+1)\pi T & \text{for fermions.} \end{cases}$$

W -boson :

$$c_W = 2$$

$$i\mathcal{D}_W^{-1\mu\nu}(k; \varphi) = (-k^2 + m_W^2(\varphi))\eta^{\mu\nu} + (1 - \frac{1}{\xi})k^\mu k^\nu$$

with $m_W(\varphi) = \frac{1}{2}g\varphi$

Dirac fermion :

$$c_f = -2$$

$$i\mathcal{D}_f^{-1}(k; \varphi) = \not{k} - m_f(\varphi)$$

with $m_f(\varphi) = y_f\varphi/\sqrt{2}$

Higgs boson:

$$m_H^2(\varphi) = 3\lambda\varphi^2 - \mu^2 \text{ — negative for small } \varphi$$

⇒ complex V_{eff}

← sum over *daisy diagrams*, or *improved perturbation*

neglecting the Higgs contribution

$$V_{\text{eff}}(\varphi; T) = V_0(\varphi) + \bar{V}(\varphi; T)$$

where

$$V_0(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2Bv_0^2\varphi^2 + B\varphi^4 \left[\log\left(\frac{\varphi^2}{v_0^2}\right) - \frac{3}{2} \right]$$

$$\bar{V}(\varphi; T) = \frac{T^4}{2\pi^2} [6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)]$$

$$B = \frac{3}{64\pi^2 v_0^4} (2m_W^4 + m_Z^4 - 4m_t^4)$$

$$I_{B,F}(a^2) \equiv \int_0^\infty dx x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right)$$

with

$v_0 = 246\text{GeV}$ is the minimum of $V_0(\varphi)$

$$a_A = m_A(\varphi)/T$$

high-temperature expansion [$m/T \ll 1$]

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16} \log \frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16} \left(\gamma_E - \frac{3}{4} \right) \frac{a^4}{2} + O(a^6)$$

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{16} \log \frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16} \left(\gamma_E - \frac{3}{4} \right) + O(a^6)$$

For $T > m_W, m_Z, m_t$,

$$V_{\text{eff}}(\varphi; T) \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2)$$

$$E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2)$$

$$\lambda_T = \lambda$$

$$- \frac{3}{16\pi^2 v_0^4} \left(2m_W^4 \log \frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log \frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log \frac{m_t^2}{\alpha_F T^2} \right)$$

with $\log \alpha_B = 2 \log 4\pi - 2\gamma_E$ and $\log \alpha_F = 2 \log \pi - 2\gamma_E$

$$\text{At } T_C, \varphi_C = \frac{2ET_C}{\lambda_{T_C}}$$

$$\Gamma_{\text{sph}}^{(b)}/T_C^3 < H(T_C) \iff \frac{\varphi_C}{T_C} \gtrsim 1$$

\implies upper bound on λ

$$[m_H = \sqrt{2}\lambda v_0]$$

$$m_H \lesssim 46 \text{ GeV}$$

\iff inconsistent with the lower bound $m_H > 89.8 \text{ GeV}$

2-doublet extension of the MSM or MSSM :
 more scalars \longrightarrow more φ^3 -terms
 \implies stronger first-order EWPT

MSSM with light stop [de Carlos and Espinosa, NPB503 ('97)]

stop mass-squared matrix :

$$\begin{pmatrix} m_{\tilde{q}}^2 + \left(\frac{g_1^2}{24} - \frac{g_2^2}{8} \right) (\rho_u^2 - \rho_d^2) + \frac{1}{2} y_t^2 \rho_u^2 & \frac{y_t}{\sqrt{2}} (\mu \rho_d + A \rho_u e^{-i\theta}) \\ * & m_{\tilde{t}}^2 - \frac{g_1^2}{6} (\rho_u^2 - \rho_d^2) + \frac{1}{2} y_t^2 \rho_u^2 \end{pmatrix}$$

where

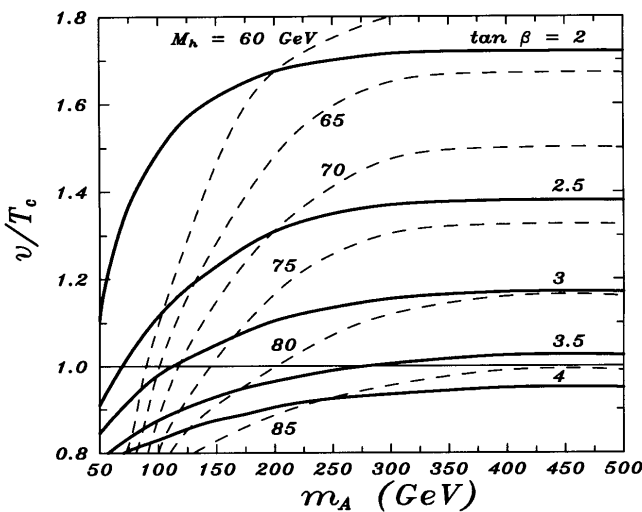
$$\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_d \\ 0 \end{pmatrix}, \quad \langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho_u e^{i\theta} \end{pmatrix},$$

$m_{\tilde{q}} = 0$ or $m_{\tilde{t}} = 0 \implies$ smaller eigenvalue $\equiv m_-^2 \sim O(\rho^2)$

\therefore high- T expansion

$$\bar{V}_{\tilde{t}}(\rho_i, \theta; T) \implies -\frac{T}{6\pi} (m_-^2)^{3/2}$$

φ_C/T_C vs m_A for various $\tan \beta$ (m_h)



$m_t = 175 \text{ GeV}$
 $m_{\tilde{q}} = 250 \text{ GeV}$
 $m_{\tilde{t}} = 0$

Figure 5: Same as fig. 4 but for $M_t = 175 \text{ GeV}$, $m_U = 0$, $m_Q = 250 \text{ GeV}$.

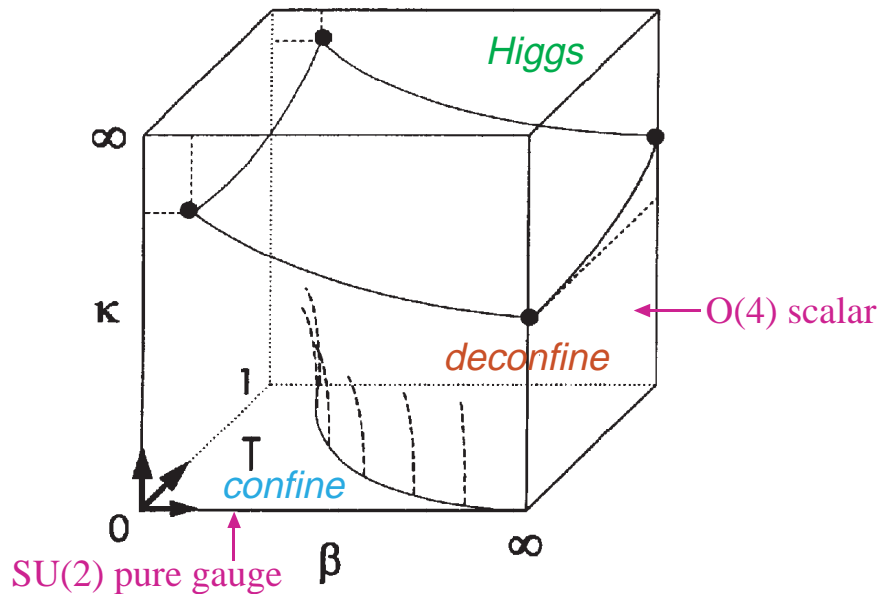
△ Monte Carlo Simulations [MSM][Jansen, N.P.B.Suppl.47('96)]

effective fermion mass : $m_f(T) \sim O(T) \leftarrow$ nonzero modes

- 4-dim. $SU(2)$ system with a Higgs doublet
- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet \uparrow
time-component of the gauge field

only zero-freq. modes of the bosons survive as $T \rightarrow$ large
 matching finite- T Green's functions with 4-dim. theory
 $\Rightarrow T$ -dependent parameters

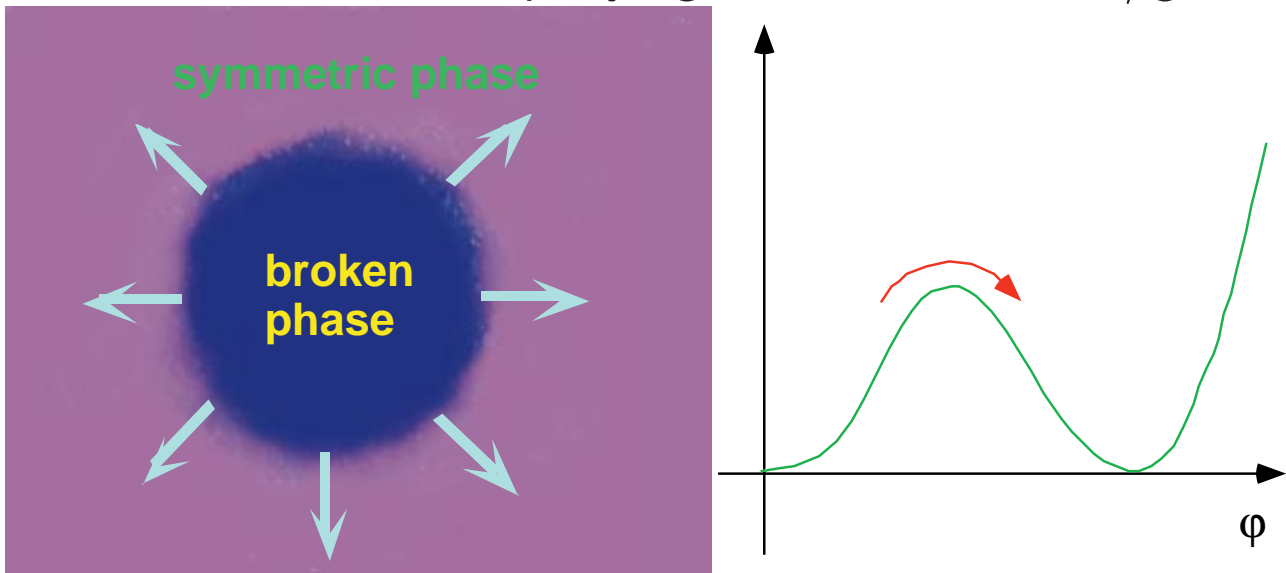
schematic finite- T phase diagram [λ -fixed]



- the latest 4d lattice MC study [Csikor, et al. hep-lat/9809293]
 EWPT is of first order for $m_h \leq 66.5 \pm 1.4 \text{ GeV}$
 endpoint of the EWPT : $m_h = 72.4 \pm 1.7 \text{ GeV}$
- $\varphi_C/T_C > 1$ is not satisfied for $m_h \geq 50 \text{ GeV}$
- The numerical results coincide with those of the continuum two-loop perturbation theory, for $m_h \leq 70 \text{ GeV}$.

III-2. Dynamics of the phase transition

first-order EWPT accompanying bubble nucleation/growth



Suppose that $V_{\text{eff}}(\varphi; T_C)$ is known.

nucleation rate per unit time and unit volume:

$$I(T) = I_0 e^{-\Delta F(T)/T}$$

where

$$\Delta F(T) = \frac{4\pi}{3} r^3 [p_s(T) - p_b(T)] + 4\pi r^2 \sigma$$

with

$$p_s(T) = -V_{\text{eff}}(0; T), \quad p_b(T) = -V_{\text{eff}}(\varphi(T); T)$$

supercooling $\longrightarrow p_s(T) < p_b(T)$

$\sigma \simeq \int dz (d\varphi/dz)^2$: surface energy density

$$\text{radius of the critical bubble} : r_*(T) = \frac{2\sigma}{p_b(T) - p_s(T)}$$

$$s = \frac{\partial V_{\text{eff}}}{\partial T} \quad : \text{entropy density}$$

$$\rho = V_{\text{eff}} - Ts \quad : \text{energy density}$$

How the EWPT proceeds ?

$f(t)$: fraction of space converted to the broken phase

$$f(t) = \int_{t_C}^t dt' I(T(t')) [1 - f(t')] V(t', t)$$

where

$V(t', t)$: volume of a bubble at t which was nucleated at t'

$$V(t', t) = \frac{4\pi}{3} \left[r_*(T(t')) + \int_{t'}^t dt'' v(T(t'')) \right]^3$$

$T = T(t) \Leftarrow \rho = (\pi^2/30) g_* T^4 \propto R^{-4}$ for RD universe

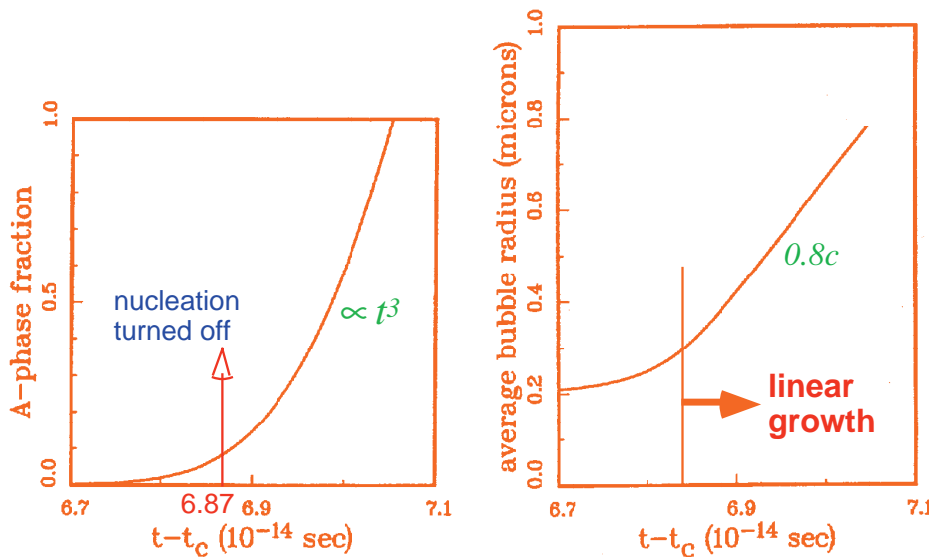
$v(T)$: wall velocity

one-loop V_{eff} of MSM with $m_H = 60\text{GeV}$ and $m_t = 120\text{GeV}$

[Carrington and Kapsta, P.R.D47('93)]

At $6.5 \times 10^{-14}\text{sec}$, bubbles began to nucleate.

[A characteristic time scale of the EW processes is $O(10^{-26})\text{sec}$.]



very small supercooling : $\frac{T_C - T_N}{T_C} \simeq 2.5 \times 10^{-4}$

90% of the universe is converted by bubble growth

weakly first order \iff small φ_C and/or lower barrier height

\implies $\left\{ \begin{array}{l} \text{nucleation dominance over growth} \\ \text{large fluctuation between the two phases} \end{array} \right.$

IV. Mechanism of Electroweak Baryogenesis

At $T \simeq T_C$, hierarchy of time scales :

EW	$t_{EW} \simeq 1\text{GeV}^{-1}$
Yukawa	$t_Y \simeq (m_W/m_f)^4\text{GeV}^{-1}$
QCD	$t_s \simeq 0.1\text{GeV}^{-1}$
Hubble	$H^{-1} \simeq 10^{13}\text{GeV}^{-1}$
sphaleron	$t_{\text{sph}} = (\kappa\alpha_W^4 T)^{-1} \simeq \kappa^{-1} \cdot 10^4\text{GeV}^{-1}$
wall motion	$t_{\text{wall}} \simeq 0.01 \sim 4\text{GeV}^{-1}$

$$t_{\text{wall}} = \frac{\text{wall width}}{\text{wall velocity}} \simeq \frac{0.01 \sim 0.04\text{GeV}^{-1}}{0.1 \sim 0.8}$$

▷ $t_{EW} \ll H^{-1} \Rightarrow$

all particles are in **kinetic equilibrium** at the same temperature

▷ for $m_f \lesssim 0.1\text{GeV}$, $t_Y \sim H^{-1}$

∴ Yukawa int. of light fermions are **out of chemical equil.**

▷ some of flavor-changing int. are **out of chemical equil.**

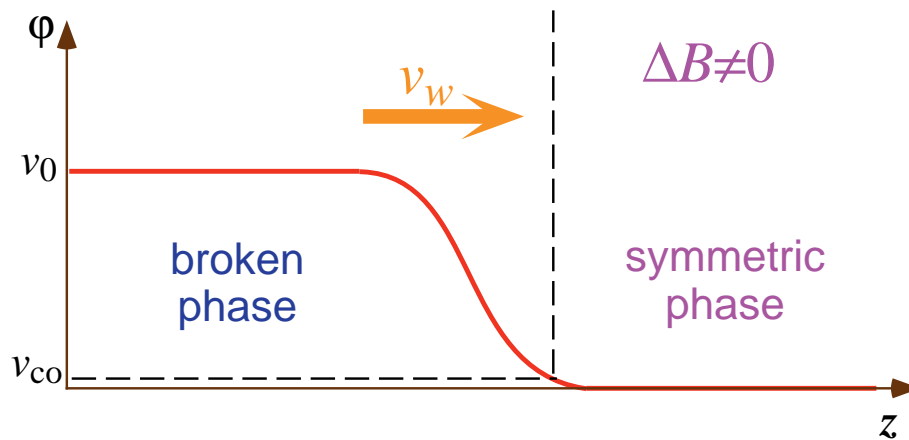
$$\therefore |V_{ub}|, |V_{cb}|, |V_{td}|, |V_{ts}| \ll 1$$

▷ $t_{\text{wall}} \ll t_{\text{sph}}$

\Rightarrow

sphaleron process is **out of chemical equil.** near the bubble wall even in the symmetric phase

∴ Nonequilibrium state is realized near expanding bubble walls.



$$v_{co} \simeq 0.01 v_0 \iff E_{\text{sph}}/T_C \simeq 1$$

bubble wall \iff classical config. of gauge-Higgs system

Effects of CP violation :

- interactions between the particles and the bubble wall
- propagation of the particles in the plasma



generation of baryon number through **sphaleron process**



decoupling of sphaleron process in the broken phase

two scenarios to realize EW baryogenesis:

- **spontaneous baryogenesis + diffusion**
classical, adiabatic
- **charge transport scenario**
quantum mechanical, nonlocal

Both need CP violation other than KM matrix

\iff extension of the MSM

two-Higgs-doublet model, MSSM, . . .

IV-1. Charge transport mechanism

[Nelson, et al. N.P.B373('92)]

CP violation in the Higgs sector [spacetime-dependent]



difference in reflections of chiral fermions and antifermions



net chiral charge flux into the symmetric phase



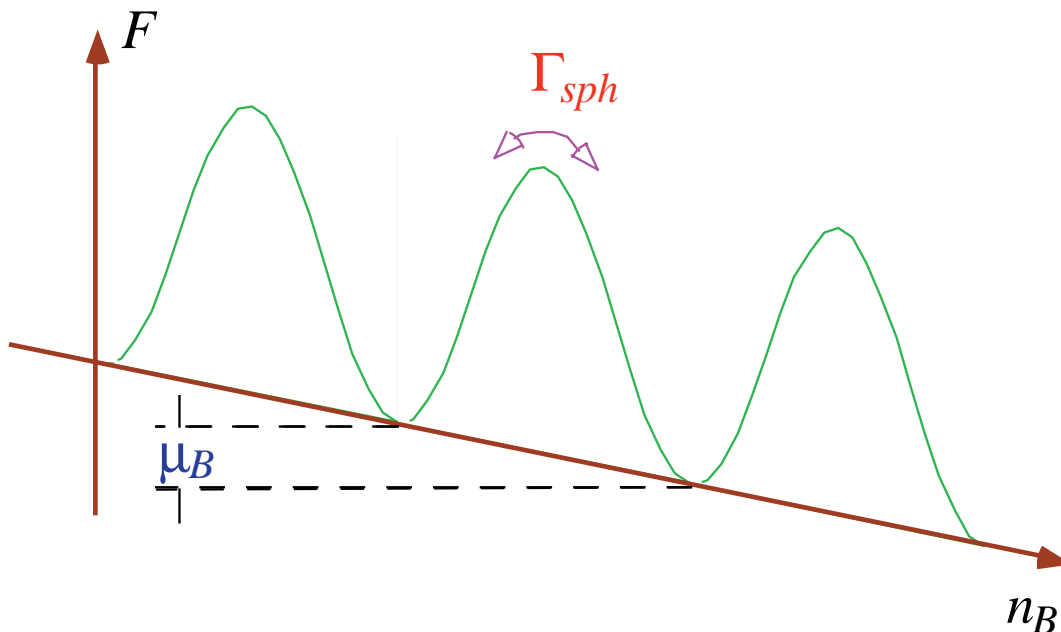
sphaleron transition converts the charge into B

change of distribution functions by the chiral charge flux

⇐ Boltzmann equations

bubble wall velocity \simeq const. \Rightarrow constant chiral charge flux

\Rightarrow bias on free energy along B [stationary nonequilibrium]



$$\dot{n}_B \simeq -\frac{\mu_B \Gamma_{sph}}{T}$$

$Q_{L(R)}^i$: charge of a **left(right)**-handed fermion of species i
 $R^s_{R \rightarrow L}$: reflection coeff. for the **right**-handed fermion incident from the **symmetric phase** region
 $\bar{R}^s_{R \rightarrow L}$: the same as above for the **right**-handed antifermion **<injected charge into symmetric phase>** brought by the fermions and antifermions in the symmetric phase :

$$\Delta Q_i^s$$

$$\begin{aligned}
 = & [(Q_{R^i} - Q_{L^i})R^s_{L \rightarrow R} + (-Q_{L^i} + Q_{R^i})\bar{R}^s_{R \rightarrow L} \\
 & + (-Q_{L^i})(T^s_{L \rightarrow L} + T^s_{L \rightarrow R}) - (-Q_{R^i})(\bar{T}^s_{R \rightarrow L} + \bar{T}^s_{R \rightarrow R})]f^s_{Li} \\
 & + [(Q_{L^i} - Q_{R^i})R^s_{R \rightarrow L} + (-Q_{R^i} + Q_{L^i})\bar{R}^s_{L \rightarrow R} \\
 & + (-Q_{R^i})(T^s_{R \rightarrow L} + T^s_{R \rightarrow R}) - (-Q_{L^i})(\bar{T}^s_{L \rightarrow L} + \bar{T}^s_{L \rightarrow R})]f^s_{Ri}
 \end{aligned}$$

the same brought by transmission from the broken phase :

$$\begin{aligned}
 \Delta Q_i^b = & Q_{L^i}(T^b_{L \rightarrow L}f^b_{Li} + T^b_{R \rightarrow L}f^b_{Ri}) \\
 & + Q_{R^i}(T^b_{L \rightarrow R}f^b_{Li} + T^b_{R \rightarrow R}f^b_{Ri}) \\
 & + (-Q_{L^i})(\bar{T}^b_{R \rightarrow L}f^b_{Li} + \bar{T}^b_{L \rightarrow L}f^b_{Ri}) \\
 & + (-Q_{R^i})(\bar{T}^b_{R \rightarrow R}f^b_{Li} + \bar{T}^b_{L \rightarrow R}f^b_{Ri})
 \end{aligned}$$

by use of

$$\text{unitarity: } R^s_{L \rightarrow R} + T^s_{L \rightarrow L} + T^s_{L \rightarrow R} = 1, \quad \text{etc.}$$

$$\text{reciprocity: } T^s_{R \rightarrow L} + T^s_{R \rightarrow R} = T^b_{L \rightarrow L} + T^b_{R \rightarrow L}, \quad \text{etc.}$$

$$f_{iL}^{s(b)} = f_{iR}^{s(b)} \equiv f_i^{s(b)}$$

we obtain

$$\Delta Q_i^s + \Delta Q_i^b = (Q_{L^i} - Q_{R^i})(f_i^s - f_i^b)\Delta R$$

where

$$\Delta R \equiv R^s_{R \rightarrow L} - \bar{R}^s_{R \rightarrow L}$$

total flux injected into the *symmetric phase* region

$$F^i_Q = \frac{Q_L^i - Q_R^i}{4\pi^2\gamma} \int_{m_0}^{\infty} dp_L \int_0^{\infty} dp_T p_T \times [f_i^s(p_L, p_T) - f_i^b(-p_L, p_T)] \Delta R\left(\frac{m_0}{a}, \frac{p_L}{a}\right)$$

where

$$f_i^s(p_L, p_T) = \frac{p_L}{E \exp[\gamma(E - v_w p_L)/T] + 1}$$

$$f_i^b(-p_L, p_T) = \frac{p_L}{E \exp[\gamma(E + v_w \sqrt{p_L^2 - m_0^2})/T] + 1}$$

are the fermion flux densities in the symmetric and broken phases.

m_0 : fermion mass in the broken phase

v_w : wall velocity

$$\gamma = 1/\sqrt{1 - v_w^2}$$

p_T : transverse momentum

$$E = \sqrt{p_L^2 + p_T^2}$$

$1/a$: wall width

$\Delta R \implies$ effects of CP violation

- MSM — KM matrix

dispersion relation of the fermion $\sim O(\alpha_W)$

[Farrar and Shaposhnikov, P.R.D50('94)]

— decoherence by QCD effects (short range)

- CP violation in mass matrices — Higgs sector & SUSY-br. *tree-level* quantum scattering by the bubble wall

choich of the charge :

$$\left. \begin{array}{l} Q_L - Q_R \neq 0 \\ \text{conserved in the symmetric phase} \end{array} \right\} \implies Y, \quad I_3$$

change of the state by the injection of the flux

assume :

- bubble is macroscopic and expand with const. velo.
- deep in the sym. phase, elementary processes are fast enough to realize a new stationary state

⇒ chemical potential argument

charged-current interaction :

$$\mu_W = \mu_0 + \mu_- = -\mu_{t_L} + \mu_{b_L} = -\mu_{\nu_\tau} + \mu_{\tau_L}$$

Yukawa interaction :

$$\mu_0 = -\mu_{t_L} + \mu_{t_R} = -\mu_{b_L} + \mu_{b_R} = -\mu_{\tau_L} + \mu_{\tau_R}$$

no further independent relations

chem. potentials of **conserved** or **almost conserved** quantum numbers :

$$\mu_{B-L}, \quad \mu_Y, \quad \mu_{I_3}; \quad \mu_B$$

We assume the sphaleron process is **out of equilibrium**.

$$\mu_{t_L(b_L)} = \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} + \frac{1}{6}\mu_Y + (-)\frac{1}{2}\mu_{I_3},$$

$$\mu_{t_R} = \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} + \frac{2}{3}\mu_Y$$

$$\mu_{b_R} = \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} - \frac{1}{3}\mu_Y$$

$$\mu_{\tau_L(\nu_\tau)} = -\mu_{B-L} - \frac{1}{2}\mu_Y + (-)\frac{1}{2}\mu_{I_3}$$

$$\mu_{0(-)} = +(-)\frac{1}{2}\mu_Y - \frac{1}{2}\mu_{I_3}$$

$$\mu_W = -\mu_{I_3}$$

baryon and lepton number densities:

$$n_B = 3 \cdot \frac{1}{3} \cdot \frac{T^2}{6} (\mu_{t_L} + \mu_{t_R} + \mu_{b_L} + \mu_{b_R})$$

$$= \frac{T^2}{9} (2\mu_B + 2\mu_{B-L} + \mu_Y)$$

$$n_L = \frac{T^2}{6} (\mu_{\nu_\tau} + \mu_{\tau_L} + \mu_{\tau_R}) = \frac{T^2}{6} (-3\mu_{B-L} - 2\mu_Y)$$

If $n_B = n_L = 0$ before the injection of the hypercharge flux,

$$\mu_{B-L} = -\frac{2}{3}\mu_Y, \quad \mu_B = \frac{1}{6}\mu_Y$$

hypercharge density:

$$\frac{Y}{2} = \frac{T^2}{6} \left\{ 3 \left[\frac{1}{6} (\mu_{t_L} + \mu_{b_L}) + \frac{2}{3} \mu_{t_R} - \frac{1}{3} \mu_{b_R} \right] - \frac{1}{2} (\mu_{\nu_\tau} + \mu_{\tau_L}) - \mu_{\tau_R} \right\} + \frac{T^2}{3} \frac{1}{2} (\mu_0 - \mu_-) m$$

$$= \frac{T^2}{6} (m + \frac{5}{3}) \mu_Y \quad [m = \#(\text{Higgs doublets})]$$

$$\therefore \mu_B = \frac{Y}{2(m + 5/3)T^2}$$

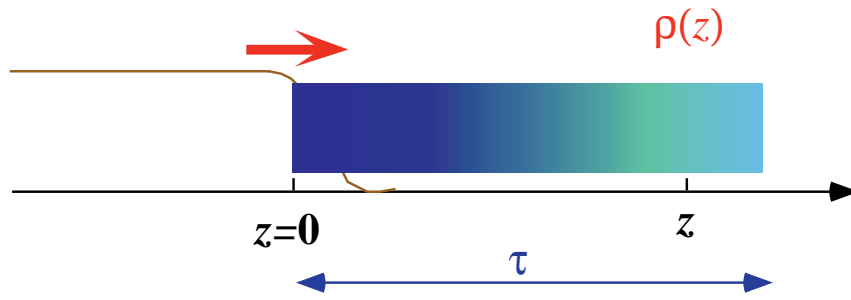
Integrating the equation for \dot{n}_B ,

$$n_B = -\frac{\Gamma_{\text{sph}}}{T} \int dt \mu_B = -\frac{\Gamma_{\text{sph}}}{2(m + 5/3)T^3} \int dt Y$$

where

$$\int dt Y = \int_{-\infty}^{z/v_w} dt \rho_Y(z - v_w t) = \frac{1}{v_w} \int_0^\infty dz \rho_Y(z).$$

[$z = \text{distance from the bubble wall}$]



The last integral is approximated as

$$\frac{1}{v_w} \int_0^\infty dz \rho_Y(z) \simeq \frac{F_Y \tau}{v_w}$$

where

τ = transport time within which the scattered fermions are captured by the wall

generated BAU :

$$\frac{n_B}{s} \simeq \mathcal{N} \frac{100}{\pi^2 g_*} \cdot \kappa \alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T$$

where

$$\mathcal{N} \sim O(1)$$

$$\tau T \simeq \begin{cases} 1 & \text{for quarks} \\ 10^2 \sim 10^3 & \text{for leptons} \end{cases}$$

MC simulation \Rightarrow forward scattering enhanced :

for top quark

$$\tau T \simeq 10 \sim 10^3 \quad \text{max. at } v_w \simeq 1/\sqrt{3}$$

for this optimal case [top quark]

$$\frac{n_B}{s} \simeq 10^{-3} \cdot \frac{F_Y}{v_w T^3}$$

$\Rightarrow F_Y / (v_w T^3) \sim O(10^{-7})$ would be sufficient to explain the BAU.

charge carriers :

$$(\tau T)_{\text{quark}} \ll \tau T \text{ for leptons, higgsino, Wino, Bino}$$

- Calculation of $\Delta R \longrightarrow$ chiral charge flux

relative phase of $\langle \Phi_1 \rangle$ and $\langle \Phi_2 \rangle \Rightarrow$ CP violating angle θ
 \implies Dirac equation through Yukawa coupling

$$-f \langle \phi(x) \rangle = m(x) \in \mathbf{C}$$

$$i \not{\partial} \psi(x) - m(x) P_R \psi(x) - m^*(x) P_L \psi(x) = 0$$

- (i) perturbative method [FKOTT, P.R.D50('94)]
- (ii) numerical method [CKN, N.P.B373('92), FKOT, P.T.P.95('96)]

\Downarrow
 ΔR as a function of (m, a, p_L)
 \Downarrow
 chiral charge flux $F_Q(T, m, a, v_w)$

Example [CKN, N.P.B373('92)]

$$m(z) = m_0 \frac{1 + \tanh(az)}{2} \exp \left(-i\pi \frac{1 - \tanh(az)}{2} \right)$$

— no CP violation in the broken phase [$z \sim \infty$]

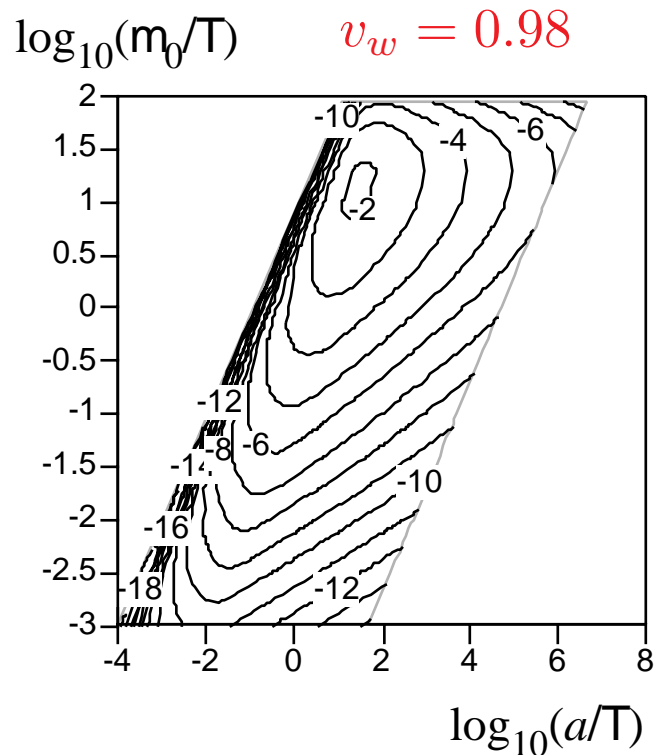
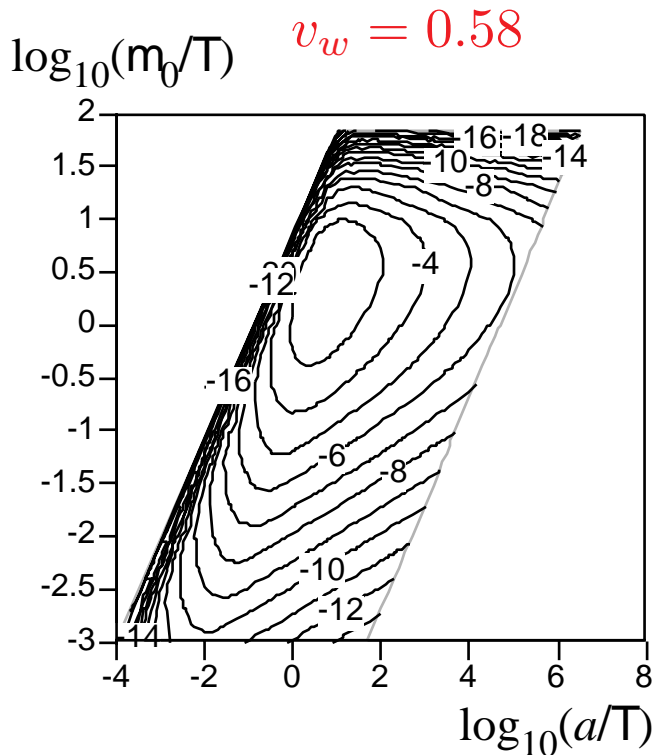
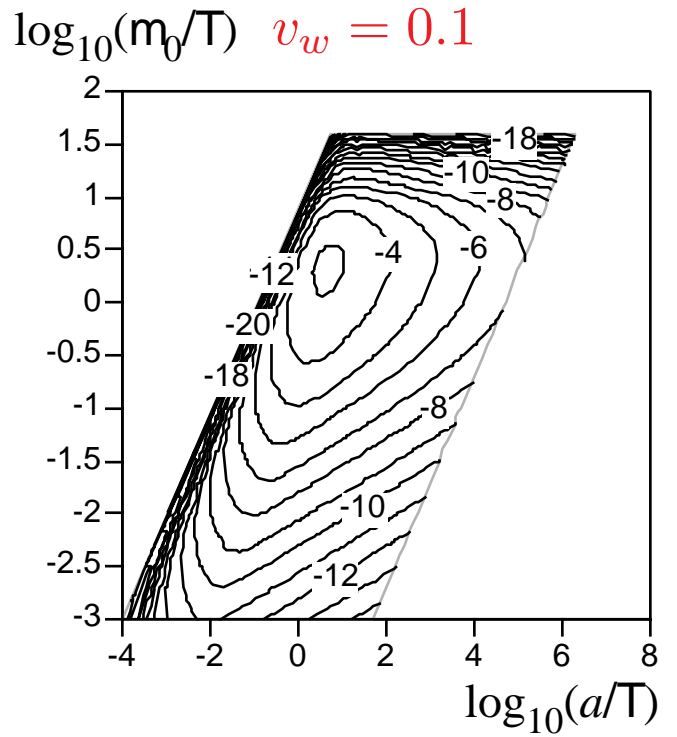
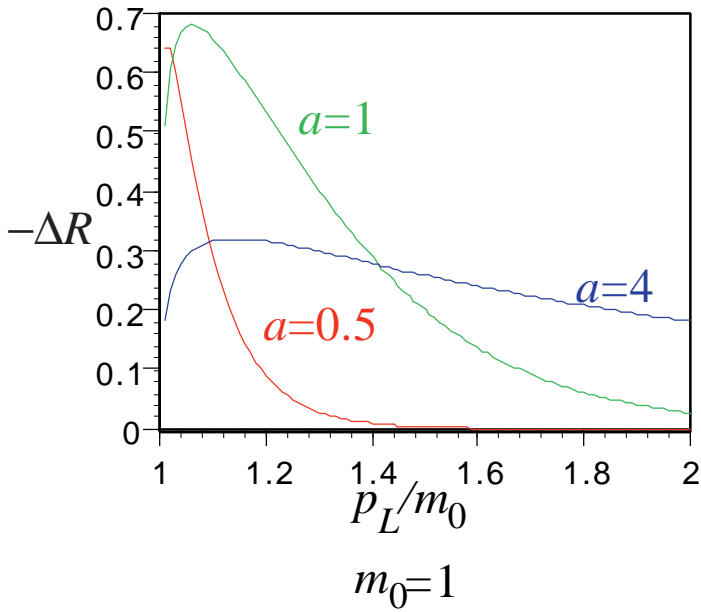
- $\Delta R \equiv R^s_{R \rightarrow L} - \bar{R}^s_{R \rightarrow L}$
 wall width \simeq wave length of the carrier $\implies \Delta R \sim O(1)$
 for larger energy, ΔR decays exponentially

- chiral charge flux
 normalized as

$$\frac{F_Q}{T^3(Q_L - Q_R)} \quad [\text{dimensionless}]$$

Numerical results

$T = 100 \text{ GeV}$



$$\frac{n_B}{s} \simeq \mathcal{N} \frac{100}{\pi^2 g_*} \cdot \kappa \alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T$$

$$\simeq 10^{-3} \cdot \frac{F_Y}{v_w T^3}$$

for an optimal case (top quark)

IV-2. Spontaneous baryogenesis

(i) in two-Higgs-doublet model [at $T = 0$]

$$\Delta\mathcal{L}_{\text{eff}} = -\frac{g^2 N_f}{24\pi^2} \theta(x) F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x)$$

— CP -even $\Leftarrow \theta(x)$, $F\tilde{F}$: CP -odd

$\Rightarrow \dot{\theta} \sim$ chem.pot. for N_{CS}

At high- T , suppressed by $\left(\frac{m_t}{T}\right)^2$.

(ii) bias for the hypercharge instead of N_{CS} [CKN,P.L.B263('91)]
neutral comp. of 2 Higgs scalars :

$$\phi_j^0(x) = \rho_j(x) e^{i\theta_j}, \quad (j = 1, 2)$$

Suppose **only** ϕ_1 couples to the fermions.

Eliminate θ_1 in Yukawa int. by **anomaly-free** $U(1)_Y$ trf.

fermion kinetic term induces:

$$2\partial_\mu\theta_1(x) \left[\frac{1}{6}\bar{q}_L(x)\gamma^\mu q_L(x) + \frac{2}{3}\bar{u}_R(x)\gamma^\mu u_R(x) \right. \\ \left. - \frac{1}{3}\bar{d}_R(x)\gamma^\mu d_R(x) - \frac{1}{2}\bar{l}_L(x)\gamma^\mu l_L(x) - \bar{e}_R(x)\gamma^\mu e_R(x) \right]$$

$\langle \dot{\theta}_1 \rangle \neq 0$ during EWPT \Rightarrow **charge potential**

★ criticism by Dine-Thomas [P.L.B328('94)]

▷ The current is not the conserved Y -current, but the fermionic part of it.

Nonconservation of Y in the broken phase leads to

$$\partial_\mu \theta_1 \cdot j_Y^\mu \propto \frac{m_t^2}{T^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

▷ The bias for Y exists where $(v/T)^2 > 0$.

The sphaleron process is effective for $v < v_{co}$

∴ The generated B is suppressed by $(v_{co}/T^2 \sim O(10^{-6}))$.

★ enhancement by diffusion [CKN, P.L.B336('94)]

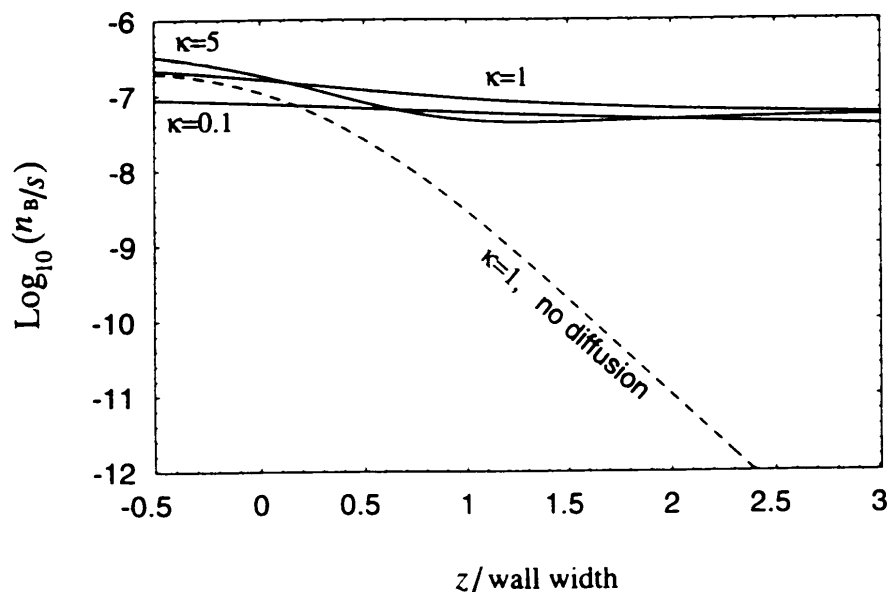
Diffusion carries Y into the symmetric phase.

→ nonlocal baryogenesis

for the profile

$$\langle \phi(z) \rangle = v \frac{1 - \tanh(az)}{2} \exp \left[-i \frac{\pi}{2} \frac{1 - \tanh(az)}{2} \right]$$

z_{co} VS $\log_{10}[(n_B/s)(g_*/100)]$ with $v_{co} = \varphi(z_{co})$



— almost independent of z_{co}

V. EWPT and CP Violation in the MSSM

[K.F., hep-ph/9809517, published in PTP101('99)]

Minimal Supersymmetric Standard Model

$$= \left\{ \begin{array}{l} \text{the particles in the SM} \\ + \text{ a second Higgs doublet} \end{array} \right\} \oplus \begin{array}{l} \text{superpartner} \\ \text{for each species} \end{array}$$

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}$$

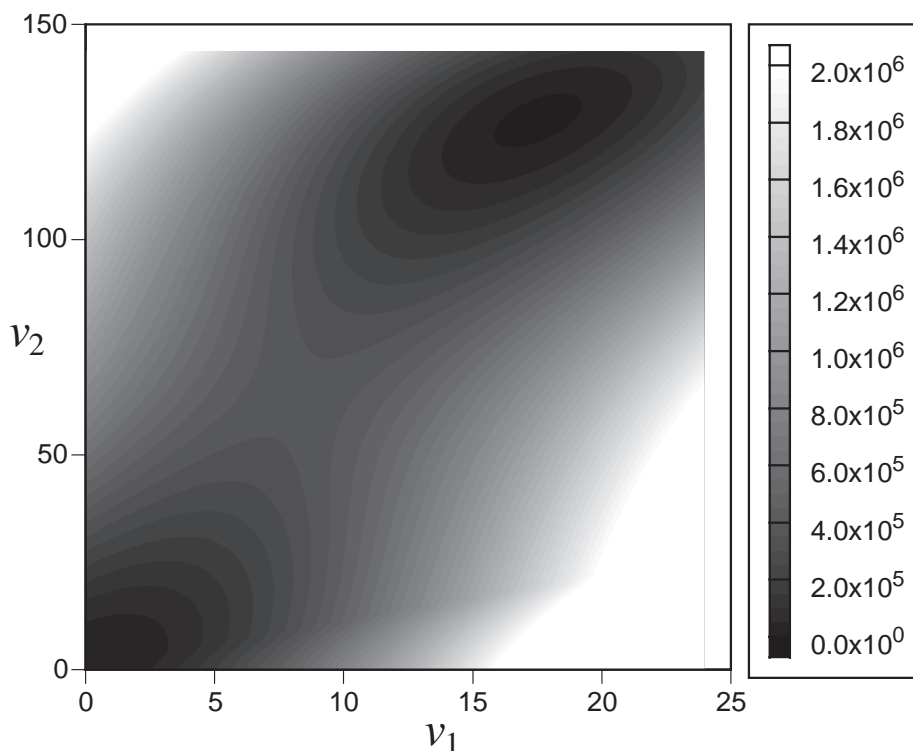
• more scalar fields \Rightarrow stronger (first-order) PT

★ one-loop effective potential [K.F.]

$$\text{first order with } v_C/T_C > 1 \Leftarrow \left\{ \begin{array}{l} \text{lighter stop } m_{\tilde{t}_1} \lesssim m_t \\ m_h \lesssim 90\text{GeV} \end{array} \right.$$

★ 3d lattice MC [Laine et al. hep-lat/9809045]

strong 1st order for $m_{\tilde{t}_1} \lesssim m_t$ and $m_h \leq 110\text{GeV}$



$$\tan \beta = 6$$

$$m_h = 82.3\text{GeV}$$

$$m_A = 118\text{GeV}$$

$$m_{\tilde{t}_1} = 168\text{GeV}$$

$$T_C = 93.4\text{GeV}$$

$$v_C = 129\text{GeV}$$

- many complex parameters \Rightarrow explicit CP violation
 - two Higgs doublets \Rightarrow possibility of spontaneous CP viol.
- CP violation effective for the chiral charge at the lowest order
- relative phases of μ, M_2, M_1, A_t
 chargino, neutralino, stop transport
 [Huet and Nelson, PRD53('96); Aoki, et al. PTP98('97)]
 - relative phase θ of the two Higgs doublets

$$\langle \Phi_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho_i e^{i\theta_i} \end{pmatrix}$$

quarks and leptons \Leftarrow Yukawa coupl. $\propto \rho_i e^{i\theta}$
 chargino, neutralino, stop mass matrix
 N.B. $\theta = \theta(x) \therefore$ it cannot be rotated away
 [Nelson et al. NPB373('92); FKOTT, PRD50('94), PTP95('96)]

a scenario to have large θ near the bubble wall

= spontaneous CP violation in the transient region

+ small explicit CP violation

to resolve degeneracy between CP conjugate bubbles

net BAU

$$\frac{n_B}{s} = \frac{\sum_j \left(\frac{n_B}{s}\right)_j N_j}{\sum_j N_j}$$

with

$N_j = \exp(-4\pi R_C^2 \mathcal{E}_j / T_C)$ nucleation rate

$\mathcal{E}_j =$ energy density of the type- j bubble

♠ Transitional CP violation

Suppose that at $T \simeq T_C$,

$$\begin{aligned}
 & V_{\text{eff}}(\rho_i, \theta = \theta_1 - \theta_2) \\
 = & \frac{1}{2}m_1^2\rho_1^2 + \frac{1}{2}m_2^2\rho_2^2 - m_3^2\rho_1\rho_2 \cos \theta + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\
 + & \frac{\lambda_3 + \lambda_4}{4}\rho_1^2\rho_2^2 + \frac{\lambda_5}{4}\rho_1^2\rho_2^2 \cos 2\theta - \frac{1}{2}(\lambda_6\rho_1^2 + \lambda_7\rho_2^2)\rho_1\rho_2 \cos \theta \\
 - & [A\rho_1^3 + \rho_1^2\rho_2(B_0 + B_1 \cos \theta + B_2 \cos 2\theta) \\
 & \quad + \rho_1\rho_2^2(C_0 + C_1 \cos \theta + C_2 \cos 2\theta) + D\rho_2^3] \\
 = & \left[\frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2) \right] \\
 & \times \left[\cos \theta - \frac{2m_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)} \right]^2 \\
 & + \theta\text{-independent terms}
 \end{aligned}$$

where all the parameters are real

conditions for spontaneous CP violation for a given (ρ_1, ρ_2)

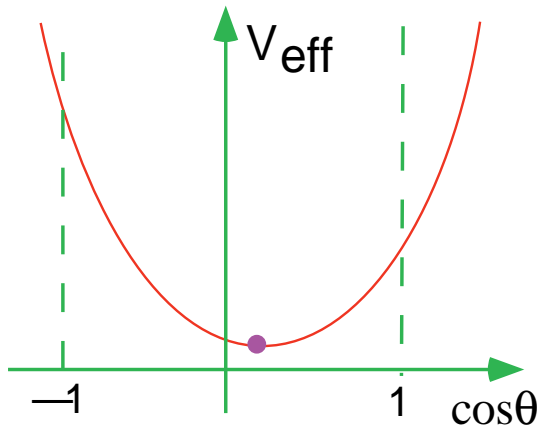
$$\begin{aligned}
 F(\rho_1, \rho_2) & \equiv \frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2) > 0, \\
 -1 < G(\rho_1, \rho_2) & \equiv \frac{2m_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)} < 1
 \end{aligned}$$

At $T \simeq T_C$, around the EW bubble wall

$$(\rho_1, \rho_2) : (0, 0) \longrightarrow (v_C \cos \beta_C, v_C \sin \beta_C)$$

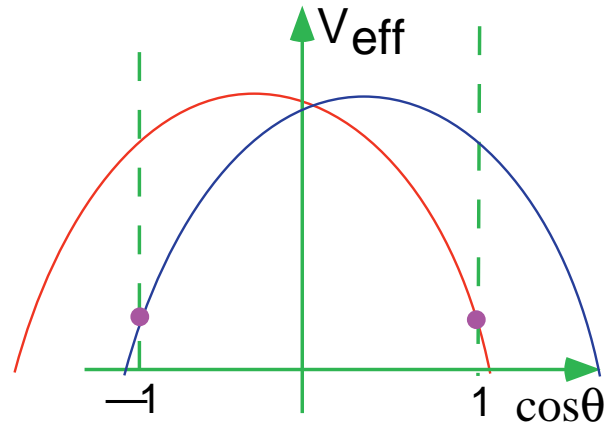
There may be a chance to satisfy the conditions in the transient region.

$$F(\rho_1, \rho_2) > 0$$

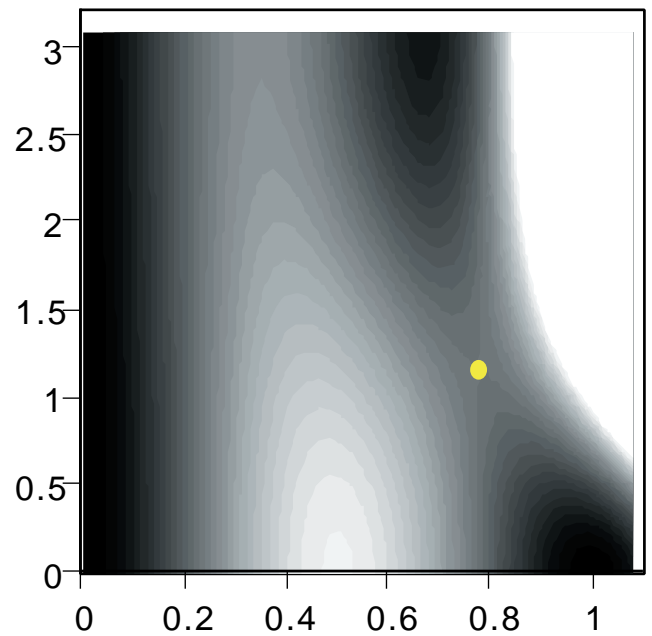
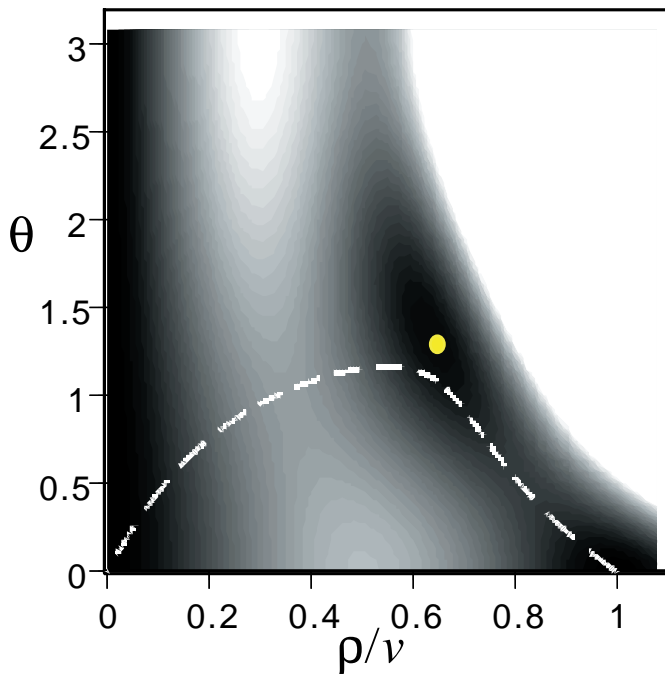


CP-violating
local minimum

$$F(\rho_1, \rho_2) < 0$$



CP-violating
saddle point



♠ Enhancement of an explicit CP violation

$$\alpha = \text{Arg}(\mu M_2) = \text{Arg}(\mu M_1), \quad \beta = \text{Arg}(\mu A_t^*),$$

then

$$(m_3^2)_{\text{eff}} = m_3^2 + \Delta_{\phi^\pm}^{(0)} m_3^2 + e^{i\alpha} \Delta_\chi^{(0)} m_3^2 + e^{i\beta} \Delta_{\tilde{t}}^{(0)} m_3^2,$$

$$\lambda_5 = \Delta_{\phi^\pm}^{(0)} \lambda_5 + e^{i2\alpha} \Delta_\chi^{(0)} \lambda_5 + e^{i2\beta} \Delta_{\tilde{t}}^{(0)} \lambda_5,$$

$$\lambda_{6,7} = \Delta_{\phi^\pm}^{(0)} \lambda_{6,7} + e^{i\alpha} \Delta_\chi^{(0)} \lambda_{6,7} + e^{i\beta} \Delta_{\tilde{t}}^{(0)} \lambda_{6,7}$$

$\Delta^{(0)} \equiv$ correction without explicit CP violation

If $\Delta_\chi^{(0)} \gg \Delta_{\tilde{t}}^{(0)}, \Delta_{\phi^\pm}^{(0)}$, by rephasing, $\lambda_{5,6,7} \in \mathbf{R}$ and

$$e^{-i\alpha} (m_3^2)_{\text{eff}} = e^{-i\alpha} m_3^2 + \Delta_\chi^{(0)} m_3^2 \equiv e^{-i\delta} |(m_3^2)_{\text{eff}}|$$

with

$$\tan \delta = -\frac{m_3^2 \sin \alpha}{m_3^2 \cos \alpha + \Delta_\chi^{(0)} m_3^2}.$$

N.B $|m_3^2 + \Delta_\chi^{(0)} m_3^2| \ll m_3^2$ for transitional CP violation

for some parameter set, we have at $T \simeq T_C$

$$\Delta_{\tilde{t}}^{(0)} m_3^2 = 140.69, \quad \Delta_\chi^{(0)} m_3^2 = -2356.73,$$

so that even for $\alpha = 10^{-3}$,

$$\begin{aligned} \tan \delta &= -\frac{(m_3^2 + \Delta_{\tilde{t}}^{(0)} m_3^2) \sin \alpha}{(m_3^2 + \Delta_{\tilde{t}}^{(0)} m_3^2) \cos \alpha + \Delta_\chi^{(0)} m_3^2} \\ &\simeq \frac{10^{-3}}{6.8 \times 10^{-3}} = 0.147 \end{aligned}$$

\implies only the lowest-energy bubble survives

VI. Summary

♣ strong 1st-order EWPT
sufficient CP violation } $\Rightarrow B$ -genesis by MSM \times

♣ MSSM with $m_{\tilde{t}_1} \leq m_t$ and $m_h \leq 110\text{GeV}$ may allow EW baryogenesis, if CP violation is efficient at T_C .

\uparrow
transitional CP violation ?
constraints on $\tan\beta$, μ , m_A , etc.

♣ other extensions of the MSM, e.g. 2HDM
many parameters \longrightarrow broad allowed region ??

♣ If EW baryogenesis is impossible,

- ν -Majorana mass + CP viol. \Rightarrow leptogenesis

sphaleron \downarrow
BAU

- Affleck-Dine mechanism
- GUTs
- ???

near-future experiments:

determination of $m_h, m_A, m_{\tilde{t}_1}, \dots$
discovery of the superpartners
discovery of a new CP violation
other than the KM phase } \Rightarrow possibility of
EW Baryogenesis