New Results from the Isospin Analysis of $B \to \pi\pi$ Decays

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We present an *algebraic* isospin approach towards a more straightforward and model-independent determination of the weak phase α (or ϕ_2) and QCD penguin pollution in $B \to \pi\pi$ decays. The world averages of current experimental data allow us to impose some useful constraints on the isospin parameters of $B \to \pi\pi$ transitions. We find that the magnitude of α (or ϕ_2) extracted from the indirect CP violation in $\pi^+\pi^-$ mode is in agreement with the standard-model expectation from other indirect measurements, but its four-fold discrete ambiguity has to be resolved in the near future.

$\boxed{1}$ The major goal of KEK and SLAC B-meson factories is to test the Kobayashi-Maskawa mechanism of CP violation [1] within the standard model and to detect possible new sources of CP violation beyond the standard model. An elegant description of CP violation in B physics is the unitarity triangle defined by the following orthogonality relation of six quark mixing matrix elements in the complex plane [2]:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \;=\; 0 \; . \tag{1}$$

Three inner angles of this triangle are denoted as α, β and γ by the BaBar Collaboration, or equivalently ϕ_1, ϕ_2 and ϕ_3 by the Belle Collaboration.

So far β has been rather precisely determined from the measurement of CP violation in B^0_d vs $\overline{B}^0_d \to J/\psi K_{\rm S}$ transitions [3], and its value $\beta \approx 23^\circ$ is compatible very well with the standard-model expectation. The next experimental step is to measure α and γ to a good degree of accuracy at B-meson factories, such that one may cross-check the Kobayashi-Maskawa picture of CP violation and probe possible new physics in the B-meson system.

It is well known that $\overline{B}^0_d \to \pi^+\pi^-$, $\overline{B}^0_d \to \pi^0\pi^0$ and $B^-_u \to \pi^0\pi^-$ decays can be used to extract the weak angle α in a model-independent way, because the isospin relation of their transition amplitudes allows us to remove the QCD penguin pollution [4]. Note, however, that the experimentally-reported branching fractions of $B \to \pi\pi$ decays are all charge-averaged:

$$\begin{aligned}
\mathcal{B}_{+-} &\equiv \frac{1}{2}\left[\mathcal{B}(B^0_d \to \pi^+\pi^-) + \mathcal{B}(\overline{B}^0_d \to \pi^+\pi^-)\right] \; , \\
\mathcal{B}_{00} &\equiv \frac{1}{2}\left[\mathcal{B}(B^0_d \to \pi^0\pi^0) + \mathcal{B}(\overline{B}^0_d \to \pi^0\pi^0)\right] \; , \\
\mathcal{B}_{0\pm} &\equiv \frac{1}{2}\left[\mathcal{B}(B^+_u \to \pi^0\pi^+) + \mathcal{B}(B^-_u \to \pi^0\pi^-)\right] \; .
\end{aligned} \tag{2}$$

In addition, the *direct* CP-violating asymmetries between $\overline{B}^0_d \to \pi^+\pi^-$, $\overline{B}^0_d \to \pi^0\pi^0$, $B^-_u \to \pi^0\pi^-$ and their CP-conjugate decays can be defined as

$$\begin{aligned}
\mathcal{C}_{+-} &\equiv \frac{\mathcal{B}(B^0_d \to \pi^+\pi^-) - \mathcal{B}(\overline{B}^0_d \to \pi^+\pi^-)}{\mathcal{B}(B^0_d \to \pi^+\pi^-) + \mathcal{B}(\overline{B}^0_d \to \pi^+\pi^-)} \; , \\
\mathcal{C}_{00} &\equiv \frac{\mathcal{B}(B^0_d \to \pi^0\pi^0) - \mathcal{B}(\overline{B}^0_d \to \pi^0\pi^0)}{\mathcal{B}(B^0_d \to \pi^0\pi^0) + \mathcal{B}(\overline{B}^0_d \to \pi^0\pi^0)} \; , \\
\mathcal{A}_{0\pm} &\equiv \frac{\mathcal{B}(B^-_u \to \pi^0\pi^-) - \mathcal{B}(B^+_u \to \pi^0\pi^+)}{\mathcal{B}(B^-_u \to \pi^0\pi^-) + \mathcal{B}(B^+_u \to \pi^0\pi^+)} \; .
\end{aligned} \tag{3}$$

Both BaBar [5] and Belle [6] collaborations have actually measured the time-dependent rates of B^0_d vs $\overline{B}^0_d \to \pi^+\pi^-$ decays on the $\Upsilon(4S)$ resonance:

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$$\Gamma[B^0_d(\Delta t) \to \pi^+\pi^-] = \frac{e^{-|\Delta t|/\tau_0}}{4\tau_0} \left[1 - \mathcal{S}_{+-} \sin(\Delta m_d \Delta t) + \mathcal{C}_{+-} \cos(\Delta m_d \Delta t)\right] ,$$
$$\Gamma[\overline{B}^0_d(\Delta t) \to \pi^+\pi^-] = \frac{e^{-|\Delta t|/\tau_0}}{4\tau_0} \left[1 + \mathcal{S}_{+-} \sin(\Delta m_d \Delta t) - \mathcal{C}_{+-} \cos(\Delta m_d \Delta t)\right] , \tag{4}$$

where τ_0 is the lifetime of neutral B mesons, and $\mathcal{S}_{+-}$ signifies the *indirect* CP violation arising from the interplay between decay and B^0_d-$\overline{B}^0_d$ mixing [7]. A similar time-dependent measurement can be done for B^0_d vs $\overline{B}^0_d \to \pi^0\pi^0$ decays, whose rates consist of $\mathcal{C}_{00}$ and $\mathcal{S}_{00}$ corresponding to $\mathcal{C}_{+-}$ and $\mathcal{S}_{+-}$ in Eq. (4). Until now, only $\mathcal{S}_{+-}$ has been determined from the KEK and SLAC experiments.

Although the relevant experimental data on direct CP violation remain quite preliminary, they can be used to do a quantitative analysis of $B \to \pi\pi$ decays. The main purpose of this paper is to recommend an *algebraic* isospin approach, which allows us to determine the weak phase α (or ϕ_2) from $\mathcal{S}_{+-}$ and (or) $\mathcal{S}_{00}$ in a more straightforward and model-independent way. We find that the allowed region of α is in agreement with the standard-model expectation from other indirect measurements, but its four-fold discrete ambiguity has to be resolved in the near future.

$\boxed{2}$ Under isospin symmetry and in the neglect of electroweak penguin contributions [8], the amplitudes of $B^0_d \to \pi^+\pi^-$, $B^0_d \to \pi^0\pi^0$ and $B^+_u \to \pi^0\pi^+$ decays (or their CP-conjugate processes) form a triangle in the complex plane:

$$A(B^0_d \to \pi^+\pi^-) + \sqrt{2}A(B^0_d \to \pi^0\pi^0) = \sqrt{2}A(B^+_u \to \pi^0\pi^+) ,$$
$$A(\overline{B}^0_d \to \pi^+\pi^-) + \sqrt{2}A(\overline{B}^0_d \to \pi^0\pi^0) = \sqrt{2}A(B^-_u \to \pi^0\pi^-) . \tag{5}$$

The magnitudes of $A(B^-_u \to \pi^0\pi^-)$ and $A(B^+_u \to \pi^0\pi^+)$ are identical to each other in this safe approximation [4,9], hence the CP-violating asymmetry $\mathcal{A}_{0\pm}$ vanishes.

The CP-violating parameters $\mathcal{S}_{+-}$ and $\mathcal{S}_{00}$ in Eq. (4) are related to the weak angle α (or ϕ_2) as follows [10]:

$$\mathcal{S}_{+-} = (1+\mathcal{C}_{+-}) \mathrm{Im}\left[\frac{q}{p} \cdot \frac{A(\overline{B}^0_d \to \pi^+\pi^-)}{A(B^0_d \to \pi^+\pi^-)}\right] = (1+\mathcal{C}_{+-})|R| \sin[2(\alpha+\Theta)] ,$$
$$\mathcal{S}_{00} = (1+\mathcal{C}_{00}) \mathrm{Im}\left[\frac{q}{p} \cdot \frac{A(\overline{B}^0_d \to \pi^0\pi^0)}{A(B^0_d \to \pi^0\pi^0)}\right] = (1+\mathcal{C}_{00})|\overline{R}| \sin[2(\alpha+\overline{\Theta})] , \tag{6}$$

where $q/p \approx (V_{td}V^*_{tb})/(V^*_{td}V_{tb})$ denotes the weak phase of B^0_d-$\overline{B}^0_d$ mixing [7], and

$$R = |R|e^{2i\Theta} = \frac{1-\overline{r}}{1-r} ,$$
$$\overline{R} = |\overline{R}|e^{2i\overline{\Theta}} = \frac{2+\overline{r}}{2+r} , \tag{7}$$

where

$$r = |r|e^{i\theta} \equiv \frac{A_0}{A_2} ,$$
$$\overline{r} = |\overline{r}|e^{i\overline{\theta}} \equiv \frac{\overline{A}_0}{\overline{A}_2} . \tag{8}$$

If there were no direct CP violation in $B \to \pi\pi$ transitions (i.e., $\mathcal{C}_{+-} = \mathcal{C}_{00} = 0$ or $\overline{r} = r$), we would arrive at $\overline{R} = R = 1$ from Eqs. (7)–(8). In this case, Eq. (6) would be simplified to $\mathcal{S}_{+-} = \mathcal{S}_{00} = \sin 2\alpha$. It is therefore necessary to pin down $\mathcal{C}_{+-}$ and $\mathcal{C}_{00}$ to a reasonable degree of accuracy, in order to extract the weak angle α from $\mathcal{S}_{+-}$ and (or) $\mathcal{S}_{00}$.

$\boxed{3}$ Now we carry out a numerical analysis of $B \to \pi\pi$ decays and CP violation by using the isospin formulas obtained above and current experimental data. The 1σ, 2σ and 3σ confidence regions of $(\alpha, \mathcal{S}_{+-})$ are shown in Fig. 1. Because of the four-fold ambiguity associated with Θ or $\overline{\Theta}$, the result of α determined from $\mathcal{S}_{+-}$ involves the four-fold discrete ambiguity too, as illustrated in Fig. 1.

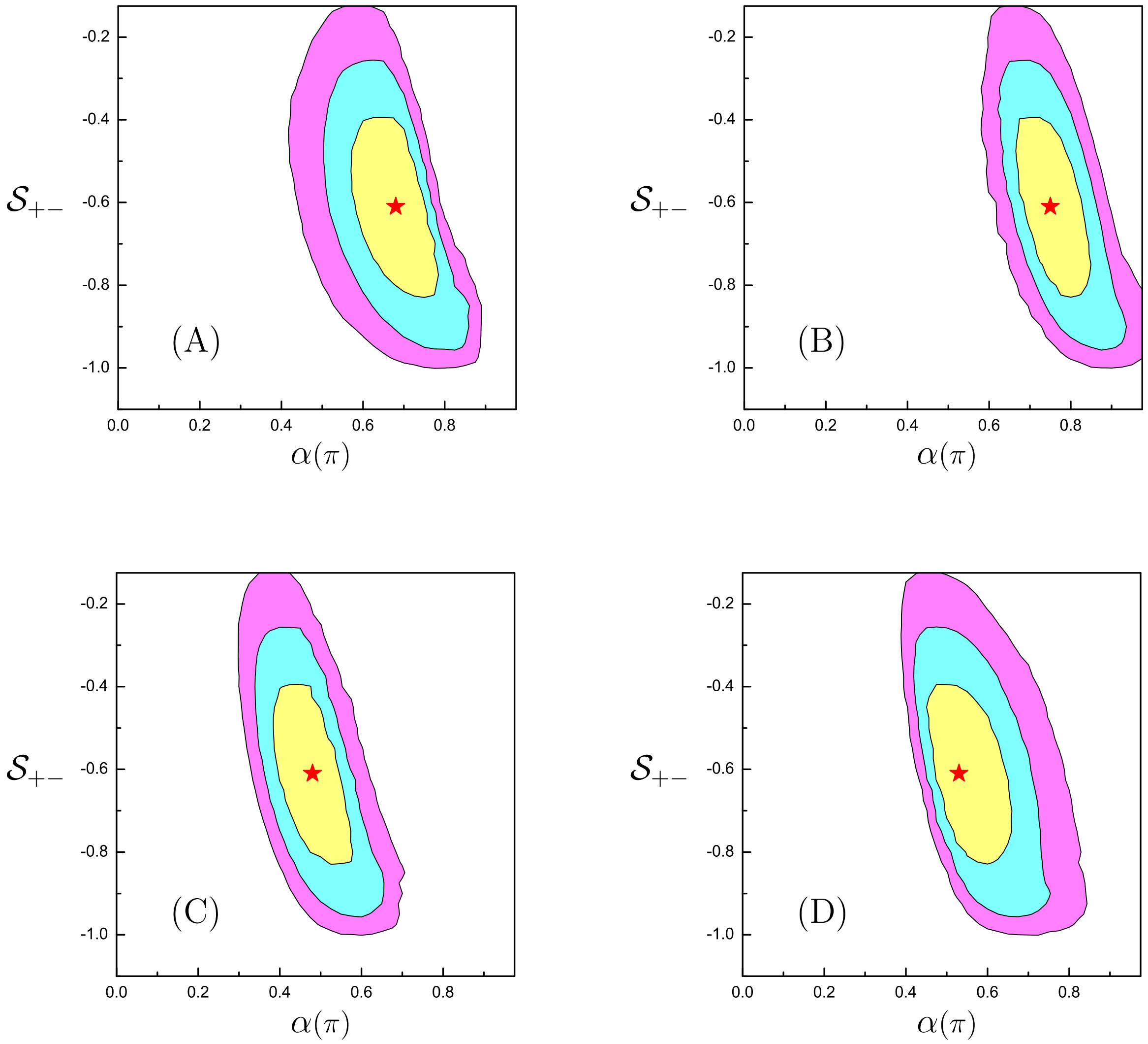

FIG. 1. The 1σ, 2σ and 3σ confidence regions of α with four-fold discrete ambiguity, extracted from current experimental data on $\mathcal{S}_{+-}$.

We can see the result is in agreement with the standard-model expectation of α (i.e., $\alpha \sim 90^\circ$) from other indirect measurements [2]. Once the CP-violating asymmetry $\mathcal{S}_{00}$ is also measured, it will be possible to completely or partly remove the discrete ambiguity of α [4]. Then one may constrain the weak phase α and QCD penguin pollution in $B \to \pi\pi$ decays at a much better confidence level.

It is worth remarking that the validity of our isospin analysis relies on the assumption of negligible electroweak penguin effects. The electroweak penguin contribution to $B \to \pi\pi$ decays is in general expected to be insignificant [8]. This expectation would be problematic or incorrect, if $\mathcal{A}_{0\pm} \neq 0$ were experimentally established [10]. Note also that final-state interactions in $B \to \pi\pi$ transitions consist of both elastic $\pi\pi \rightleftharpoons \pi\pi$ rescattering and some possible inelastic rescattering effects. Whether the latter is negligibly small or not remains an open question [11]. To answer this question requires more precise measurements of both branching fractions and CP-violating asymmetries of $B \to \pi\pi$

decays.

$\boxed{4}$ We have presented an *algebraic* isospin analysis of rare $B \to \pi\pi$ decays by taking account of the fact that the experimentally-reported branching fractions are charge-averaged and large direct CP violation may exist in them. This approach is more straightforward than the originally-proposed *geometric* approach, from which the weak phase α (or ϕ_2) and QCD penguin pollution are determined through the reconstruction of two isospin triangles. Therefore, our method is expected to be very useful to analyze the future experimental data on $B \to \pi\pi$ transitions and CP violation in a model-independent way.

Although the present experimental data (in particular, those on direct and indirect CP violation in $B \to \pi\pi$ decays) are not sufficiently precise, they can impose some instructive constraints on the parameter space of QCD penguin effects. Furthermore, we find that the allowed region of α (or ϕ_2) is actually in agreement with the standard-model expectation from other indirect measurements. To resolve the four-fold discrete ambiguity associated with the magnitude of α (or ϕ_2) determined from the indirect CP-violating asymmetry in $\pi^+\pi^-$ mode, a measurement of the similar CP-violating asymmetry in $\pi^0\pi^0$ mode is necessary. We expect that more accurate measurements of such charmless B decays will help us to test the consistency of the Kobayashi-Maskawa mechanism of CP violation and to probe possible new physics beyond the standard model.

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