Baryon Asymmetry of the Universe

Koichi Funakubo

Department of Physics, Saga University
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§1. Evidence of the BAU

Baryon Asymmetry of the Universe

\[
\frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} = (0.48 - 0.98) \times 10^{-10}
\]

\[
s = \frac{\pi^2}{45} g_* T^3 \approx 7.04 n_\gamma: \text{entropy density}
\]

\[
g_* = \text{massless degrees of freedom}
\]

— constant during the expansion of the universe, once \( \Delta B \neq 0 \) is turned off

\[
n_b, n_{\bar{b}} \propto a(t)^{-3} \propto T^3 \text{ [entropy } a^3 s \propto a^3 T^3 = \text{const.}, \text{ where } a = \text{scale factor]}
\]

This value is determined by the data of

★ WMAP — observation of fluctuation in the Cosmic Microwave Background Radiation

The evolution of the fluctuation is determined by the components of the universe (baryon, CDM, electron, photon) at the decoupling.

\[T_0, H_0 \rightarrow \rho_C = 3H_0^2/(8\pi G), \Omega_B = \rho_B/\rho_C \ (\rho = \text{energy density})\]

★ Big Bang Nucleosynthesis
- $T \gg 1\text{MeV}$
  \[ n + \nu_e \leftrightarrow p + e^- \quad \therefore \quad n_\nu \simeq n_p \]

- $T = T_F \simeq 1\text{MeV}$ ($\Gamma_{n \leftrightarrow p} \simeq H$)
  \[ \frac{n_n}{n_p} \simeq e^{-(m_n-m_p)/T_F} \simeq \frac{1}{6} \]

- $T = 0.3 - 0.1\text{MeV}$
  depending on $\eta = \frac{n_B}{n_\gamma}$,
  \[ \frac{n_n}{n_p} = \frac{1}{7} - \frac{1}{6} \]

$^4\text{He}$ mass fraction
\[ Y \equiv \frac{2n_n}{n_n + n_p} \quad (= 0.25 \leftrightarrow \frac{n_n}{n_p} = \frac{1}{7}) \]
1. observed antiparticles in cosmic rays = secondary antiparticles

2. clusters of galaxies are stable

mass of a cluster: \( M_{\text{cluster}} \approx 10^{12-14} M_\odot \) (\( M_\odot \) =solar mass)

In the universe, is there net baryon number?

Starting from a Baryon-symmetric universe, can we extract finite \( n_B \)?

At \( T < 1\text{GeV} \), a bit of nucleons and anti-nucleons exist with the same number because of thermal fluctuation: 

\[
n_N = n_{\bar{N}} = (m_N T)^{3/2} e^{-m_N / T}
\]

\[
\frac{n_b}{s} = \frac{n_{\bar{b}}}{s} \approx 8 \times 10^{-11} \quad \text{at} \ T = 38\text{MeV}
\]

At this \( T \), total energy within a causal volume (\(~ H^{-3} \)) is \( 10^{-7} M_\odot \ll 10^{12} M_\odot \)
At a higher $T$, there are more $n_b/s$ and $n_{\bar{b}}/s$, but

$$E_{\text{causal}} \sim \rho(T)H^{-3}(T) \sim T^4(T^2/m_{Pl})^{-3} = m_{Pl}^3T^{-2} < 10^{-7}M_\odot,$$

where

$$H = \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{8\pi G}{3}}\rho(T) \quad \text{with} \quad \rho(T) = g_*\frac{\pi^2}{30}T^4$$

Even the Maxwell's demon could not separate matter and antimatter to create the present universe, without violating causality.

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**Before the era of nucleosynthesis, we must have**

$$\frac{n_B}{s} = (0.48 - 0.98) \times 10^{-10}$$

**over a region broader than a causal region at $T = 1\text{MeV}$.**
§2. Requirements for the baryogenesis

= generating $B$ starting from $B$-symmetric universe

Sakharov’s 3 conditions

(1) Baryon number violating process
(2) Violation of $C$ and $CP$ symmetries
(3) Out of equilibrium

(1) is obvious.

(3) In equilibrium, $\Delta B \neq 0$-process and its inverse occur with the same probability. Then no $B$ is generated.
Let us consider the case in which (2) is not satisfied.

Suppose that a local state of the spatially uniform universe is described by the density operator

$$\rho(t) = \sum_n p_n |n(t)\rangle \langle n(t)| \quad (p_n = \text{prob. to find the universe in the state } |n(t)\rangle)$$

Expectation value of some observable $\mathcal{O}$:

$$\langle \mathcal{O} \rangle(t) = \text{Tr} [\rho(t) \mathcal{O}] = \sum_n p_n \langle n(t)|\mathcal{O}|n(t)\rangle$$

time evolution of $\rho(t)$ is governed by the Liouville equation:

$$i\hbar \frac{\partial \rho(t)}{\partial t} + [\rho(t), H] = 0$$

($\Leftarrow$ Schrödinger eq. $i\hbar \frac{\partial}{\partial t} |n(t)\rangle = H |n(t)\rangle)$
$\rho_0$: density operator of an initial state with $n_B = 0$  \[ \langle n_B \rangle_0 \equiv \text{Tr} [\rho_0 \, n_B] = 0 \]

density operator $\rho(t)$ at a later time is given by the solution to

\[
i\hbar \frac{\partial \rho(t)}{\partial t} + [\rho(t), H] = 0
\]

with the initial condition $\rho(t_0) = \rho_0$

The solution is formally written in terms of $H$ and $\rho_0$

$\Rightarrow$ If $H$ is invariant under $C$- or $CP$-trf.  \[ \implies [\rho, C] = 0 \quad or \quad [\rho, CP] = 0 \]

$\quad CB^{-1} = -B, \quad CPB(\overline{CP})^{-1} = -B \quad (i.e., \quad B \text{ is vectorlike and odd under } C')$

\[
\begin{align*}
\langle n_B \rangle &= \text{Tr}[\rho n_B] = \text{Tr}[\rho \, C n_B C^{-1}] = -\text{Tr}[\rho n_B] = 0 \\
\text{or} \\
\langle n_B \rangle &= \text{Tr}[\rho \, CP n_B (CP)^{-1}] = -\text{Tr}[\rho n_B] = 0
\end{align*}
\]

$\therefore \ \text{Both } C \text{ and } CP \text{ must be violated to have } \langle n_B \rangle \neq 0$
§3. Scenarios of baryogenesis

— models of particle physics realizing baryogenesis

(1) $\Delta B \neq 0$ process

- quark-lepton mixing — in Grand Unified Theories (GUTs)

- in Supersymmetric theories,

  \[
  \begin{align*}
  \text{quark: } q & \leftrightarrow \tilde{q} &: \text{scalar quark (squark)} \\
  \text{lepton: } l & \leftrightarrow \tilde{l} &: \text{scalar lepton (slepton)}
  \end{align*}
  \]

  \[
  \implies \langle \tilde{q} \rangle \neq 0, \langle \tilde{l} \rangle \neq 0
  \]

- axial $U(1)$ anomaly in the $(B + L)$-current

  — exists in the Standard Model
\[ \partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}[g_2^2 \text{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu}\tilde{B}^{\mu\nu}] \neq 0 \]

for some specific configuration of the SU(2)-gauge field

We shall discuss this process later.

All these processes must be suppressed at \( T = 0 \), in order for a proton not to decay.

- in GUTs, \( m_X \) must be heavier than some value \( \sim 10^{16}\text{GeV} \)
- The potential for the squark fields must have such a form that \( \langle \tilde{q} \rangle \neq 0 \) is realized in the early universe, while \( \langle \tilde{q} \rangle = 0 \) at \( T = 0 \)
- at \( T = 0 \), \( \Delta(B + L) \neq 0 \) process occurs by quantum tunneling

As shown later, the probability is negligible.
(2) $C$ and $CP$ violation

$\star \ C$ violation $\leftrightarrow$ **chiral** gauge interactions in EW theory and GUTs

$SU(2)_L$ gauge int., different $U(1)_Y$ charges for $L$- and $R$-fermions

$\star \ CP$ violation $\leftrightarrow$ **complex parameters** with irreducible phases

In renormalizable field theories, only some types of interactions can violate $CP$.

- In the SM, **KM phase** and $F_{\mu\nu}\tilde{F}^{\mu\nu}$ ($\theta$-term)
  
  \[ |\theta_{QCD}| \sim 0 \text{ by neutron EDM experiment} \]

- Extended models:
  - SUSY models: relative phases of $\mu$, gaugino mass, $A$, $B$ (soft-SUSY-br.)
  - extended Higgs sector: complex Higgs self-couplings

magnitude of $CP$ violation is limited by experiments

- e.g., B-factory, EDM of neutron and lepton, K-rare processes
(3) Out of equilibrium

“equilibrium in the expanding universe” ⇔ time scale of process: $\tau < H(T)^{-1}$

for relativistic particles, $\tau \simeq \lambda$: mean free path

$s$: total cross section
$n(T)$: particle number density

$\sigma \cdot \lambda = \frac{1}{n(T)}$

$H(T) = \sqrt{\frac{8\pi G}{3}} \rho(T) \simeq 1.66\sqrt{g_*} \frac{T^2}{m_{Pl}}$

$\rho(T) = \frac{\pi^2}{30} g_* T^4$

relativistic degrees of freedom for SM with $N_f$ generation, $N_H$ Higgs doublets

$g_* = 24 + 4N_H + \frac{7}{8} \cdot 30 N_f^{\text{MSM}} = 106.75$
number density: \( n(T) = g \int \frac{d^3 k}{(2\pi)^3} \frac{1}{e^{\sqrt{k^2+m^2}/T} \mp 1} \approx g \left\{ \begin{array}{l} \frac{\zeta(3)}{\pi^2} T^3  \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} T^3 \end{array} \right\} \)  

\( \zeta(3) \approx 1.2020569 \ldots \)  

for relativistic particles, \( \sigma \approx \frac{\alpha^2}{s} \approx \frac{\alpha^2}{T^2} \)  

\( H(T) \approx 10^{14} \text{ GeV}^{-1} \) expansion  
\( \lambda_s \approx \frac{1}{\alpha_s^2 T} \approx 1 \text{ GeV}^{-1} \) strong interactions  
\( \lambda_{EW} \approx \frac{1}{\alpha_{EW}^2 T} \approx 10 \text{ GeV}^{-1} \) EW interactions  
\( \lambda_Y \approx \left( \frac{m_W}{m_f} \right)^4 \lambda_{EW} \) Yukawa interactions  

\( T = 100 \text{ GeV} \)
time scale of EW interaction vs that of the expansion

\[ \log a \sim \log(T^{-1}) \]

- Hubble
- electroweak

10^{15}\text{GeV} \quad 100\text{GeV}
Possible out-of-equilibrium processes

- Expansion of the universe
  
  EW or GUTs interaction in the radiation dominated universe at $T > 10^{15}\text{GeV}$
  
  Heavy particle (e.g., heavy Majorana neutrino) at $T > 10^{10}\text{GeV}$

- First-order phase transition
  
  some rare process runs out of equil. even at $T = 100\text{GeV}$
  
  $-\ (B + L)$-violating process in EW theory

- Reheating or preheating after inflation
  
  accompanying production of a large amount of entropy
  
  $\rightarrow$ dilution of baryon number density
An example — GUTs

minimal $SU(5)$ model:

$$\begin{align*}
\text{matter:} & \quad \begin{cases} 
5^* : \psi^i_L & \ni d^c_R, l_L \\
10 : \chi_{[ij]}L & \ni q_L, u^c_R, e^c_R 
\end{cases} \\
i = 1 - 5 & \rightarrow (\alpha = 1 - 3, a = 1, 2)
\end{align*}$$

gauge: $A_\mu = \left( \begin{array}{cc} G_\mu, & B_\mu \\ X^{a\alpha}_\mu, & W_\mu, & B_\mu \end{array} \right)$

$$\begin{align*}
L_{\text{int}} & \ni g\bar{\psi}\gamma^\mu A_\mu \psi + g\text{Tr} [\bar{\chi}\gamma^\mu \{A_\mu, \chi\}] \\
& \ni gX^a_{\alpha\mu} [\varepsilon^{\alpha\beta\gamma} \bar{u}^c_R \gamma^\mu q_L \beta_a + \varepsilon_{ab} (\bar{q}^\alpha_{Lb} \gamma^\mu e^c_R + \bar{l}_{Lb} \gamma^\mu d^{c\alpha}_R)]
\end{align*}$$

Expectation value of $\Delta B$ in decays of $X$-$\bar{X}$ pairs generated from the plasma

$$\langle \Delta B \rangle = \frac{2}{3}r - \frac{1}{3}(1 - r) - \frac{2}{3}\bar{r} + \frac{1}{3}(1 - \bar{r}) = r - \bar{r}$$

$$\therefore C \text{ or } CP \text{ is conserved}(r = \bar{r})$$

$$\implies \Delta B = 0$$

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If the inverse process is suppressed, $B \propto r - \bar{r}$ is generated.

Indeed, at $T \simeq m_X$, the decay rate of $X$: $\Gamma_D \simeq \alpha m_X$ ($\alpha \sim 1/40$)

$\Rightarrow \Gamma_D \simeq H(T = m_X)$  \quad \therefore \text{annihilation and production of } X\bar{X} \text{ out of equil.}$

$H(T) \simeq 1.66\sqrt{g_*} \frac{T^2}{m_{Pl}}$

SU(5) GUT conserves $B - L \to (B + L)$-genesis

\[
\text{EW } \Delta(B + L) \neq 0 \text{ process in equilibrium } \rightarrow B + L \rightarrow 0
\]

\[
\Downarrow
\]

new possibility of baryogenesis

$B - L \neq 0$ before EW $\Delta(B + L) \neq 0$ process in equil.

Leptogenesis: $\Delta L \neq 0 \rightarrow B = -L$
\[ \Delta(B + L) \neq 0 \text{ process} \iff \text{chiral } U(1) \text{ anomaly in } (B + L)\text{-current} \]

\[
\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2} [g_2^2 \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu} \tilde{B}^{\mu\nu}]
\]

\[
\partial_\mu j^\mu_{B-L} = 0
\]

\[ N_f = \# \text{ of generations} \]

\[ \tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \]

Integrating the sum of these eqs.

\[
B(t_f) - B(t_i) = \frac{N_f}{32\pi^2} \int_{t_i}^{t_f} d^4x \left[ g_2^2 \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu} \tilde{B}^{\mu\nu} \right] = N_f \left[ N_{CS}(t_f) - N_{CS}(t_i) \right]
\]

where \( N_{CS} \) is the Chern-Simons number: in the \( A_0 = 0 \)-gauge

\[
N_{CS}(t) = \frac{1}{32\pi^2} \int d^3x \epsilon_{ijk} \left[ g_2^2 \text{Tr} \left( F_{ij} A_k - \frac{2}{3} g A_i A_j A_k \right) - g_1^2 B_{ij} B_k \right]_{t}
\]
classical vacuum of the gauge fields: $\mathcal{E} = \frac{1}{2}(E^2 + B^2) = 0 \iff F_{\mu\nu} = B_{\mu\nu} = 0$

$\iff A = iU^{-1}dU, B = dv$ with $U \in SU(2)$

$U(x) : S^3 \rightarrow U \in SU(2) \simeq S^3$

$\pi_3(S^3) \simeq \mathbb{Z} \implies U(x)$ is classified by the integer $N_{CS}$

\[ \Delta(B + L) \neq 0 \text{ process} \begin{cases} \triangleright \text{Quantum Tunneling} \quad \text{low-}T \\ \triangleright \text{Thermal Activation} \quad \text{high-}T \end{cases} \]

tunnel. prob. $\sim e^{-2S_{\text{instanton}}} = e^{-8\pi^2/g_2^2} \simeq e^{-164} \ll 1$

$\therefore$ no proton-decay problem
\[ (B + L)\text{-transition (sphaleron transition)} \text{ rate} \]

- \( T < T_C \): EW phase transition temperature (broken phase)
  \[ \Gamma_{\text{sph}}^{(b)} \simeq T e^{-E_{\text{sph}}/T} \]
- \( T > T_C \) (symmetric phase)
  \[ \Gamma_{\text{sph}}^{(s)} \simeq \kappa \alpha_W^4 T \quad \text{with} \quad \kappa \sim O(1) \]

**sphaleron** = static saddle-point solution of \( SU(2)\)-gauge-Higgs system

\[ E_{\text{sph}} = O(1) \text{TeV} \]
first found by Manton \[ \text{[Phys. Rev. D28 ('83)]} \]
Now found in the MSSM (2HDM), the Next-to-MSSM
time scale of the sphaleron transition vs that of the expansion

\[ \log \tau \]

\[ 10^{15} \text{GeV} \quad 10^{12} \text{GeV} \]

\[ T_c \]

\[ \log a \sim \log \left( T^{-1} \right) \]
For $T_C < T < 10^{12}$ GeV, $\Gamma_{\text{sph}}^{(s)} > H(T)$

$$\implies B \propto (B - L)_{\text{primordial}} \quad \text{at } T \leq T_C$$

∴ For the matter in the Universe to be left at present, either

(i) $B - L \neq 0$ must exist before the sphaleron process decouples, or

(ii) $B + L$ must be generated at the EW phase transition and the sphaleron process becomes ineffective after that.

(i) $\Rightarrow$ $(B - L)$-violating GUTs, Leptogenesis

(ii) $\Rightarrow$ Electroweak baryogenesis

sphaleron decoupling condition: $\Gamma_{\text{sph}}^{(b)}(T_C) < H(T_C)$

$\implies$ lower bound on $E_{\text{sph}}(T_C) \propto v(T_C) \quad (v(T) = \langle \Phi \rangle; \Phi = \text{Higgs doublet})$

$\implies$ upper bound on the Higgs mass (see below)
§4. Electroweak baryogenesis
— closely related to the particle physics at the weak scale

(i) Baryon number violation \(\rightarrow\) sphaleron process in the symmetric phase
must decouple just after the EW phase transition

(ii) \(C\) and \(CP\) violation \(\rightarrow\) KM phase, or complex parameters in extended models

(iii) Out of equilibrium \(\rightarrow\) sphaleron process can be out of equilibrium at the EWPT
if \(E_{\text{sph}}(T_C)/T_C\) is sufficiently large
— strongly first-order PT

As we shall see, the conditions (ii) and (iii) require an extension of the standard model, because

★ KM phase is insufficient to produce the present BAU
★ EWPT is not of first order for the Higgs mass \(m_h > 114\text{GeV}\) (LEP bound)

for reviews, see
• KF, Prog. Theor. Phys. 96 (1996) 475
Electroweak phase transition (EWPT)

Standard Model

order parameter:

\[ \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix} \]

\[ v_C \equiv \lim_{T \uparrow T_C} \varphi(T) \neq 0 \]

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— Baryon Asymmetry of the Universe — 25/30
Minimal SM — perturbation at the 1-loop level ($W$, $Z$, top quark loop)

$$V_{\text{eff}}(\varphi; T) = -\frac{1}{2} \mu^2 \varphi^2 + \frac{\lambda}{4} \varphi^4 + 2B v_0^2 \varphi^2 + B \varphi^4 \left[ \log \left( \frac{\varphi^2}{v_0^2} \right) - \frac{3}{2} \right] + \tilde{V}(\varphi; T)$$

where $B = \frac{3}{64 \pi^2 v_0^4} (2m_W^4 + m_Z^4 - 4m_t^4)$,

$$\tilde{V}(\varphi; T) = \frac{T^4}{2 \pi^2} \left[ 6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t) \right], \quad (a_A = m_A(\varphi)/T)$$

$$I_{B,F}(a) \equiv \int_0^{\infty} dx x^2 \log \left( 1 \mp e^{-\sqrt{x^2 + a^2}} \right).$$

high-temperature expansion $[m/T \ll 1]$ \hspace{1cm} $\gamma_E = 0.5772 \cdots$

$$I_B(a) = -\frac{\pi^4}{45} + \frac{\pi^2}{12} a^2 - \frac{\pi}{6} (a^2)^{3/2} - \frac{a^4}{16} \log \frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16} \left( \gamma_E - \frac{3}{4} \right) + O(a^6)$$

$$I_F(a) = \frac{7\pi^4}{360} - \frac{\pi^2}{24} a^2 - \frac{a^4}{16} \log \frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16} \left( \gamma_E - \frac{3}{4} \right) + O(a^6)$$
Applying the high-T expansion assuming $T > m_W, m_Z, m_t$, 

$$V_{\text{eff}}(\varphi; T) \simeq D(T^2 - T_0^2)\varphi^2 - E T \varphi^3 + \frac{\lambda_T}{4} \varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2), \quad E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4} \left( 2m_W^4 \log \frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log \frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log \frac{m_t^2}{\alpha_F T^2} \right)$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2), \quad \log \alpha_{F(B)} = 2\log (4)\pi - 2\gamma_E$$

At $T_C$, 3 degenerate minima:

$$\varphi_C = \frac{2E T_C}{\lambda_{TC}}$$

$$\Gamma_{\text{sph}}^{(br)} < H(T_C) \iff \frac{\varphi_C}{T_C} \gtrsim 1 \implies \text{upper bound on } \lambda$$

$$[m_H = \sqrt{2}\lambda v_0]$$

$$m_H \lesssim 46\text{GeV} \implies \text{Minimal SM is excluded!}$$
Nonperturbative study by Lattice Gauge Theory:

\[ m_h > 73\text{GeV} \Rightarrow \text{PT disappears (crossover)} \]

\[ \therefore \text{The EWBG does not work in the Standard Model with } m_h \geq 114\text{GeV}. \]

If the EWPT is of first order, it proceeds accompanying nucleation and growth of the bubble walls:

\[ \rightarrow \text{sphaleron process becomes out of equilibrium} \]
Difference in the reflection rate for $\psi_L$ and $\psi_R$

\[ \downarrow \]

**chiral charge flux** into the sym. phase

$Q_L \neq Q_R$, conserved in sym. phase: $Y, I_3$

\[ \downarrow \]

**bias** on the sphaleron process

\[ \downarrow \]

$B + L \neq 0$ in the symmetric phase

\[ \downarrow \]

$B + L$ frozen in the broken phase

★ Minimal SM — CKM phase

\[ m_i \neq m_j: \]

$O(\alpha_W)$ effect in the dispersion

[Farrar, Shaposhnikov, Phys. Rev. 50 ('94)]

▷ decoherence by QCD correction

\[ \longrightarrow |\frac{n_B}{s}| < 10^{-26} \]

effective $CP$ violation at the tree level

- relative phase of Higgs doublets in the extended models
  
  $$m_f(x) = y_f |\phi(x)| e^{i\theta(x)} \quad \text{— spacetime-dependent phase}$$

- complex parameters in the mass matrices of SUSY particles
  
  e.g. chargino: $M_{\chi^\pm} = \begin{pmatrix} M_2 & -\frac{i}{\sqrt{2}}g_2 v_u e^{-i\theta} \\ -\frac{i}{\sqrt{2}}g_2 v_d & -\mu \end{pmatrix}$

  $x$-dependent $v_d$ and $v_u \rightarrow$ effectively $x$-dependent phases

For the EWBG to work, we need an extension of the Standard Model.

- Higgs sector — MSSM: $m_{H_1} \leq 120\text{GeV}$ and a light stop $m_{\tilde{t}_1} \leq m_t$

  2HDM, NMSSM: possible for $m_h > 130\text{GeV}$

- CP violation — Higgs doublets more than 2

  complex parameters in the SUSY models