Geometric Neutrino Mass Hierarchy and Leptogenesis

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Recent neutrino oscillation experiments have provided us with very compelling evidence that neutrinos have nonvanishing masses and their mixing involves two large angles and one small angle. These important results imply that the minimal standard electroweak model, in which neutrinos are massless Weyl particles, is actually incomplete. A very simple extension of the standard model is to include one right-handed neutrino in each of three lepton families, while the Lagrangian of electroweak interactions keeps invariant under $\mathrm{SU(2)_L \times U(1)_Y}$ gauge transformation. In this case, the Lagrangian relevant for lepton masses can be written as

$$-\mathcal{L}_{\rm lepton} \; = \; \bar{l}_{\rm L} \tilde{\phi} Y_l E + \bar{l}_{\rm L} \phi Y_\nu N + \frac{1}{2} \overline{N^{\rm c}} M_{\rm R} N + {\rm h.c.} \; . \tag{1}$$

After spontaneous gauge symmetry breaking, we obtain the charged lepton mass matrix $M_l \equiv Y_l \langle \phi \rangle$ and the Dirac neutrino mass matrix $M_{\rm D} \equiv Y_\nu \langle \phi \rangle$ with $\langle \phi \rangle \approx 174$ GeV. The scale of $M_{\rm R}$ may be considerably higher than $\langle \phi \rangle$, because right-handed neutrinos are $\mathrm{SU(2)_L}$ singlets and their mass term is not subject to electroweak symmetry breaking. As a consequence, the effective (light and left-handed) neutrino mass matrix M_ν can be derived from $M_{\rm D}$ and $M_{\rm R}$ via the well-known seesaw mechanism: $M_\nu \approx M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$.

Note that lepton number violation induced by the third term of $\mathcal{L}_{\rm lepton}$ allows decays of the heavy Majorana neutrinos N_i (for $i = 1, 2, 3$) to happen. Because the decay can occur at both tree and one-loop levels, their interference may lead to a CP-violating asymmetry ε_i between the CP-conjugated $N_i \rightarrow l + \phi^\dagger$ and $N_i \rightarrow l^{\rm c} + \phi$ processes. If the masses of N_i are hierarchical (i.e., $M_1 < M_2 < M_3$), the interactions of N_1 can be in thermal equilibrium when N_2 and N_3 decay. The asymmetries ε_2 and ε_3 are therefore erased before N_1 decays, and only the asymmetry ε_1 produced by the out-of-equilibrium decay of N_1 survives. In the flavor basis where $M_{\rm R}$ is diagonal and positive, we have

$$\varepsilon_1 \; \approx \; -\frac{3 M_1}{16\pi} \left[\frac{{\rm Im} \left[({\bf Y}^\dagger_\nu {\bf Y}_\nu)_{12} \right]^2}{M_2 ({\bf Y}^\dagger_\nu {\bf Y}_\nu)_{11}} + \frac{{\rm Im} \left[({\bf Y}^\dagger_\nu {\bf Y}_\nu)_{13} \right]^2}{M_3 ({\bf Y}^\dagger_\nu {\bf Y}_\nu)_{11}} \right] , \tag{2}$$

where ${\bf Y}_\nu \equiv Y_\nu U_{\rm R}$ with $U_{\rm R}$ being a unitary matrix defined to diagonalize $M_{\rm R}$ (namely, $U^T_{\rm R} M_{\rm R} U_{\rm R} = {\rm Diag}\{M_1, M_2, M_3\}$), and the condition $M^2_1 \ll M^2_2 \ll M^2_3$ has been taken. The point of leptogenesis is that ε_1 may result in a net lepton number asymmetry $Y_{\rm L} \equiv (n_{\rm L} - n_{\bar{\rm L}})/{\bf s} = d\varepsilon_1/g_*$, where ${\bf s}$ denotes the entropy of the early universe, $g_* = 106.75$ is an effective number characterizing the relativistic degrees of freedom which contribute to ${\bf s}$, and d measures the dilution effects induced by the lepton-number-violating wash-out processes. This lepton number asymmetry is eventually converted into a net baryon number asymmetry $Y_{\rm B}$ via nonperturbative sphaleron processes: $Y_{\rm B} \equiv (n_{\rm B} - n_{\bar{\rm B}})/{\bf s} \approx -0.35 Y_{\rm L}$. Such an elegant mechanism may therefore interpret the cosmological matter-antimatter asymmetry, $7 \times 10^{-11} \lesssim Y_{\rm B} \lesssim 10^{-10}$, which is drawn from the recent observational data.

The purpose of this talk is to propose a novel and viable phenomenological model of lepton mass matrices, from which both the cosmological baryon number asymmetry and neutrino mixing parameters can be calculated.

First of all, I conjecture that three light (left-handed) neutrinos have a geometric mass hierarchy at low energies: $m_1/m_2 = m_2/m_3 \equiv r$. Given the neutrino mass-squared differences $\Delta m^2_{21} \equiv m^2_2 - m^2_1 = (5.4 - 9.5) \times 10^{-5}$ eV^2 and $\Delta m^2_{31} \equiv m^2_3 - m^2_1 = (1.4 - 3.7) \times 10^{-3}$ eV^2, which have been determined from a global analysis of current neutrino oscillation data, it is easy to obtain

$$m_1 = \frac{r^2}{\sqrt{1-r^4}}\sqrt{\Delta m^2_{31}} \;\; , \quad m_2 = \frac{r}{\sqrt{1-r^4}}\sqrt{\Delta m^2_{31}} \;\; , \quad m_3 = \frac{1}{\sqrt{1-r^4}}\sqrt{\Delta m^2_{31}} \;\; , \tag{3}$$

where $r = \sqrt{\Delta m^2_{21}/(\Delta m^2_{31} - \Delta m^2_{21})}$ lies in the range $0.122 \lesssim r \lesssim 0.270$. I stress that the geometric mass hierarchy of three light neutrinos can essentially be extrapolated up to the lowest mass scale of three heavy (right-handed) Majorana neutrinos. Taking the normal mass hierarchy $M_1 < M_2 < M_3$, one may establish a simple relation for m_i (with $i = 1, 2, 3$) between the electroweak scale $\mu = M_Z$ and the seesaw mass scale $\mu = M_1$:

$$m_i(M_1) \; \approx \; m_i(M_Z)\mathcal{I}_\alpha \; , \tag{4}$$

where $\mathcal{I}_\alpha = \exp\left[\int_{\ln M_Z}^{\ln M_1} \alpha(\tau) {\rm d}\tau\right]$ with $\alpha \approx (\lambda_H - 3g^2_2 + 6f^2_t)/(16\pi^2)$ measures the overall renormalization-group effect of the Higgs self-coupling, gauge coupling and top-quark Yukawa coupling from M_Z to M_1. Setting $\mathcal{I}_\alpha = 1$ at M_Z, we shall get $\mathcal{I}_\alpha > 1$ at any energy scales higher than M_Z. Because Eq. (4) is universally valid for three light neutrinos, we arrive at the geometric hierarchy $m_1(M_1)/m_2(M_1) \approx m_2(M_1)/m_3(M_1)$ to a good degree of accuracy at the seesaw scale.

I further conjecture that three heavy right-handed neutrinos have the same geometric mass hierarchy as three light neutrinos do: $M_1/M_2 = M_2/M_3 = r$. As a consequence of $r \sim 0.2$, the condition $M^2_1 \ll M^2_2 \ll M^2_3$ holds to guarantee the validity of Eq. (2). One may then take advantage of baryogenesis via leptogenesis to fix the seesaw mass scale M_1 (or M_3). To do so, a specific texture of the Dirac neutrino coupling matrix Y_ν must be assumed. I find that it is natural to assume three eigenvalues of Y_ν, denoted as D_i (for $i = 1, 2, 3$), to have the same geometric hierarchy as three heavy Majorana neutrinos do: $D_1/D_2 = D_2/D_3 = r$. Now the question is how to incorporate such a common geometric relation into the formula $M_\nu \approx Y_\nu M^{-1}_{\rm R} Y^T_\nu \langle\phi\rangle^2$ at the seesaw scale.

I consider a simple ansatz of lepton mass matrices to accommodate the conjectures made above. At the scale $\mu = M_1$, the textures of Y_ν and $M_{\rm R}$ are required to have a universal Fritzsch form: $Y_\nu = D_3 F_{\rm D}$ and $M_{\rm R} = M_3 F_{\rm R}$ with

$$F_\lambda \; = \; \begin{pmatrix} \mathbf{0} & c e^{i\varphi_\lambda} & \mathbf{0} \\ c e^{i\varphi_\lambda} & \mathbf{0} & b e^{i\phi_\lambda} \\ \mathbf{0} & b e^{i\phi_\lambda} & a \end{pmatrix} \; , \tag{5}$$

where $\lambda = {\rm D}$ or R, $a = 1 - r + r^2$, $b = (1-r)\sqrt{r(1+r^2)/a}$ and $c = r\sqrt{r/a}$. It is easy to check that three eigenvalues of F_λ have the geometric hierarchy $r^2 : r : 1$. The effective neutrino mass matrix M_ν at low energies may also take the Fritzsch texture: $M_\nu(M_Z) = m_3 F_\nu$, where F_ν has the same form as F_λ defined in Eq. (5) but its two phases (denoted by ϕ_ν and φ_ν) are free parameters. Since the running effects of three lepton flavor mixing angles and three CP-violating phases are negligibly small in the standard model with a normal neutrino mass hierarchy we extrapolate $M_\nu(M_Z)$ up to the scale $\mu = M_1$ in a good approximation: $M_\nu(M_1) \approx m_3 \mathcal{I}_\alpha F_\nu$, where $\mathcal{I}_\alpha \sim 1.4$ for $M_1 \sim 10^{10}$ GeV. It is then straightforward to obtain the seesaw relation $M_\nu(M_1) \approx Y_\nu M_{\rm R}^{-1} Y_\nu^T \langle\phi\rangle^2$, only if the phase condition $\phi_{\rm D} - \phi_{\rm R} = \varphi_{\rm D} - \varphi_{\rm R}$ is satisfied. From this seesaw formula, $D_3 = \sqrt{m_3 M_3 \mathcal{I}_\alpha}/\langle\phi\rangle$, $\phi_\nu = 2\phi_{\rm D} - \phi_{\rm R}$ and $\varphi_\nu = 2\varphi_{\rm D} - \varphi_{\rm R}$ can be derived. Our result shows that the Fritzsch texture must be seesaw-invariant, if the eigenvalues of Y_ν and $M_{\rm R}$ share a universal geometric hierarchy and their phase parameters satisfy a very simple relation. Indeed,

$$\phi_\nu - \varphi_\nu = \phi_{\rm D} - \varphi_{\rm D} = \phi_{\rm R} - \varphi_{\rm R} \tag{6}$$

holds in this phenomenological scenario. We argue that the texture zeros of F_λ could stem from an underlying horizontal flavor symmetry. Although the dynamical reason for a universal geometric mass hierarchy of light and heavy Majorana neutrinos is completely unknown, we do have observed the approximate geometric hierarchy of up- and down-quark masses (i.e., $m_u/m_c \sim m_c/m_t$ and $m_d/m_s \sim m_s/m_b$).

I proceed to take the charged-lepton Yukawa coupling matrix Y_l to have a generic Fritzsch texture with no geometric mass hierarchy:

$$Y_l = \begin{pmatrix} \mathbf{0} & \tilde{c}e^{i\varphi_l} & \mathbf{0} \\ \tilde{c}e^{i\varphi_l} & \mathbf{0} & \tilde{b}e^{i\phi_l} \\ \mathbf{0} & \tilde{b}e^{i\phi_l} & \tilde{a} \end{pmatrix} , \tag{7}$$

where $\tilde{a} \approx m_\tau/\langle\phi\rangle$, $\tilde{b} \approx \sqrt{m_\mu m_\tau}/\langle\phi\rangle$ and $\tilde{c} \approx \sqrt{m_e m_\mu}/\langle\phi\rangle$. Because of $m_e/m_\mu \sim 0.005$ and $m_\mu/m_\tau \sim 0.06$, the contribution of Y_l (or M_l) to lepton flavor mixing is not expected to be very significant. A careful numerical analysis made before has shown that the Fritzsch texture of lepton mass matrices is compatible with current neutrino oscillation data at the 3σ level. We find that the same conclusion can be drawn when the geometric mass hierarchy of three light Majorana neutrinos is taken. In this case, the allowed values of r are restricted to the range $0.215 \lesssim r \lesssim 0.270$.

Now let me turn to the cosmological baryon number asymmetry. To calculate the CP-violating asymmetry ε_1 in Eq. (2), we need to figure out the unitary matrix $U_{\rm R}$ which is defined to diagonalize $M_{\rm R}$. The result is

$$U_{\rm R} = \frac{1}{(1+r)\sqrt{a}} \begin{pmatrix} e^{i(\phi_{\rm R} - \varphi_{\rm R})} & 0 & 0 \\ 0 & e^{-i\phi_{\rm R}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -i\sqrt{r(1+r^2)} & r^2 \\ \sqrt{ra} & i\sqrt{(1+r^2)a} & \sqrt{ra} \\ -r\sqrt{1+r^2} & -i(1-r)\sqrt{r} & \sqrt{1+r^2} \end{pmatrix} , \tag{8}$$

where $a = 1 - r + r^2$ has been given below Eq. (5). After a straightforward calculation, we arrive at

$$\varepsilon_1 = -\frac{3 M_1 m_3 \mathcal{I}_\alpha (1-r)^2 (1+r^2) \sin^3\omega \cos\omega}{2\pi \langle\phi\rangle^2 \left[r^2 (1+r)^2 + 4(1-r)(1+r^2)\sin^2\omega \right]} , \tag{9}$$

where $\omega \equiv (\phi_{\rm D} - \phi_{\rm R})/2$ and m_3 has been given in Eq. (3). Note that ε_1 does not depend on the phase difference $(\varphi_{\rm D} - \varphi_{\rm R})$, because the latter is cancelled out in $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)$ due to the phase relation in Eq. (6). At the seesaw scale, the effective neutrino mass parameter $\tilde{m}_1 \equiv (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{11} \langle \phi \rangle^2 / M_1$ explicitly reads

$$\tilde{m}_1 = m_1 \mathcal{I}_\alpha \left[1 + \frac{4(1-r)(1+r^2)\sin^2\omega}{r^2(1+r)^2} \right] , \tag{10}$$

where m_1 has been given in Eq. (3). If $\tilde{m}_1$ lies in the range 10^{-2} eV $\lesssim \tilde{m}_1 \lesssim 1$ eV (the so-called strong washout regime), one may estimate the dilution factor d by using the approximate formula $d \approx 0.02 \times (0.01 \text{ eV}/\tilde{m}_1)^{1.1}$. This simple power law leads to $d \approx 2.0 \times 10^{-2}$ at $\tilde{m}_1 \approx 0.01$ eV and $d \approx 1.3 \times 10^{-4}$ at $\tilde{m}_1 \approx 1$ eV. Given appropriate inputs of ω and M_1, the observed baryon number asymmetry $Y_{\rm B}$ can then be reproduced from this simple leptogenesis scenario.

For the purpose of illustration, I typically take $\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2, $r = 0.25$ and $m_H = 144$ GeV (Higgs masss) to evaluate $Y_{\rm B}$. Restricting the output of $Y_{\rm B}$ to the generous range $7 \times 10^{-11} \lesssim Y_{\rm B} \lesssim 10^{-10}$, we can arrive at the allowed region of ω and M_1. We find that the lower bound of M_1 is about 3.0×10^{10} GeV and the lower limit of ω is about 11.4°. When M_1 is much larger than 10^{12} GeV, the value of ω approaches 90°. This result is a straightfoward consequence of $Y_{\rm B} \propto \varepsilon_1 \propto M_1 \sin^3\omega \cos\omega$. We have noticed that our numerical outputs are insensitive to the Higgs mass m_H, which slightly affects the running function $\mathcal{I}_\alpha$. The effective neutrino mass $\tilde{m}_1$ is found to monotonically increase with ω, from 0.01 eV to 0.16 eV in the interval $\omega \in [11.4^\circ, 90^\circ)$. Hence the lepton-number-violating wash-out processes (measured by d) are quite efficient in our scenario.

It is worth mentioning that the above calculation of $Y_{\rm B}$ can simply be extended to the minimal supersymmetric standard model. The sizes of ε_1 and g_* in the supersymmetric case are twice as large as in the standard model, thus the two effects tend to cancel in the estimate of $Y_{\rm L}$. For mild values of $\tan\beta$ (from 10 to 50, for example), the supersymmetric running function $\mathcal{I}_\alpha$ is essentially comparable in magnitude with its standard model counterpart. Therefore, the obtained numerical result is roughly valid for the minimal supersymmetric standard model.

It is also worth remarking that the CP-violating asymmetry ε_1 has no direct connection with the low-energy CP violation in neutrino oscillations. The reason is simply that the former is governed by the phase difference $(\phi_{\rm D} - \phi_{\rm R})$, while the latter is associated with the phase differences $(\phi_\nu - \phi_l)$ and $(\varphi_\nu - \varphi_l)$. This feature is certainly dependent on the phase choice of lepton mass matrices. If Y_l in Eq. (7) is taken to be real (i.e., $\phi_l = \varphi_l = 0$ or π), for instance, CP violation at low energies turn out to rely on the phase parameters $\phi_\nu = 2\phi_{\rm D} - \phi_{\rm R}$ and $\varphi_\nu = 2\varphi_{\rm D} - \varphi_{\rm R}$. In this case, there appears an indirect connection between CP violation at high scales and that at low scales.

In summary, I have proposed a simple but viable seesaw model of lepton mass matrices based on the phenomenological conjecture that left- and right-handed Majorana neutrinos have a universal geometric mass hierarchy. The cosmological baryon number asymmetry and current neutrino oscillation data can simultaneously be interpreted in this model with very few free parameters.

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