Family Asymmetry and Neutrino Matrix

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Abstract

Seesaw model is an attractive model as leptogenesis model which may explain neutrino mass. There are many CP violating phases which are not thoroughly studied. In the leptogenesis scenario, the total lepton number is given by a sum of a family number. $L = L_e + L_\mu + L_\tau$. The family asymmetries $L_{e,\mu,\tau}$ can vary by changing CP phases and depend on the form of the Dirac matrix and Majorana mass of the model. In this work, the relation between the low energy CP violation and the family asymmetries is studied.

- We develop the method with which one can obtain MNS matrix, $\Delta \sim$ Jarlskog invariant, and the light neutrino mass eigenvalues, for a given set of heavy Majorana masses and m_D.[5] With the formulae, one can study the relation of CP violation of low energy and leptogenesis (family asymmetries [1]) in general. Also one can impose the constraints from the lepton mixings and neutrino mass squared differences of experiments.

- As an example which shows strong correlation bet. low energy CP violation and leptogenesis, one can find a scenario "one family dominant leptogenesis" (e.g., $L = L_\tau$). [1] [5] We identify the two zero textures which leads to one family dominant leptogenesis scenarios.

- As an example which shows there is no correlation, we find the case that that even if all the lepton asymmetries are zero i.e., $L = L_e = L_\mu = L_\tau = 0$, the low energy CP violation of neutrino oscillation Δ can be non-zero. We identify the texture which has such property and study the unitarity triangle of the leptonic matrix for this case.

1 Notation and framework

We study the minimal seesaw model with two heavy Majorana neutrino. The Dirac mass term is given by 3×2 matrix,

$$\begin{pmatrix} \mathbf{m_{D1}}, & \mathbf{m_{D2}} \end{pmatrix} = \begin{pmatrix} m_{De1} & m_{De2} \\ m_{D\mu1} & m_{D\mu2} \\ m_{D\tau1} & m_{D\tau2} \end{pmatrix} = \begin{pmatrix} \mathbf{u_1}, \mathbf{u_2} \end{pmatrix} \begin{pmatrix} m_{D1} & 0 \\ 0 & m_{D2} \end{pmatrix}, \tag{1}$$

where two unit vectors are introduced, $\mathbf{u_i} = \frac{\mathbf{m_{Di}}}{m_{Di}}$ and $m_{Di} = |\mathbf{m_{Di}}|$.

$$\mathbf{u_1} = \begin{pmatrix} u_{e1} \\ u_{\mu1} \\ u_{\tau1} \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} u_{e2} \\ u_{\mu2} \\ u_{\tau2} \end{pmatrix}. \tag{2}$$

The heavy Majorana mass matrix M is real and diagonal, $M_{ij} = M_i \delta_{ij}$. Using the definitions above, one can write effective majorana mass term as,

$$-m_{eff} = m_D \frac{1}{M} m_D^T = \mathbf{u_1} \mathbf{u_1^T} X_1 + \mathbf{u_2} \mathbf{u_2^T} X_2, \tag{3}$$

with $X_1 = \frac{m_{D1}^2}{M_1}$ $X_2 = \frac{m_{D2}^2}{M_2}$. The light neutrino mass spectra (n_1, n_2, n_3) is obtained from the eigenvalu equation: $\det(m_{eff} m_{eff}^\dagger - n^2) = 0$. For the normal hierarchical case, i.e., $n_1 = 0$ $n_2^2 = m_{sol}^2, n_3^2 = m_{sol}^2 + m_{atm}^2$, we have,

$$\begin{aligned} n_2^2 + n_3^2 &= X_1^2 + X_2^2 + 2X_1 X_2 Re\left((\mathbf{u_1^\dagger} \cdot \mathbf{u_2})^2\right), \\ n_2 n_3 &= X_1 X_2 (1 - |\mathbf{u_1^\dagger} \cdot \mathbf{u_2}|^2). \end{aligned} \tag{4}$$

Since the mass spectrum depend on the innner product $\mathbf{u_1^\dagger}\cdot\mathbf{u_2}$, we parametrize,

$$(\mathbf{u_1^\dagger}\cdot\mathbf{u_2}) = \cos\beta\exp(i\gamma_R). \tag{5}$$

There are two interesting cases about the mass formulae Eq.(4),

$$\begin{aligned} &\mathbf{u_1}\perp\mathbf{u_2} \quad \mathbf{u_1^\dagger}\cdot\mathbf{u_2} = 0 \to n_2 = Min(X_1, X_2) \quad n_3 = Max(X_1, X_2) \\ &\mathbf{u_1}//\mathbf{u_2} \quad \mathbf{u_1^\dagger}\cdot\mathbf{u_2} = \exp(i\gamma_R) \quad n_2 = 0 \quad n_3 = \sqrt{X_1^2 + X_2^2 + 2X_1X_2\cos[2\gamma_R]}. \end{aligned} \tag{6}$$

Therefore when u_1 and u_2 are orthogonal, X_1 and X_2 are mass eigenvalues. When u_1 and u_2 are not linearly independent, we have two massless states and the mass of the massive state depend on CP phase γ_R strongly as,

$$\begin{aligned} n_3 &= |X_1 - X_2| \quad (\gamma_R = \frac{\pi}{2}) \\ n_3 &= X_1 + X_2 \quad (\gamma_R = 0). \end{aligned} \tag{7}$$

When $2\gamma_R = \pi$, X_1 and X_2 can be very different from the mass eigenvalues.

2 low energy CP violation

The low energy CP violation in neutrino oscillation is related to the following quantity,[4] [3] [2]

$$\begin{aligned} \Delta \quad &= J(n_1^2 - n_2^2)(n_2^2 - n_3^2)(n_3^2 - n_1^2) \\ &= \mathrm{Im}\left((m_{eff}m_{eff}^\dagger)_{e\mu}(m_{eff}m_{eff}^\dagger)_{\mu\tau}(m_{eff}m_{eff}^\dagger)_{\tau e}\right). \end{aligned} \tag{8}$$

We have computed Δ without further assumption.[5]

$$\begin{aligned} \Delta = \quad &\left(1 - |\mathbf{u_1^\dagger}\cdot\mathbf{u_2}|^{\mathbf{2}}\right)\times \\ &\Big(X_1^4X_2^2\left(\mathrm{Im}[\left(u_{e1}^*u_{e2}u_{\mu1}u_{\mu2}^*\right)|u_{\tau1}|^2 + \left(u_{\mu1}^*u_{\mu2}u_{\tau1}u_{\tau2}^*\right)|u_{e1}|^2 + (u_{\tau1}^*u_{\tau2}u_{e1}u_{e2}^*)\,|u_{\mu1}|^2]\right) \\ &+X_1^3X_2^3\left(\mathrm{Im}[(u_{e1}^*u_{e2})(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})(|u_{\tau1}u_{\mu2}|^2 - |u_{\mu1}u_{\tau2}|^2)\right. \\ &\qquad +(u_{\mu1}^*u_{\mu2})(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})(|u_{e1}u_{\tau2}|^2 - |u_{\tau1}u_{e2}|^2) \\ &\qquad \left.+(u_{\tau1}^*u_{\tau2})(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})(|u_{\mu1}u_{e2}|^2 - |u_{e1}u_{\mu2}|^2)]\right) \\ &-X_1^2X_2^4\left(\mathrm{Im}[\left(u_{e1}^*u_{e2}u_{\mu1}u_{\mu2}^*\right)|u_{\tau2}|^2 + \left(u_{\mu1}^*u_{\mu2}u_{\tau1}u_{\tau2}^*\right)|u_{e2}|^2 + (u_{\tau1}^*u_{\tau2}u_{e1}u_{e2}^*)\,|u_{\mu2}|^2]\right)\Big). \end{aligned} \tag{9}$$

This is the most generalization of FGY scenarios [3]. Let us study the two zero textures model of m_D. The authors of ref.[3] studied two of them presented below. Two zero textures model corresponding to τ leptogenesis model have $u_{e1} = 0, u_{\mu2} = 0$ or $u_{e2} = 0, u_{\mu1} = 0$, which satisfy τ family dominant condition for leptogenesis, $\mathbf{u_1^\dagger}\cdot\mathbf{u_2} = u_{1\tau}^*u_{2\tau}$.

$$m_D = \begin{pmatrix} 0 & * \\ * & 0 \\ * & * \end{pmatrix} \ or \ \begin{pmatrix} * & 0 \\ 0 & * \\ * & * \end{pmatrix} \quad \Delta = (1 - |u_{\tau1}u_{\tau2}|^2)X_1^3X_2^3Im[(u_{\tau1}^*u_{\tau2})^2](|u_{\mu1}u_{e2}|^2 - |u_{e1}u_{\mu2}|^2). \tag{10}$$

These cases correspond to τ leptogenesis $L = L_\tau$ and $\Delta \sim \pm L_\tau$.
We have other two zero textures with $\mathbf{u_1^\dagger}\cdot\mathbf{u_2} = u_{a1}^*u_{a2}$. $(a = e, \mu, \tau)$. All the cases show one to one correlation bet. one family dominant leptogenesis and low energy CP. i.e., $L = L_a$ and $\Delta \sim \pm L_a$.
(1) **e-leptogenesis** $L = L_e$ (2) μ-**leptogenesis** $L = L_\mu$

$$\begin{pmatrix} * & * \\ * & 0 \\ 0 & * \end{pmatrix} \text{or} \begin{pmatrix} * & * \\ 0 & * \\ * & 0 \end{pmatrix}. \qquad \begin{pmatrix} * & 0 \\ * & * \\ 0 & * \end{pmatrix} \text{or} \begin{pmatrix} 0 & * \\ * & * \\ * & 0 \end{pmatrix}.$$

3 Neutrino Mixings

Neutrino mixings, U_{MNS} can be obtained by the fiagonalization, $U_{MNS}^\dagger m_{eff} U_{MNS}^*$. We perform the diagonalization with taking two steps. 1)Isolate zero mass state by unitary rotation U.
2) Perform two by two diagonalization for massive states which determines another unitary matrix K. U_{MNS} is given by their product as $U_{MNS} = UK$.

$$\begin{aligned}
U_{MNS} &= \begin{pmatrix} \frac{\mathbf{u_2^*}\times\mathbf{u_1^*}}{\sqrt{1-|\mathbf{u_1^\dagger}\cdot\mathbf{u_2}|^2}} & \frac{\mathbf{u_2}-(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})\mathbf{u_1}}{\sqrt{1-|\mathbf{u_1^\dagger}\cdot\mathbf{u_2}|^2}} & \mathbf{u_1} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & s_\theta e^{(-i\phi)} \\ 0 & -s_\theta e^{i\phi} & c_\theta \end{pmatrix} P, \\
\phi &= arg.(X_1(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})^* + X_2(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})), \\
\tan 2\theta &= \frac{2X_2\sqrt{1-|\mathbf{u_1^\dagger}\cdot\mathbf{u_2}|^2}|X_1(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})^* + X_2\mathbf{u_1^\dagger}\cdot\mathbf{u_2}|}{X_1^2+X_2^2(2|\mathbf{u_1^\dagger}\cdot\mathbf{u_2}|^2-1)+2X_1X_2\mathrm{Re.}(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})^2}. \qquad (11)
\end{aligned}$$

To work out the special case, we consider the case with $\mathbf{u_1^\dagger}\cdot\mathbf{u_2}=\mathbf{0}$, which corresponds to there is no leptogenesis at all because $Im[(u_1^\dagger \cdot u_2)^2]=0$. As we will see even cosmological CP violation vanishes, the low energy CP violation can be non-zero. Mass eigenvalues and MNS matrix can be easily obtained for this case,

$$\begin{aligned}
&(n_1, n_2, n_3) = (0, X_2, X_1) \\
&U_{MNS} = \begin{pmatrix} \mathbf{u_2^*}\times\mathbf{u_1^*}, & \mathbf{u_2}, & \mathbf{u_1} \end{pmatrix}.
\end{aligned}$$

The orthogonality condition reduces the the numbers of parameters in $\mathbf{u_1}, \mathbf{u_2}$ from the general case with 4 real + 3 imaginary parts to 3 real + 2 imaginary parts. We can impose two mixing angles from experiments. Then MNS matrix have three parameters (1 real + 2 imag.) to be determined. We set, $u_{e1}=s_{13}$, $\arg(u_{e2})=\theta_e$, $\mathrm{Im}(u_{\mu 2}\exp(-i\theta_e))=x$.

$$U_{MNS} = \begin{pmatrix} (\mp\sqrt{\frac{2}{3}-2x^2}-i\sqrt{2}x)c_{13} & \sqrt{\frac{1}{3}}e^{i\theta_e}c_{13} & s_{13} \\ * & (\mp\sqrt{\frac{1}{3}-x^2}-\sqrt{\frac{1}{6}}s_{13}+ix)e^{i\theta_e} & \sqrt{\frac{1}{2}}c_{13} \\ * & (\pm\sqrt{\frac{1}{3}-x^2}-\sqrt{\frac{1}{6}}s_{13}-ix)e^{i\theta_e} & \sqrt{\frac{1}{2}}c_{13} \end{pmatrix}.$$

The three unknown parameters s_{13},x and θ_e can be determined from the low energy CP violation, neutrinoless double β decay $\sim m_{eff\,ee}$ and U_{e3}^{MNS}.

$$\begin{aligned}
&|(m_{eff})_{ee}| = |s_{13}^2X_1 + \frac{1}{3}c_{13}^2\exp(2i\theta_e)X_2|, \quad |U_{e3}^{MNS}| = s_{13}, \\
&\Delta = X_1^2X_2^2(X_1^2-X_2^2)Im\,(u_{\tau 1}^*u_{\tau 2}u_{e1}u_{e2}^*) \\
&= n_2^2n_3^2(n_3^2-n_2^2)\sqrt{\frac{1}{2}}s_{13}c_{13}\sqrt{\frac{1}{3}}\times(-x). \qquad (12)
\end{aligned}$$

Acknowledgement
T. M. Thank Z. Xing for discussion. The work is supported by Kakenhi of JSPS, Japan N0.16028213.

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