Introduction to Higgs Physics

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Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC

Phys. Lett. B716, 1 (2012)

Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC

Phys. Lett. B716, 30 (2012)

Confirmed in 2013 that its spin and parity is 0^+.

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Study of the Mass and Spin-Parity of the Higgs Boson Candidate via Its Decays to *Z* Boson Pairs

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A study is presented of the mass and spin-parity of the new boson recently observed at the LHC at a mass near 125 GeV. An integrated luminosity of 17.3 fb^{-1}, collected by the CMS experiment in proton-proton collisions at center-of-mass energies of 7 and 8 TeV, is used. The measured mass in the ZZ channel, where both Z bosons decay to e or μ pairs, is $126.2 \pm 0.6(\text{stat}) \pm 0.2(\text{syst})$ GeV. The angular distributions of the lepton pairs in this channel are sensitive to the spin-parity of the boson. Under the assumption of spin 0, the present data are consistent with the pure scalar hypothesis, while disfavoring the pure pseudoscalar hypothesis.

The Higgs field in the Standard Model provides
the masses of all the weak gauge boson and fermions.

$$\mathcal{L}_{\rm SM} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_Y + \mathcal{L}_H$$

$$\mathcal{L}_g = -\frac{1}{4}G^s_{\mu\nu}(x)G^{s\mu\nu}(x) - \frac{1}{4}F^a_{\mu\nu}(x)F^{a\mu\nu}(x) - \frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x)$$

$$G^s_{\mu\nu}(x) = \partial_\mu G^s_\mu(x) - \partial_\nu G^s_\mu(x) - g_3 f^{stu} G^t_\mu(x) G^u_\mu(x)$$

$$\mathcal{L}_f = \bar{q}_L(x) i\gamma^\mu \left(\partial_\mu - ig_3 \frac{\lambda^s}{2} G^s_\mu(x) - ig_2 \frac{\tau^a}{2} A^a_\mu(x) - \frac{i}{6} g_1 B_\mu(x)\right) q_L(x) + \cdots$$

$$\mathcal{L}_Y = \bar{q}_L(x) \boldsymbol{Y}_u u_R(x) \tilde{\Phi}(x) + \bar{q}_L(x) \boldsymbol{Y}_d d_R(x) \Phi(x) + \bar{l}_L(x) \boldsymbol{Y}_l e_R(x) + \text{h.c.}$$

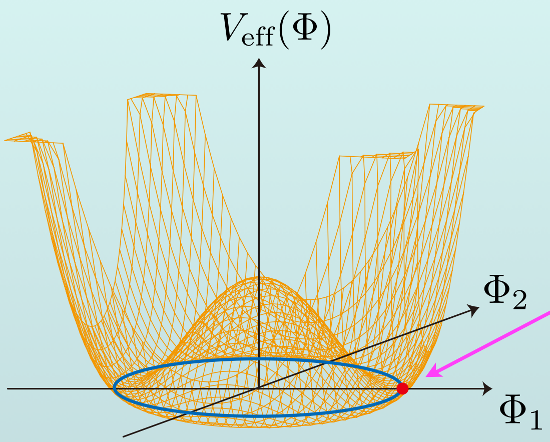
Yukawa coupling matrix

Higgs field $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$ $\tilde{\Phi}(x) = i\tau^2 \Phi^*(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}$

No mass scale in the gauge and fermion sector.

A part of the gauge symmetry of the SM is
spontaneously broken by $\langle \Phi \rangle \neq 0$

A symmetry of the lagrangian is broken by the ground state.



classically
degenerate ground state

Picking up a point
from the degenerate states
breaks the symmetry.

The only mass scale arises from the Higgs sector.

$$\mathcal{L}_H = \left| \left(\partial_\mu - i g_2 \frac{\tau^a}{2} A_\mu^a(x) - \frac{i}{2} g_1 B_\mu(x) \right) \Phi(x) \right|^2 - V(\Phi)$$

$$\Phi \longrightarrow \langle \Phi \rangle = \begin{pmatrix} 0 \\ v_0/\sqrt{2} \end{pmatrix} \qquad m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + 0 A_\mu A^\mu$$

where

$$m_W^2 = \frac{1}{4} g_2^2 v_0^2, \qquad m_Z^2 = \frac{1}{4}(g_2^2 + g_1^2) v_0^2 = \frac{m_W^2}{\cos^2\theta_W}$$

$$W_\mu^\pm(x) = \frac{1}{\sqrt{2}} \left(A_\mu^1(x) \mp i A_\mu^2(x) \right)$$

$$Z_\mu(x) = A_\mu^3(x) \cos\theta_W - B_\mu(x) \sin\theta_W$$

$$A_\mu(x) = A_\mu^3(x) \sin\theta_W + B_\mu(x) \cos\theta_W$$

Similarly for the fermions,

setting $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 + h(x) \end{pmatrix}$ unitary gauge

$$\mathcal{L}_Y = \frac{v_0 + h(x)}{\sqrt{2}} \left[\bar{u}_L \mathbf{Y}_u u_R + \bar{d}_L \mathbf{Y}_d d_R + \bar{e}_L \mathbf{Y}_e e_R + \text{h.c.} \right]$$

bi-unitary transformation by f_L and f_R

$$\left(1 + \frac{h(x)}{v_0} \right) \left(m_{u_A} \bar{u}_A u_A + m_{d_A} \bar{d}_A d_A + m_{e_A} \bar{e}_A e_A \right)$$

$A = 1, 2, 3:$ generation

Higgs boson

The couplings of the Higgs boson to the gauge bosons and fermions are proportional to their masses.

The effect of the unitary transformation resides only in the quark charged-current interaction.

CKM matrix

masses of the SM particles

except for the higgs boson

	electric charge	1st gen.	2nd gen.	3rd gen.
quark	$+\frac{2}{3}$	u	c	t
		2–3MeV	1.27GeV	174GeV
	$-\frac{1}{3}$	d	s	b
		4–6MeV	101MeV	4.2GeV
charged lepton	-1	e	μ	τ
		0.51MeV	106MeV	1.8GeV

Weak boson	
W^+, W^-	Z
80.4GeV	91.2GeV

N.B.

$$m_u + m_u + m_d \sim \frac{1}{10} m_{\text{proton}}$$

90% of the nucleon mass comes from QCD dynamics.

--- confirmed by lattice MC calculation

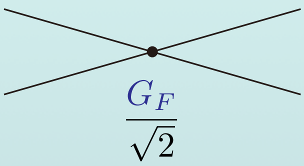
What determines the value of the Higgs VEV v_0 ?

4-Fermi effective theory vs W-exchange interaction

e.g. weak decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

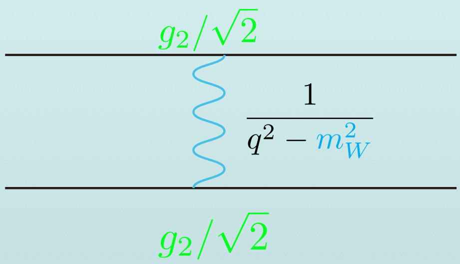
$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} \bar{e}\gamma_\rho(1-\gamma_5)\nu_e \, \bar{\nu}_\mu \gamma^\rho (1-\gamma_5)\mu + \cdots$$

(V–A)-type current-current interaction



$$\mathcal{L}_{CC} = \frac{g_2}{\sqrt{2}} \left(J^-_\mu W^{+\mu} + J^+_\mu W^{-\mu} \right)$$

$$J^-_\mu = \bar{\nu}_A \gamma_\mu \frac{1-\gamma_5}{2} e_A + \bar{u}_A \gamma_\mu \frac{1-\gamma_5}{2} U_{AB} d_B$$



$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_W^2} = \frac{1}{2v_0^2} \quad \Rightarrow \quad v_0 = \left(\frac{1}{\sqrt{2}\,G_F} \right)^{\frac{1}{2}} = 246.26\text{GeV}$$

In this lecture, we shall focus on
the properties of the Higgs boson(s) in the SM and its extensions.

mass, branching ratio (BR) | 2-Higgs-doublet Model (2HDM)
Minimal SUSY SM (MSSM)

SM 1 Higgs doublet $(I, Y) = \left(\frac{1}{2}, \frac{1}{2}\right)$ cf. $Q = I_3 + Y$

to break $SU(2)_L \times U(1)_Y \longrightarrow U(1)_{\rm em}$

If it were a triplet, extra U(1) sym. would be left.

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v_0 + h(x) + ia(x)) \end{pmatrix}$$

Nambu-Goldstone mode "gauged away" ($\phi^+(x)$, $a(x)$)

physical Higgs field ($h(x)$)

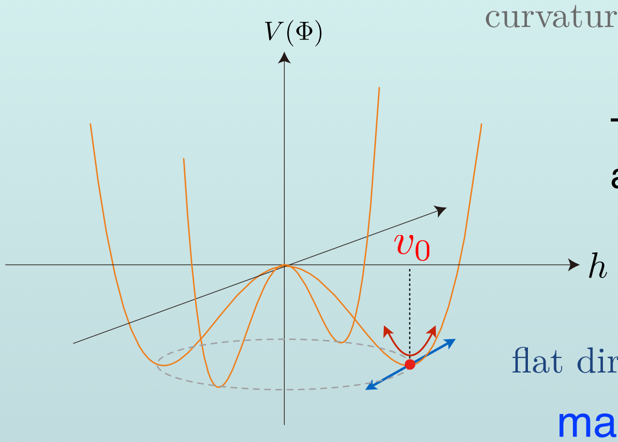
1 complex + 2 real scalar fields ⟶ 1 real scalar is physical

gauge-invariant, renormalizable potential

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$
$$= \frac{1}{2}\cdot 2\lambda v_0^2\, h^2 + \lambda v_0 h^3 + \frac{\lambda}{4}h^4$$

minimum at v_0
$\mu^2 = \lambda v_0$

$2\lambda v_0^2 = m_h^2$ tree level mass
curvature of $V(\Phi)$ at v_0



The 3- and 4-point self-interactions are related only to the Higgs mass.

flat direction(s) of $V(\Phi)$ at v_0
massless NG modes

Go on to extend the Higgs sector.

Suppose that we add a scalar field in repr. $(I^{(r)}, Y_r)$ of $SU(2)_L \times U(1)_Y$.

Only its neutral component can acquire VEV $v_r \neq 0$.

m_W and m_Z depend on the repr. and the value of v_r.

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1.0002^{+0.0024}_{-0.0009} \qquad (95\%\text{CL})$$

$$= \frac{1}{2}\frac{\sum_r v_r^2 \left[I^{(r)}(I^{(r)}+1) - (I_3^{(r)})^2\right]}{\sum_r v_r^2 (I_3^{(r)})^2}$$

$$I(I+1) - I_3^2 = 2I_3^2 \longrightarrow \rho = 1$$

$$(I, I_3) = (0,0),\ \left(\tfrac{1}{2}, \pm\tfrac{1}{2}\right),\ (3, \pm 2), \cdots$$

mass of the weak gauge bosons

$$\mathcal{L}_H \sim \sum_r \langle\Phi_r\rangle^\dagger \left|g_2 T_r^a A_\mu^a + g_1 Y_r B_\mu\right|^2 \langle\Phi_r\rangle \qquad \langle\Phi_r\rangle = \begin{pmatrix} 0 \\ \vdots \\ v_r \\ \vdots \\ 0 \end{pmatrix}$$

$$\frac{g_2}{\cos\theta_W}(T_r^3 - Q_r \sin^2\theta_W) Z_\mu + e Q_r A_\mu + \frac{g_2}{\sqrt{2}}\left(T_r^+ W_\mu^+ + T_r^- W_\mu^-\right)$$

$$T^\pm \equiv T^1 \pm i T^2 = \left(T^\mp\right)^\dagger \qquad Q = T^3 + Y$$

$$T^+T^- + T^-T^+ = 2(T^1)^2 + 2(T^2)^2 = 2\left[\boldsymbol{T}^2 - (T^3)^2\right]$$

$$Q\langle\Phi\rangle = 0 \qquad \langle\Phi\rangle^\dagger T^3 T^\pm \langle\Phi\rangle = \langle\Phi\rangle^\dagger T^\pm T^3 \langle\Phi\rangle = 0$$

$$= \sum_r v_r^2 \left[\underbrace{\frac{g_2}{\cos^2\theta_W}(I_3^{(r)})^2}_{\frac{1}{2}m_Z^2} Z_\mu Z^\mu + \underbrace{g_2^2\left(I^{(r)}(I^{(r)}+1) - (I_3^{(r)})^2\right)}_{m_W^2} W_\mu^+ W^{-\mu}\right]$$

To keep ρ=1, we shall add a doublet or a singlet.

One can introduce *any* multiplet with $v_r = 0$.

$$\Phi_1(x),\ \Phi_2(x)\ \in\ \left(\frac{1}{2},+\frac{1}{2}\right) \quad \text{of } SU(2)_L \times U(1)_Y$$

the most general gauge-invariant, renormalizable potential

$$\begin{aligned} V(\Phi_1,\Phi_2) &\\ = &\ m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 + (m_3^2\Phi_1^\dagger\Phi_2 + \text{h.c.}) \\ &+\frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+\left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\} \end{aligned}$$

cf. Fierz identity : $(\Phi_i^\dagger\tau^a\Phi_j)(\Phi_k^\dagger\tau^a\Phi_l) = 2(\Phi_i^\dagger\Phi_l)(\Phi_k^\dagger\Phi_j) - (\Phi_i^\dagger\Phi_j)(\Phi_k^\dagger\Phi_l)$

Yukawa interaction

$$\mathcal{L}_Y \sim \bar{q}_L \left(\boldsymbol{Y}_d^{(1)}\Phi_1 + \boldsymbol{Y}_d^{(2)}\Phi_2\right) d_R$$

After diagonalizing the mass matrix $\boldsymbol{Y}_d^{(1)}\langle\Phi_1\rangle + \boldsymbol{Y}_d^{(2)}\langle\Phi_1\rangle$

we are left with the (tree-level Higgs mediated) FCNC.

Impose the discrete symmetry:

(I) $\Phi_1 \mapsto \Phi_1, \quad \Phi_2 \mapsto -\Phi_2 : \quad u_R \mapsto u_R, \quad d_R \mapsto d_R, \quad e_R \mapsto e_R$

(II) $\Phi_1 \mapsto \Phi_1, \quad \Phi_2 \mapsto -\Phi_2 : \quad u_R \mapsto -u_R, \quad d_R \mapsto d_R, \quad e_R \mapsto e_R$

then $\lambda_6 = \lambda_7 = 0$ in the Higgs potential

(I) $\mathcal{L}_Y = \bar{q}_L\boldsymbol{Y}_d d_R\Phi_1 + \bar{q}_L\boldsymbol{Y}_u u_R\tilde{\Phi}_1 + \bar{l}_L\boldsymbol{Y}_e e_R\Phi_1 + \text{h.c.}$

(II) $\mathcal{L}_Y = \bar{q}_L\boldsymbol{Y}_d d_R\Phi_1 + \bar{q}_L\boldsymbol{Y}_u u_R\tilde{\Phi}_2 + \bar{l}_L\boldsymbol{Y}_e e_R\Phi_1 + \text{h.c}$

$\tilde{\Phi}(x) = i\tau^2\Phi^*(x)$

Higgs potential and the Yukawa interaction of
Minimal Supersymmetric Standard Model (MSSM)

||

Type-II 2HDM with

$$\begin{cases} \tilde{\Phi}_1 \to \Phi_d, \qquad \Phi_2 \to \Phi_u, \\ \lambda_1 = \lambda_2 = -\lambda_3 = \frac{g_2^2 + g_1^2}{4}, \quad \lambda_4 = -\frac{g_2^2}{4}, \quad \lambda_{5,6,7} = 0 \end{cases}$$

Of course, there are introduced
new interactions containing SUSY particles.

VEV and physical modes of the Higgs fields *in the MSSM*

Vacuum Expectation Value

$$\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle \Phi_u \rangle = \frac{e^{i\theta}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}$$

Here we assume $U(1)_{em}$ is not broken.

fluctuation around the vacuum

$$\Phi_d(x) = \langle \Phi_d \rangle + \begin{pmatrix} \frac{1}{\sqrt{2}}(h_d(x) + i a_d(x)) \\ \phi_d^-(x) \end{pmatrix}$$

$$\Phi_u(x) = \langle \Phi_u \rangle + e^{i\theta} \begin{pmatrix} \phi_u^+(x) \\ \frac{1}{\sqrt{2}}(h_u(x) + i a_u(x)) \end{pmatrix},$$

1 charged and 1 neutral NG modes

1 charged and 3 neutral physical modes

N.B.

- In the general 2HDM, $U(1)_{em}$ can be broken if the charged component of the Higgs field acquire a nonvanishing VEV.

One must ***tune*** the parameters in the potential.

In the MSSM, VEV of the charged component =0.

- **Mass eigenstates** are some linear combinations of the physical modes.

Shown in the MSSM below.

- $\theta \neq 0$ → CP violation

relative phase of the VEVs

This occurs when m_3^2 and λ_5 are complex and $\mathrm{Arg}\lambda_5 \neq 2\mathrm{Arg}(m_3^2)$.

Even when all the parameters are *real*, V can be minimized at $|\cos\theta| \neq 1$.

$\lambda_5 > 0$ and $m_3^2 < \lambda_5 v_1 v_2$ → spontaneous CP violation

Mass eigenstates

minimizing the Higgs potential (MSSM at the tree level)

$$\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_4 \\ v_2 + i v_3 \end{pmatrix}$$

$$\begin{aligned} V(\boldsymbol{v}) &= m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u - m_3^2 (\Phi_d \Phi_u + \mathrm{h.c.}) + \frac{g_2^2 + g_1^2}{8} (\Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u)^2 + \frac{g_2^2}{2} |\Phi_d^\dagger \Phi_u|^2 \\ &= \frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2^2 (v_2^2 + v_3^2 + v_4^2) - m_3^2 v_1 v_2 \\ &\quad + \frac{1}{32} (g_2^2 + g_1^2)(-v_1^2 + v_2^2 + v_3^2 + v_4^2)^2 + \frac{1}{8} g_2^2 v_1^2 v_4^2 \end{aligned}$$

The values of m_1^2, m_2^2 and m_3^2 depend on SUSY breaking.

constrained by the stability and requirement of EWSB

In SM, ensured by $\lambda > 0$ $\qquad -\mu^2 < 0$

stability

The quartic terms vanish along $v_1^2 = v_2^2 + v_3^2$ and $v_4 = 0$, for which

$$V = \frac{1}{2}(m_1^2 + m_2^2)v_1^2 - m_3^2 v_1 v_2 \geq \frac{1}{2}(m_1^2 + m_2^2)v_1^2 - \left|m_3^2\right| |v_1| \sqrt{v_1^2 - v_3^2}$$
$$\geq \frac{1}{2}(m_1^2 + m_2^2 - 2\left|m_3^2\right|)v_1^2.$$

$\therefore \quad m_1^2 + m_2^2 \geq 2\left|m_3^2\right| \Longrightarrow V$ is bounded from below

EWSB

curvature of V at $\boldsymbol{v} = 0$: $$\left.\frac{\partial^2 V(\boldsymbol{v})}{\partial v_i \partial v_j}\right|_{\boldsymbol{v}=0} = \begin{pmatrix} m_1^2 & -m_3^2 & 0 & 0 \\ -m_3^2 & m_2^2 & 0 & 0 \\ 0 & 0 & m_2^2 & 0 \\ 0 & 0 & 0 & m_2^2 \end{pmatrix}$$

This matrix must have at least one negative eigenvalue.

$$m_2^2 < 0 \qquad \text{or} \qquad m_1^2 + m_2^2 - \sqrt{(m_1^2 - m_2^2)^2 + 4(m_3^2)^2} < 0$$

$$\therefore \quad m_2^2 < 0 \qquad \text{or} \qquad m_1^2 m_2^2 < (m_3^2)^2$$

Minimum of $V(\boldsymbol{v})$

solutions to

$$\frac{\partial V}{\partial v_1} = m_1^2 v_1 - m_3^2 v_2 - \frac{g_2^2 + g_1^2}{8} v_1(-v_1^2 + v_2^2 + v_3^2 + v_4^2) + \frac{g_2^2}{4} v_1 v_4^2 = 0,$$
$$\frac{\partial V}{\partial v_2} = m_2^2 v_2 - m_3^2 v_1 + \frac{g_2^2 + g_1^2}{8} v_2(-v_1^2 + v_2^2 + v_3^2 + v_4^2) = 0,$$
$$\frac{\partial V}{\partial v_3} = m_2^2 v_3 + \frac{g_2^2 + g_1^2}{8} v_3(-v_1^2 + v_2^2 + v_3^2 + v_4^2) = 0,$$
$$\frac{\partial V}{\partial v_4} = m_2^2 v_4 + \frac{g_2^2 + g_1^2}{8} v_4(-v_1^2 + v_2^2 + v_3^2 + v_4^2) + \frac{g_2^2}{4} v_1^2 v_4 = 0,$$

(A) $v_1 = v_0 \cos\beta, \quad v_2 = v_0 \sin\beta, \quad v_3 = v_4 = 0,$

v_0 and $\tan\beta$ are determined from the parameters in V by

$$m_1^2 = m_3^2 \tan\beta - \frac{1}{2} m_Z^2 \cos(2\beta)$$
$$m_2^2 = m_3^2 \cot\beta + \frac{1}{2} m_Z^2 \cos(2\beta)$$
$$m_Z^2 = \frac{1}{4}(g_2^2 + g_1^2) v_0^2$$

→ Instead, we determine m_1^2 and m_2^2 in terms of $(v_0, \tan\beta)$.

(B) $v_1 = v_2 = 0, \quad v_3^2 + v_4^2 = -\frac{8m_2^2}{g_2^2 + g_1^2}$ (for $m_2^2 < 0$)

One of the eigenvalues of $\left(\frac{\partial^2 V(\boldsymbol{v})}{\partial v_i \partial v_j}\bigg|_{(B)} \right) < 0.$ → not a minimum!!

As long as the conditions for the stability and EWSB are satisfied, (A) gives the global minimum of the potential.

the masses of the Higgs bosons

↑

curvature of V at the minimum

$$\Phi_d(x) = \langle \Phi_d \rangle + \begin{pmatrix} \frac{1}{\sqrt{2}}(h_d(x) + i a_d(x)) \\ \phi_d^-(x) \end{pmatrix} \qquad \Phi_u(x) = \langle \Phi_u \rangle + \begin{pmatrix} \phi_u^+(x) \\ \frac{1}{\sqrt{2}}(h_d(x) + i a_d(x)) \end{pmatrix}$$

Expand $V(\Phi_d, \Phi_u)$ in powers of the fluctuation fields.

$$V(\Phi_d, \Phi_u) = \frac{1}{2}(h_d\ h_u)\mathcal{M}_H^2 \begin{pmatrix} h_d \\ h_u \end{pmatrix} + \frac{1}{2}(a_d\ a_u)\mathcal{M}_A^2 \begin{pmatrix} a_d \\ a_u \end{pmatrix} + (\phi_d^+\ \phi_u^+)\mathcal{M}_{H^\pm}^2 \begin{pmatrix} \phi_d^- \\ \phi_u^- \end{pmatrix} + \cdots$$

where CP-even CP-odd charged

$$\mathcal{M}_H^2 = \begin{pmatrix} m_3^2 \tan\beta + m_Z^2 \cos^2\beta & -(m_3^2 + m_Z^2 \sin\beta\cos\beta) \\ -(m_3^2 + m_Z^2 \sin\beta\cos\beta) & m_3^2 \cot\beta + m_Z^2 \sin^2\beta \end{pmatrix}$$

$$\rightarrow \quad m_{h,H}^2 = \frac{1}{2}\left[m_Z^2 + \frac{2m_3^2}{\sin(2\beta)} \pm \sqrt{\left(m_Z^2 + \frac{2m_3^2}{\sin(2\beta)} \right)^2 - 8\frac{\cos^2(2\beta)}{\sin(2\beta)} m_3^2 m_Z^2} \right]$$

$$\mathcal{M}_A^2 = m_3^2 \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \quad \rightarrow \quad 0, \quad m_A^2 = \frac{2m_3^2}{\sin(2\beta)}$$

$$\mathcal{M}_{H^\pm}^2 = \begin{pmatrix} m_3^2 \tan\beta + m_W^2 \sin^2\beta & m_3^2 + m_W^2 \sin\beta\cos\beta \\ m_3^2 + m_W^2 \sin\beta\cos\beta & m_3^2 \cot\beta + m_W^2 \cos^2\beta \end{pmatrix}$$

$$\rightarrow \quad 0, \quad m_{H^\pm}^2 = m_W^2 + \frac{2m_3^2}{\sin(2\beta)}$$

zero eigenvalues ⇔ NG modes

SM limit $m_3^2 \longrightarrow \infty$ $m_A^2, m_{H^\pm} \longrightarrow \infty$

'decoupling limit' V becomes very steep except for h-direction

h-boson is too light irrespective of m_3^2

Noting that

$$m_{h,H}^2 = \frac{1}{2}\left[m_Z^2 + m_A^2 \pm \sqrt{(m_Z^2+m_A^2)^2 - 4m_Z^2 m_A^2 \cos^2(2\beta)}\right]$$

we find

$$m_h^2 \le \frac{1}{2}\left(m_Z^2 + m_A^2 - \left|m_Z^2 - m_A^2\right|\right) = \min\left\{m_Z^2, m_A^2\right\}$$

$$m_H^2 \ge \frac{1}{2}\left(m_Z^2 + m_A^2 + \left|m_Z^2 - m_A^2\right|\right) = \max\left\{m_Z^2, m_A^2\right\}$$

Is the MSSM excluded by LHC's discovery of 125GeV Higgs boson?

No! still survives marginally

1-loop correction is sizable compared to the tree-level m_Z

top Yukawa coupling

superpartner of the top quark (stop)

the bound is modified roughly to

$$m_h^2 \le m_Z^2 \cos^2(2\beta) + \frac{3}{2\pi^2}\frac{m_t^4}{v_0^2}\log\left(\frac{m_{\tilde{t}}^2 + m_t^2}{m_t^2}\right)$$

m_Z^2 $y_t^2 m_t^2$

m_h can be as large as 135GeV depending on the parameters in stop sector

Another loophole is the Light Higgs Scenario.

It is H that was found by LHC with $m_H = 125\text{GeV}$.

g_{ZZh} is too small to be produced in LEP2.

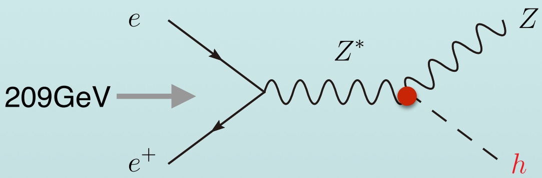
If we denote
$$\begin{pmatrix} h_d \\ h_u \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

the interaction lagrangian is expressed as

$$\mathcal{L}_{\rm gauge} = g_2 m_W \left(W_\mu^+ W^{-\mu} + \frac{Z_\mu Z^\mu}{2\cos^2\theta_W} \right) (H\cos(\beta-\alpha) + h\sin(\beta-\alpha)) + \cdots$$

$$\mathcal{L}_Y = -\frac{g_2 m_t}{2m_W}\bar{t}t \left(\frac{\sin\alpha}{\sin\beta} H + \frac{\cos\alpha}{\sin\beta} h \right) - \frac{g_2 m_b}{2m_W}\bar{b}b \left(\frac{\cos\alpha}{\cos\beta} H - \frac{\sin\alpha}{\cos\beta} h \right) + \cdots$$

LEPII



$m_h > 92.8\text{GeV}$ (95%CL)

How to calculate the radiative corrections to the Higgs masses and mixing.

from the tree-level potential to the effective potential

$$V_{\rm eff}(\boldsymbol{v}) = V(\boldsymbol{v}) + \frac{1}{64\pi^2}\sum_a c_a (\bar{m}_a^2)^2 \left(\log\frac{\bar{m}_a^2}{M^2} - \frac{3}{2} \right)$$

$a = t, \tilde{t}, b, \tilde{b}, W, Z, \cdots$

$\bar{m}_a(\boldsymbol{v})$: field-dependent mass $\quad$ M : renormalization scale $(\sim v_0)$

c_a : degrees of freedom ($c_a < 0$ for fermions)

e.g. $\bar{m}_t^2 = \frac{1}{2}y_t^2(v_2^2+v_3^2)$, $\quad \bar{m}_b^2 = \frac{1}{2}y_b^2 v_1^2$, $\quad \bar{m}_W^2 = \frac{1}{4}g_2^2(v_1^2+v_2^2+v_3^2)$

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + \left(\frac{g_2^2}{8} - \frac{g_1^2}{24}\right)(v_1^2 - v_2^2) + \bar{m}_t^2 & \frac{y_t}{\sqrt{2}}\left(A_t^*(v_2 - iv_3) - \mu v_1\right) \\ \frac{y_t}{\sqrt{2}}\left(A_t(v_2 + iv_3) - \mu^* v_1\right) & m_{\tilde{t}_R}^2 + \frac{g_1^2}{6}(v_1^2 - v_2^2) + \bar{m}_t^2 \end{pmatrix}$$

$\longrightarrow \bar{m}_{\tilde{t}_1}^2, \bar{m}_{\tilde{t}_2}^2$

Higgs fields $\longrightarrow$ VEV + fluctuation

$$\Phi_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + h_d + i a_d) \\ \phi_d^- \end{pmatrix}, \qquad \Phi_u = e^{i\theta} \begin{pmatrix} \phi_u^+ \\ \frac{1}{\sqrt{2}}(v_u + h_u + i a_u) \end{pmatrix}$$

tadpole conditions:

$$0 = \frac{1}{v_d}\left\langle \frac{\partial V_{\rm eff}}{\partial h_d} \right\rangle = m_1^2 - {\rm Re}(m_3^2 e^{i\theta})\tan\beta + \frac{1}{2}m_Z^2\cos(2\beta) + \cdots,$$

$$0 = \frac{1}{v_u}\left\langle \frac{\partial V_{\rm eff}}{\partial h_u} \right\rangle = m_2^2 - {\rm Re}(m_3^2 e^{i\theta})\cot\beta - \frac{1}{2}m_Z^2\cos(2\beta) + \cdots,$$

$$0 = \frac{1}{v_u}\left\langle \frac{\partial V_{\rm eff}}{\partial a_d} \right\rangle = \frac{1}{v_d}\left\langle \frac{\partial V_{\rm eff}}{\partial a_u} \right\rangle = {\rm Im}(m_3^2 e^{i\theta}) + \cdots.$$

$\langle \cdots \rangle$ is the value evaluated at the vacuum

$\longrightarrow$ m_1^2, m_2^2, ${\rm Im}\left(m_3^2\, e^{i\theta}\right)$ in terms of the other parameters

mass² matrix of the neutral bosons and charged boson mass

after extracting the NG modes

$$\mathcal{M}^2 = \begin{pmatrix} \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d^2} \right\rangle & \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d \partial h_u} \right\rangle & \frac{1}{\cos\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d \partial a_u} \right\rangle \\ \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d \partial h_u} \right\rangle & \left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_u^2} \right\rangle & \frac{1}{\sin\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_u \partial a_d} \right\rangle \\ \frac{1}{\cos\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_d \partial a_u} \right\rangle & \frac{1}{\sin\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial h_u \partial a_d} \right\rangle & \frac{1}{\sin\beta\cos\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial a_d \partial a_u} \right\rangle \end{pmatrix}$$

↑ CP violation in the squark/Higgsino sectors

$$m_{H^\pm}^2 = \frac{1}{\sin\beta\cos\beta}\left\langle \frac{\partial^2 V_{\rm eff}}{\partial \phi_d^+ \partial \phi_u^-} \right\rangle = \frac{1}{\sin\beta\cos\beta}{\rm Re}(m_3^2 e^{i\theta}) + m_W^2 + \cdots$$

$\longrightarrow$ input $m_{H^\pm}^2$ $\longrightarrow$ ${\rm Re}\left(m_3^2\, e^{i\theta}\right)$

mass eigenstates: H_i

$$\begin{pmatrix} h_d \\ h_u \\ a \end{pmatrix} = \mathcal{O}\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \qquad \mathcal{O}^T \mathcal{M}^2 \mathcal{O} = {\rm diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2)$$

gauge and Yukawa interactions

$$\mathcal{L}_{\rm gauge} \sim g_2 m_W\, g_{VVH_i} \left(W_\mu^+ W^{-\mu} + \frac{Z_\mu Z^\mu}{2\cos^2\theta_W} \right) H_i + \frac{g_2}{2\cos\theta_W}\, g_{ZH_iH_j} Z^\mu \left(H_i \overleftrightarrow{\partial}_\mu H_j \right)$$

$$\mathcal{L}_Y \sim -\frac{g_2 m_b}{2 m_W}\, \bar{b}(g^S_{bbh_i} + i\gamma_5 g^P_{bbh_i}) b\, H_i$$

where

$$g_{VVH_i} = O_{1i}\cos\beta + O_{2i}\sin\beta \qquad \sum_{i=1}^{3} g^2_{VVH_i} = 1$$

$$g_{ZH_iH_j} = \frac{1}{2}\left[(O_{3i}O_{1j} - O_{3j}O_{1i})\sin\beta + (O_{3i}O_{2j} - O_{3j}O_{2i})\cos\beta\right]$$

$$g^S_{bbH_i} = O_{1i}\frac{1}{\cos\beta}, \quad g^P_{bbH_i} = -O_{3i}\tan\beta, \quad g^2_{bbH_i} = \left(g^S_{bbH_i}\right)^2 + \left(g^P_{bbH_i}\right)^2$$

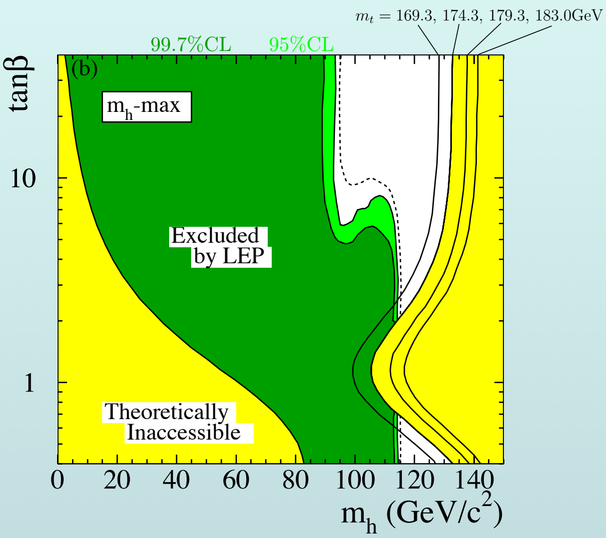
SM: $g_{VVH} = 1,\ g_{ZHH} = 0,\ g_{bbH} = 1$

CP-conserving m_h-max. scenario

$M_{\rm SUSY} \simeq m_{\tilde{q}_L} \simeq m_{\tilde{U}_R} \simeq m_{\tilde{D}_R}$

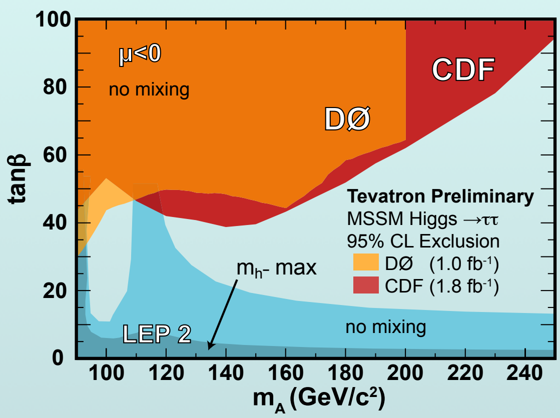
$X_t \equiv A_t - \mu\cot\beta = $ max.

allowed region for the lightest neutral Higgs boson



m_h-max benchmark scenario

allowed region for the pseudoscalar Higgs boson



[PDG: C. Amsler et al., Phys.Lett. B667, 1 (2008)]

Allowed region for
the lightest neutral Higgs boson
in the CPX scenario

$|A_t| = |A_b| = 1\text{TeV},\ \phi_A = \phi_{M_3} = \pi/2,$
$\mu = 2\text{TeV},\ M_{\text{SUSY}} = 500\text{GeV}$

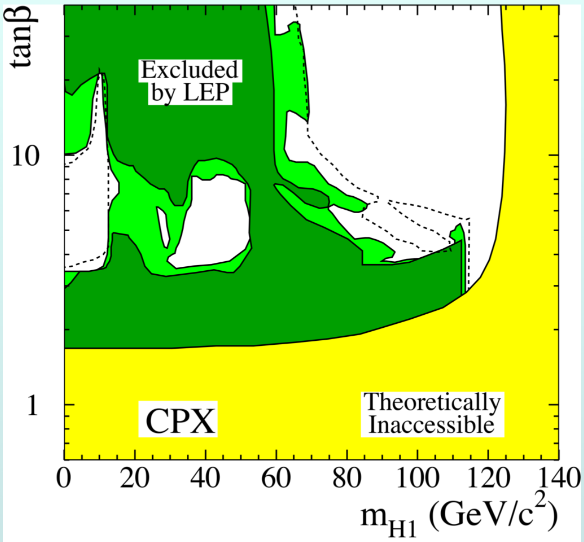
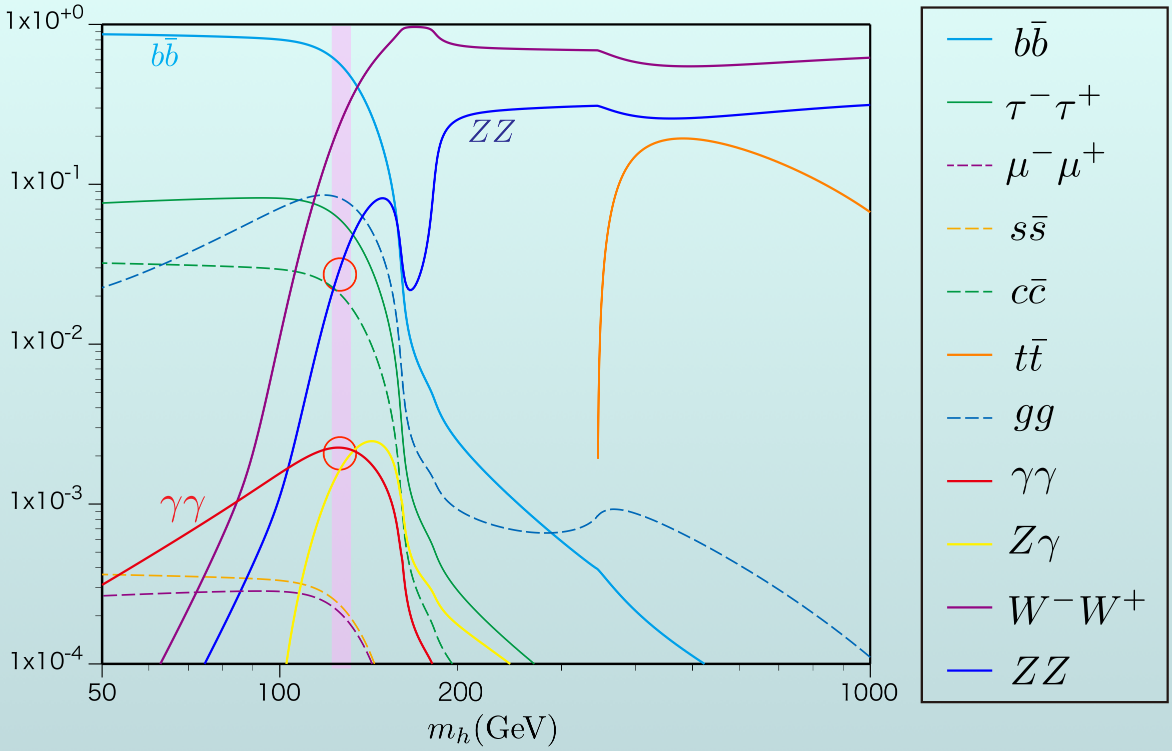


Figure 12: The MSSM exclusion contours, at the 95% C.L. (light-green) and the 99.7% CL (dark-green), obtained by LEP for the CPX scenario defined in the text. Here, $m_t = 174.3$ GeV. The figure shows the excluded and theoretically inaccessible regions in the $(m_{H_1}, \tan\beta)$ projection. The dashed lines indicate the boundary of the region which is expected to be excluded at the 95% C.L. in the absence of a signal. Color version at end of book.

To determine to which model does
the scalar boson discovered by LHC belong,
one must study **its coupling to the other particles**.

↑
decay branching ratios

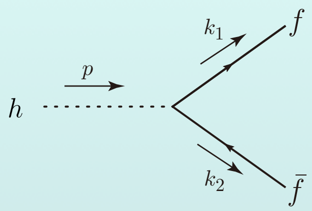
Decay Branching Ratio (SM)



Made from the data generated by H-Decay package

tree-level 2-body decays in the SM

$$\mathcal{L}_Y = -\frac{y_f}{\sqrt{2}}(v_0 + h(x))\bar{f}(x)f(x) = -m_f\bar{f}f - \frac{m_f}{2g_2 m_W}\bar{f}f\,h$$



$$i\mathcal{M}(h \to f\bar{f}) = -i\frac{g_2 m_f}{2m_W}\bar{v}^{s'}(k_2)u^s(k_1)$$

$$\sum_{s,s'}\left|\mathcal{M}(h \to f\bar{f})\right|^2 = \frac{N_C g_2^2 m_f^2}{m_W^2}(k_1 \cdot k_2 - m_f^2)$$

$$k_1 \cdot k_2 = \frac{1}{2}(p^2 - k_1^2 - k_2^2) = \frac{1}{2}(m_h^2 - 2m_f^2)$$

$$\Gamma(h \to f\bar{f}) = \frac{1}{2m_h}\int \frac{d^3\boldsymbol{k}_1}{(2\pi)^3 2E_1}\frac{d^3\boldsymbol{k}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p - k_1 - k_2)\sum_{s,s'}\left|\mathcal{M}(h \to f\bar{f})\right|^2$$

in the rest frame of h

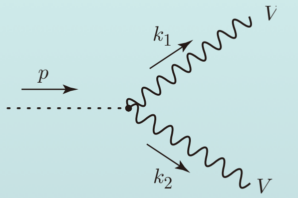
$$= \frac{N_C}{2m_h}\frac{g_2^2 m_f^2(m_h^2 - 4m_f^2)}{2m_W^2}\int \frac{d^3\boldsymbol{k}_1}{(2\pi)^3\, 2\sqrt{k_1^2 + m_f^2}}\frac{2\pi\delta\left(m_h - 2\sqrt{k_1^2 + m_f^2}\right)}{2\sqrt{k_1^2 + m_f^2}}$$

$$= \frac{N_C g_2^2 m_f^2 (m_h^2 - 4m_f^2)}{16\pi m_h m_W^2} \int \frac{dk_1\, k_1^2}{k_1^2 + m_f^2}\, \delta\Big(m_h - 2\sqrt{k_1^2 + m_f^2}\Big)$$

$$\delta\Big(m_h - 2\sqrt{k_1^2 + m_f^2}\Big) = \frac{m_h}{2\sqrt{m_h^2 - 4m_f^2}}\,\delta\left(k_1 - \sqrt{\frac{m_h^2}{4} - m_f^2}\right)$$

$$= \theta(m_h - 2m_f)\frac{N_C g_2^2 m_f^2 m_h}{32\pi m_W^2}\left(1 - \frac{4m_f^2}{m_h^2}\right)^{\frac{3}{2}}$$

$$\mathcal{L}_H = \frac{1}{2} g_2^2 v_0 h W_\mu^+ W^{-\mu} + \frac{1}{4}\frac{g_2^2}{\cos^2\theta_W} v_0 h Z_\mu Z^\mu + \cdots$$



$$i\mathcal{M}(h \to W^+W^-) = i g_2 m_W \epsilon_\mu^{r_1 *}(k_1)\epsilon^{r_2 * \mu}(k_2)$$

$$i\mathcal{M}(h \to ZZ) = \frac{i g_2 m_W}{\cos^2\theta_W}\epsilon_\mu^{r_1 *}(k_1)\epsilon^{r_2 * \mu}(k_2),$$

summation over the spins

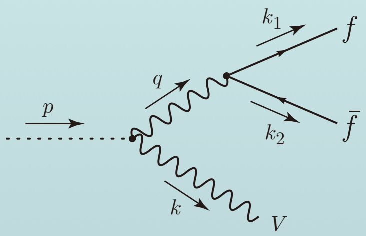
$$\sum_r \epsilon_\mu^{r*}(k)\epsilon_\nu^r(k) = -\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_V^2}\right), \qquad (V = W^\pm,\, Z)$$

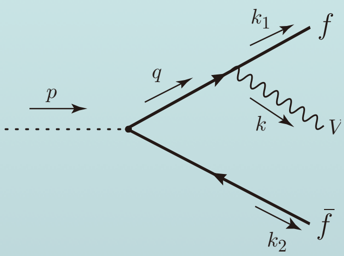
$$\sum_{r_1, r_2} |\mathcal{M}(h \to VV)|^2 = \begin{cases} g_2^2 m_W^2 \left(2 + \dfrac{(k_1 \cdot k_2)^2}{m_W^4}\right) & \text{for } W\text{-boson} \\ \dfrac{g_2^2 m_Z^2}{\cos^2\theta_W}\left(2 + \dfrac{(k_1 \cdot k_2)^2}{m_Z^4}\right) & \text{for } Z\text{-boson,} \end{cases}$$

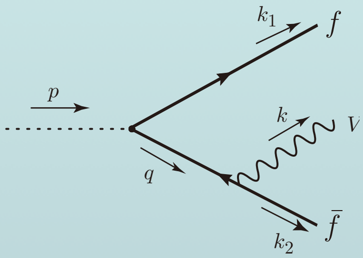
$$\Gamma(h \to W^+W^-) = \theta(m_h - 2m_W)\frac{g_2^2}{64\pi}\frac{m_h^3}{m_W^2}\left(1 - \frac{4m_W^2}{m_h^2}\right)^{1/2}\left[1 - \frac{4m_W^2}{m_h^2} + \frac{3}{4}\left(\frac{4m_W^2}{m_h^2}\right)^2\right]$$

$$\Gamma(h \to ZZ) = \theta(m_h - 2m_Z)\frac{g_2^2}{128\pi}\frac{m_h^3}{m_W^2}\left(1 - \frac{4m_Z^2}{m_h^2}\right)^{1/2}\left[1 - \frac{4m_Z^2}{m_h^2} + \frac{3}{4}\left(\frac{4m_Z^2}{m_h^2}\right)^2\right]$$

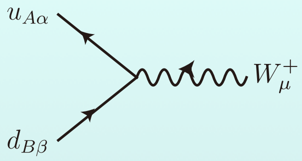
For $m_V < m_h < 2m_V$, the 3-body decay $m_h \to V f \bar{f}$ occurs.



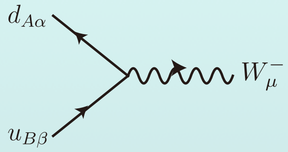




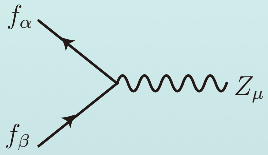
relevant Feynman rule



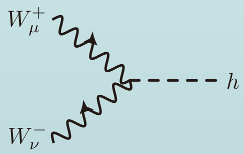
$$= i\frac{\sqrt{2}m_W}{v_0}V_{AB}(\gamma_\mu\frac{1-\gamma_5}{2})_{\alpha\beta}$$



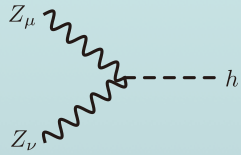
$$= i\frac{\sqrt{2}m_W}{v_0}V^\dagger_{AB}(\gamma_\mu\frac{1-\gamma_5}{2})_{\alpha\beta}$$



$$= i\frac{2m_Z}{v_0}\left(\gamma_\mu(I_3^f\frac{1-\gamma_5}{2} - Q_f\sin^2\theta_W)\right)_{\alpha\beta}$$

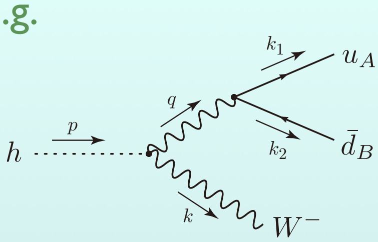


$$= \frac{2im_W^2}{v_0}g_{\mu\nu}$$



$$= \frac{2im_Z^2}{v_0}g_{\mu\nu}$$

e.g.



$$= -\frac{i\sqrt{2}m_W^2}{v_0}\frac{V_{AB}}{q^2-m_W^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2}\right)\epsilon^r_\mu(k)^*\bar{u}^{s_1}_{u_A}(k)\gamma_\nu\frac{1-\gamma_5}{2}v^{s_2}_{d_B}(k_2)$$

no summation on the generation indices A and B

After a tedious calculation, we obtain

$$\sum_{r,s_1,s_2}\left|\mathcal{M}(h\to W^-u_A\bar{d}_B)\right|^2 = \sum_{r,s_1,s_2}\left|\mathcal{M}(h\to W^+d_A\bar{u}_B)\right|^2$$

$$= \left(\frac{g_2^2 m_W\,|V_{AB}|}{q^2-m_W^2}\right)^2\Bigg[k_1\cdot k_2 + \frac{2(k\cdot k_1)(k\cdot k_2)}{m_W^2} + \frac{(m_{u_A}^2+m_{d_B}^2)k_1\cdot k_2 + 2m_{u_A}^2m_{d_B}^2}{m_W^4}\left(-p^2+\frac{(p\cdot k)^2}{m_W^2}\right)$$

$$+2\frac{m_{u_A}^2}{m_W^2}\left(p\cdot k_2 + \frac{(k\cdot k_2)(k\cdot p)}{m_W^2}\right) + 2\frac{m_{d_B}^2}{m_W^2}\left(p\cdot k_1 + \frac{(k\cdot k_1)(k\cdot p)}{m_W^2}\right)\Bigg]$$

a function of $(k\cdot k_1,\, k\cdot k_2,\, k_1\cdot k_2)$

$$q^2 = (p-k)^2 = m_h^2 + m_W^2 - 2p\cdot k$$

$$p\cdot k = (k+k_1+k_2)\cdot k = m_W^2 + k\cdot k_1 + k\cdot k_2$$

In general, to execute the phase-space integral of these quantities, we have recourse to numerical method.

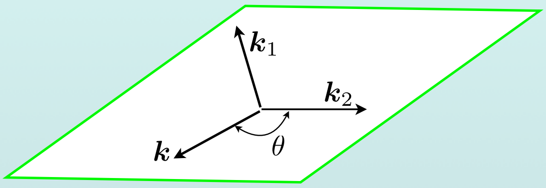
partial decay width

$$\Gamma(h\to W^- u_A \bar{d}_B) = \frac{1}{2m_h}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3 2E_k}\prod_{i=1,2}\frac{d^3\boldsymbol{k}_i}{(2\pi)^3 2E_i}(2\pi)^4\delta(p-k-k_1-k_2)\sum_{\text{spins}}\left|\mathcal{M}(h\to W^- u_A \bar{d}_B)\right|^2$$

in the rest frame of h

$$\boldsymbol{0} = \boldsymbol{k} + \boldsymbol{k}_1 + \boldsymbol{k}_2$$

δ^3-function



$$\int d^3\boldsymbol{k} d^3\boldsymbol{k}_2 = 8\pi^2\int_0^\infty dk\, dk_2\, k^2 k_2^2 \int_{-1}^{1} d(\cos\theta)$$

3-fold integral is left.

Change the variable from $\cos\theta$ to E_1 by

$$E_1 = \boldsymbol{k}_1^2 + m_u^2 = (\boldsymbol{k}+\boldsymbol{k}_2)^2 + m_u^2 = \boldsymbol{k}^2 + \boldsymbol{k}_2^2 + 2|\boldsymbol{k}||\boldsymbol{k}_2|\cos\theta + m_u^2 \longrightarrow \cos\theta = \frac{E_1^2 - \boldsymbol{k}^2 - \boldsymbol{k}_2^2 - m_u^2}{2|\boldsymbol{k}||\boldsymbol{k}_2|}$$

$$J = \left|\frac{\partial(k, k_2, \cos\theta)}{\partial(k, k_2, E_1)}\right| = \frac{\partial\cos\theta}{\partial E_1} = \frac{E_1}{kk_2}$$

Then

$$\Gamma = \frac{1}{2^6\pi^3 m_h}\int_0^\infty dk\, dk_2\, kk_2 \int \frac{dE_1}{E_k E_2}\delta(m_h - E_k - E_1 - E_2)\sum|\mathcal{M}|^2$$

'suppression by the phase space integral'

Further change the variables $(k, k_2) \to (E_k, E_2)$ $\quad E_k = \sqrt{k^2 + m_W^2} \quad E_2 = \sqrt{k_2^2 + m_d^2}$

then $\quad dk\, dk_2\, kk_2 = dE_k\, dE_2\, E_k E_2$

integration region

$$m_W \le E_k < \infty, \qquad m_d \le E_2 < \infty$$

$$\left[\left(\sqrt{E_k^2 - m_W^2} - \sqrt{E_2^2 - m_d^2}\right)^2 + m_u^2\right]^{\frac{1}{2}} \le E_1 \le \left[\left(\sqrt{E_k^2 - m_W^2} + \sqrt{E_2^2 - m_d^2}\right)^2 + m_u^2\right]^{\frac{1}{2}}$$

Once we express $(k\cdot k_1, k\cdot k_2, k_1\cdot k_2)$ in terms of (E_k, E_1, E_2), the width Γ can be calculated.

In the case of $m_u = m_d = 0$,

$$\Gamma(h\to W^- u\bar{d}) = \Gamma(h\to W^+ d\bar{u}) = \frac{m_h}{3072\pi^3}\left(g_2^2\,|V_{AB}|\right)^2\int_{2\epsilon}^{1+\epsilon^2} dx\,\frac{\sqrt{x^2-4\epsilon^2}}{(1-x)^2}\left(x^2-12\epsilon^2 x+12\epsilon^4+8\epsilon^2\right)$$

$$\epsilon \equiv \frac{m_W}{m_h}$$

Summing over the quarks and leptons,

$$\begin{aligned}\Gamma(h\to WX) &= N_C\sum_{A,B=1}^{3}\left[\Gamma(h\to W^- u_A\bar{d}_B)+\Gamma(h\to W^+ d_A\bar{u}_B)\right]\\ &\quad+\sum_{A=1}^{3}\left[\Gamma(h\to W^-\nu_A e_A^+)+\Gamma(h\to W^+ e_A\bar{\nu}_A)\right]\\ &= \frac{m_h g_2^4}{1536\pi^3}\left(N_C\sum_{A,B}|V_{AB}|^2+\sum_A\right)F(\epsilon)\\ &= \frac{m_h g_2^4}{512\pi^3}F(\epsilon)\times\begin{cases}3 & W^*\to t\bar{b}(b\bar{t})\text{ is not allowed}\\ 4 & W^*\to t\bar{b}(b\bar{t})\text{ is allowed}\end{cases}\end{aligned}$$

where

$$\begin{aligned}F(\epsilon) &\equiv \int_{2\epsilon}^{1+\epsilon^2} dx\,\frac{\sqrt{x^2-4\epsilon^2}}{(1-x)^2}\left(x^2-12\epsilon^2 x+12\epsilon^4+8\epsilon^2\right)\\ &= -(1-\epsilon^2)\left(\frac{47}{2}\epsilon^2-\frac{13}{2}+\frac{1}{\epsilon^2}\right)-3(1-6\epsilon^2+4\epsilon^4)\log\epsilon+3\frac{1-8\epsilon^2+20\epsilon^4}{\sqrt{4\epsilon^2-1}}\cos^{-1}\left(\frac{3\epsilon^2-1}{2\epsilon^3}\right)\quad \text{for } \frac{1}{2}\le\epsilon\le 1\end{aligned}$$

Similarly, for $m_f = 0$,

$$\Gamma(h\to Zf\bar{f}) = \frac{m_h}{3072\pi^3}\left(\frac{g_2^2}{\cos^2\theta_W}\right)^2\cdot 4\left(\frac{1}{2}(I_3^f)^2 - I_3^f Q_f\sin^2\theta_W + Q_f^2\sin^4\theta_W\right)F(\epsilon)$$

$$\epsilon \equiv \frac{m_Z}{m_h}$$

$$\Gamma(h\to ZX) = \frac{m_h}{2048\pi^3}\left(\frac{g_2^2}{\cos^2\theta_W}\right)^2 F(\epsilon)\begin{cases}7-\frac{40}{3}\sin^2\theta_W+\frac{160}{9}\sin^4\theta_W, & Z^*\to t\bar{t}\text{ is not allowed}\\ 8\left(1-2\sin^2\theta_W+\frac{8}{3}\sin^4\theta_W\right). & Z^*\to t\bar{t}\text{ is allowed}\end{cases}$$

one-loop 2-body decays in the SM

$h \to \gamma\gamma$

In general, the amplitude has the form of

$$i\mathcal{M}(h \to \gamma\gamma) = \epsilon^{*\mu}(k_1)\epsilon^{*\nu}(k_2)\, i\Gamma_{\mu\nu}(k_1, k_2)$$

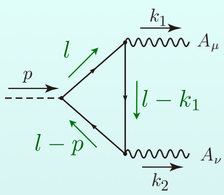
1PI vertex function

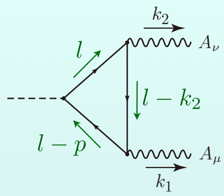
gauge invariance : $$i\Gamma_{\mu\nu}(k_1, k_2) = A\left(g_{\mu\nu} - \frac{2k_{2\mu}k_{1\nu}}{m_h^2}\right) + B\epsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma$$

$$\sum_{s_1, s_2} \left| i\Gamma^{\mu\nu}(k_1, k_2)\epsilon_\mu^{s_1 *}(k_1)\epsilon_\nu^{s_2 *}(k_2)\right|^2 = 2\left|A\right|^2 + \frac{1}{2}m_h^4\left|B\right|^2$$

$$\Gamma = \frac{1}{2}\frac{1}{16\pi m_h}\left|\mathcal{M}_{CM}\right|^2 \theta(m_h - 2m)\sqrt{1 - \frac{4m^2}{m_h^2}} = \frac{1}{32\pi m_h}\left|\mathcal{M}_{CM}\right|^2.$$

fermion loop





$d = 4 - \varepsilon$ $\qquad$ $S_f(k) = \dfrac{1}{k\cdot\gamma - m_f + i\epsilon}$

$$i\Gamma_{\mu\nu}(k_1, k_2)$$
$$= -N_C(-ieQ_f)^2\left(-\frac{ig_2 m_f}{2m_W}\right)\int\frac{d^d l}{(2\pi)^d}\mathrm{Tr}\left[\gamma_\mu iS_f(l)iS_f(l-p)\gamma_\nu iS_f(l-k_1) + \gamma_\nu iS_f(l)iS_f(l-p)\gamma_\mu iS_f(l-k_2)\right]$$

working out the trace algebra and executing the momentum integral

$$= -i\frac{N_C g_2 (eQ_f)^2 m_f^2}{8\pi^2}\left(m_h^2 g_{\mu\nu} - 2k_{2\mu}k_{1\nu}\right)\int_0^1 dx\int_0^{1-x}dy\frac{1 - 4xy}{m_f^2 - m_h^2 xy - i\epsilon}$$

$$= -i\frac{N_C g_2 (eQ_f)^2 m_f^2}{2\pi^2}\left(g_{\mu\nu} - \frac{2k_{2\mu}k_{1\nu}}{m_h^2}\right)\int_0^1 dx\int_0^{1-x}dy\frac{1-4xy}{\tau_f - 4xy - i\epsilon} \qquad \tau_f \equiv \frac{4m_f^2}{m_h^2}$$

$$= \int_0^1 dx\left(1 - x + \left[\frac{1-\tau}{-4x}\log(\tau - 4xy - i\epsilon)\right]_0^{1-x}\right)$$

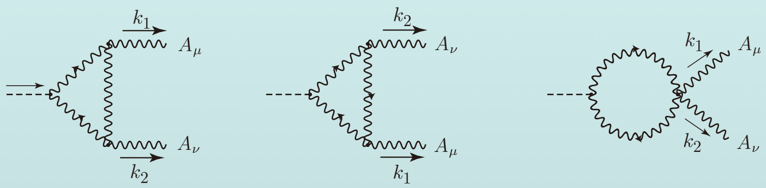
$$= \frac{1}{2} - \frac{1-\tau}{4}\int_0^1\frac{dx}{x}\log\frac{\tau - 4x(1-x) - i\epsilon}{\tau}$$

$$= \frac{1}{2}\left[1 + (1-\tau)f(\tau)\right]$$

$$i\Gamma_{\mu\nu}(k_1,k_2) = -\frac{ig_2\alpha}{4\pi}\frac{m_h^2}{m_W} N_C Q_f^2 \tau_f [1+(1-\tau_f)f(\tau_f)]\left(g_{\mu\nu} - \frac{2k_{2\mu}k_{1\nu}}{m_{H_i}^2}\right)$$

$$f(\tau) \equiv -\frac{1}{2}\int_0^1 dx\,\frac{1}{x}\log\frac{\tau - 4x + 4x^2 - i\epsilon}{\tau} = \begin{cases} \left[\sin^{-1}\left(\frac{1}{\sqrt{\tau}}\right)\right]^2 & (\tau \geq 1) \\ -\frac{1}{4}\left[\log\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & (\tau < 1) \end{cases}$$

W-boson loop



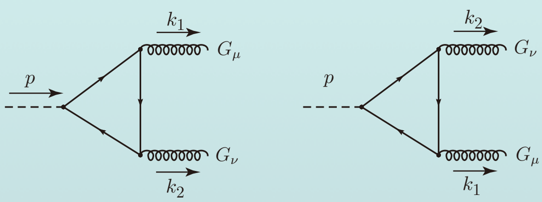
$$i\Gamma_{\mu\nu}(k_1,k_2) = \frac{ig_2\alpha}{8\pi}\frac{m_h^2}{m_W}\left[2 + 3\tau_W + 3\tau_W(2-\tau_W)f(\tau_W)\right]\left(g_{\mu\nu} - \frac{2k_{2\mu}k_{1\nu}}{m_h^2}\right)$$

Summing these contributions,

$$\Gamma(h\to\gamma\gamma) = \frac{g_2^2\alpha^2}{1024\pi^3}\frac{m_h^3}{m_W^2}\left|\sum_f N_C Q_f^2 F_{1/2}(\tau_f) + F_1(\tau_W)\right|^2$$

$$\begin{aligned} F_0(\tau) &= \tau\,(1-\tau f(\tau))\,, \\ F_{1/2}(\tau) &= -2\tau\,[1+(1-\tau)f(\tau)]\,, \\ F_1(\tau) &= 2 + 3\tau + 3\tau(2-\tau)f(\tau). \end{aligned}$$

$h \to gg$



$$\Gamma(h\to gg) = \frac{g_2^2\alpha_s^2}{512\pi^3}\frac{m_h^3}{m_W^2}\left|\sum_q F_{1/2}(\tau_q)\right|^2$$

How about the BRs in the MSSM?

vertices including the Higgs bosons are modified

mass eigenstates: H_i

$$\begin{pmatrix} h_d \\ h_u \\ a \end{pmatrix} = \mathcal{O} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \qquad \mathcal{O}^T \mathcal{M}^2 \mathcal{O} = \mathrm{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2)$$

$$\mathcal{L}_{VVH} = g_2 m_W \left(\frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu + W_\mu^+ W^{-\mu} \right) g_{VVH_i} H_i = g_2 m_W \left(\frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu + W_\mu^+ W^{-\mu} \right) (h \sin(\beta-\alpha) + H \sin(\beta-\alpha))$$

$$g_{VVH_i} = \mathcal{O}_{1i}\cos\beta + \mathcal{O}_{2i}\sin\beta$$

no CP-mixing

$$\begin{aligned}
\mathcal{L}_{VHH} &= \frac{g_2}{4\cos\theta_W} Z^\mu g_{ZH_iH_j} H_i \overleftrightarrow{\partial}_\mu H_j + i \left[\frac{g_2}{\cos\theta_W} \left(\frac{1}{2} - \sin^2\theta_W \right) Z^\mu + eA^\mu \right] H^+ \overleftrightarrow{\partial}_\mu H^- - i\frac{g_2}{2} \left[g_{WHH_i} (H_i \overleftrightarrow{\partial}_\mu H^-) W^{+\mu} - g^*_{WHI} \right. \\
&= \frac{g_2}{2\cos\theta_W} Z^\mu \left(-h\cos(\beta-\alpha) + H\sin(\beta-\alpha) \right) \overleftrightarrow{\partial}_\mu A + i \left[\frac{g_2}{\cos\theta_W} \left(\frac{1}{2} - \sin^2\theta_W \right) Z^\mu + eA^\mu \right] \left(H^+ \overleftrightarrow{\partial}_\mu H^- \right) \\
&\quad -i\frac{g_2}{2} \left[\left(-h\cos(\beta-\alpha) + H\sin(\beta-\alpha) \right) \overleftrightarrow{\partial}_\mu H^- \cdot W^{+\mu} - \left(-h\cos(\beta-\alpha) + H\sin(\beta-\alpha) \right) \overleftrightarrow{\partial}_\mu H^+ \cdot W^{-\mu} \right] \\
&\quad -\frac{g_2}{2} \left[\left((A\overleftrightarrow{\partial}_\mu H^-) W^{+\mu} + (A\overleftrightarrow{\partial}_\mu H^+) W^{-\mu} \right) \right]
\end{aligned}$$

$$\begin{aligned}
g_{ZH_iH_j} &= (\mathcal{O}_{1i}\mathcal{O}_{3j} - \mathcal{O}_{3i}\mathcal{O}_{1j})\sin\beta - (\mathcal{O}_{2i}\mathcal{O}_{3j} - \mathcal{O}_{3i}\mathcal{O}_{2j})\cos\beta \\
g_{WHH_i} &= \mathcal{O}_{1i}\sin\beta - \mathcal{O}_{2i}\cos\beta - i\mathcal{O}_{3i}.
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_Y &= -\frac{g_2}{2m_W} \left[\bar{d}(g^S_{ddH_i} - i\gamma_5 g^P_{ddH_i}) m^{(d)} d + \bar{u}(g^S_{uuH_i} - i\gamma_5 g^P_{uuH_i}) m^{(u)} u + \bar{e}(g^S_{ddH_i} - i\gamma_5 g^P_{ddH_i}) m^{(e)} e \right] H_i \\
&\quad + \frac{g_2}{\sqrt{2}\, m_W} \left[\left(\bar{e}\frac{1-\gamma_5}{2} m^{(e)} \nu \tan\beta + \bar{d}\frac{1-\gamma_5}{2} m^{(d)} V^\dagger u \tan\beta + \bar{d}\frac{1+\gamma_5}{2} V^\dagger m^{(u)} u \cot\beta \right) H^- \right. \\
&\qquad \left. + \left(\bar{\nu}\frac{1+\gamma_5}{2} m^{(e)} e \tan\beta + \bar{u}\frac{1+\gamma_5}{2} V m^{(d)} d \tan\beta + \bar{u}\frac{1-\gamma_5}{2} m^{(u)} V d \cot\beta \right) H^+ \right] \\
&= -\frac{g_2}{2m_W} \left[\bar{d} \left(\frac{\cos\alpha}{\cos\beta} H - \frac{\sin\alpha}{\cos\beta} h \right) m^{(d)} d + \bar{u} \left(\frac{\sin\alpha}{\sin\beta} H + \frac{\cos\alpha}{\sin\beta} h \right) m^{(u)} u + \cdots \right] + \cdots
\end{aligned}$$

$$\begin{aligned}
g^S_{ddH_i} &= \frac{1}{\cos\beta}\mathcal{O}_{1i}, \qquad g^P_{ddH_i} = \mathcal{O}_{3i}\tan\beta, \\
g^S_{uuH_i} &= \frac{1}{\sin\beta}\mathcal{O}_{2i}, \qquad g^P_{uuH_i} = \mathcal{O}_{3i}\cot\beta.
\end{aligned}$$

The extra factors modifies the vertex factors.

e.g.

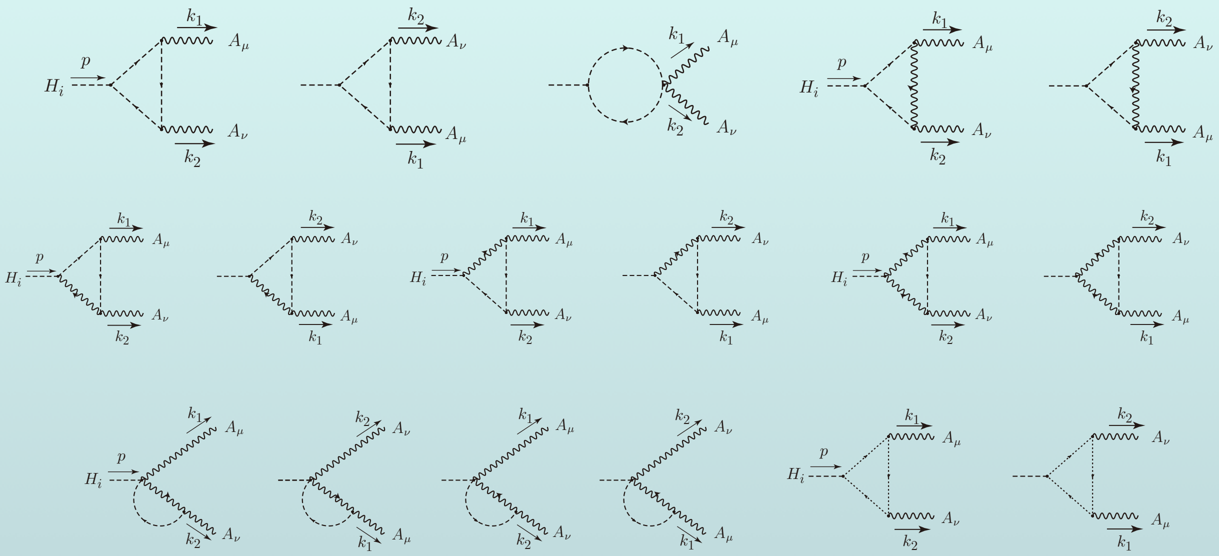
$$\Gamma(H_i \to f\bar{f}) = \theta(m_{H_i} - 2m_f) \frac{N_C}{32\pi} \frac{g_2^2 m_f^2 m_{H_i}}{m_W^2} \left(1 - \frac{4m_f^2}{m_{H_i}^2} \right)^{1/2} \left[(g^S_{ffH_i})^2 \left(1 - \frac{4m_f^2}{m_{H_i}^2} \right) + (g^P_{ffH_i})^2 \right]$$

The extra particles give rise to new loop diagrams.

$H,\ A,\ H^{\pm}$: superpartners

e.g. new contributes to $H_i \to \gamma\gamma$

$H^{\pm}$ induces the following diagrams



FP ghosts

$$\Gamma(H_i \to \gamma\gamma) = \frac{g_2^2 \alpha^2}{1024\pi^3} \frac{m_{H_i}^3}{m_W^2} \left(|A|^2 + 8\,|B|^2 \right)$$

where

$$A = \sum_f g^S_{ffH_i} N_C Q_f^2 F_{1/2}(\tau_f) + g_{VVH_i} F_1(\tau_W)$$

quark/lepton, W-boson

$$+ \frac{m_W^2}{m_{H^\pm}^2} g_{H^+H^-H_i} F_0(\tau_{H^\pm}) + \sum_{j=1,2} \frac{m_W}{m_{\chi_j^\pm}} \mathrm{Re}(g_{\chi_j^\pm \chi_j^\pm H_i})\, F_{1/2}(\tau_{\chi_j}) + \frac{m_W}{g_2} \sum_{f=u,d,e} N_C Q_f^2 \sum_{A,j} \frac{C^{f_A}_{ijj}}{m^2_{\tilde{f}_{Aj}}} F_0(\tau_{\tilde{f}_{Aj}})$$

charged Higgs — charginos — squarks

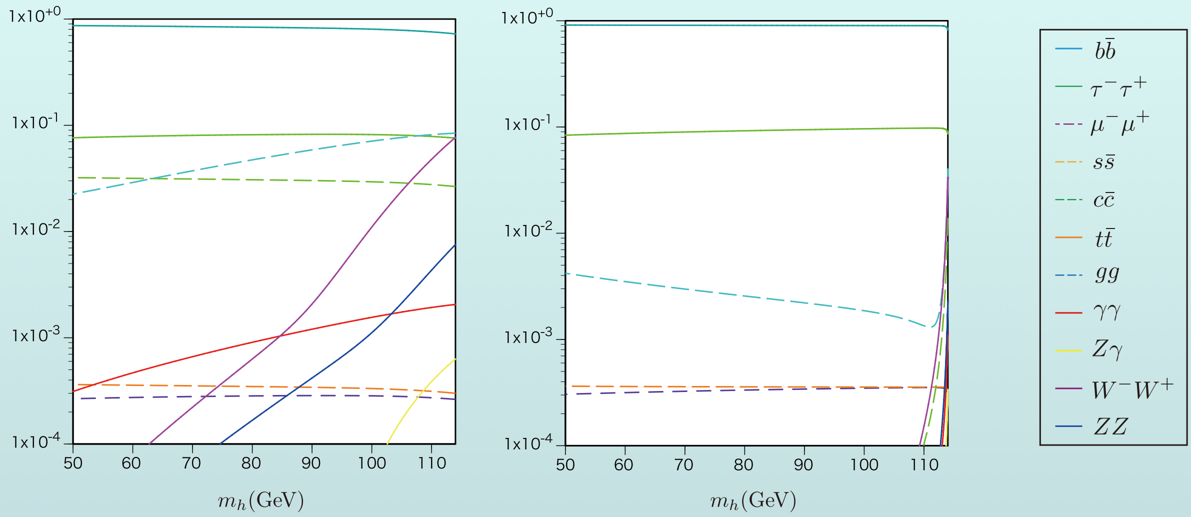
$$B = \sum_f N_C Q_f^2 g^P_{ffH_i}\, \tau_f f(\tau_f) + \sum_{j=1,2} \mathrm{Im}(g_{\chi_j^\pm \chi_j^\pm H_i})\, \tau_{\chi_j} f(\tau_{\chi_j})$$

squarks — charginos

h-BR for $50\text{GeV} \leq m_h \leq 114\text{GeV}$

SM | MSSM(mSUGRA)

$\mu = 200\text{GeV}, \quad M_{\tilde{q}} = M_2 = A_q = 1\text{TeV}, \quad \tan\beta = 20$



This is an example in which the stop loop destructively contributes to the 2-gluon and 2-photon modes.

Summary

A Higgs particle was discovered after over-50ys since it was proposed by P. Higgs.

$m_h = 125.7\text{GeV} \qquad J^P = 0^+$

the first evidence of 0+ 'elementary particle'

We still do not know its true character.

h of the SM $\qquad h$ (or H) of the 2HDM (MSSM)
neutral scalar boson in other extended models

To identify what it is, we must study its properties such as the couplings to the other particles as well as the self-coupling.

precise measurement of the decay BR, 3- and 4-point self interactions
loop effects of new particles, CP violation other than the KM phase,….

These are Your Tasks!

fin.