Maximal CP Violation Hypothesis and Phase Convention of the CKM Matrix

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Abstract

The maximal CP violation hypothesis depends on the phase convention of the CKM matrix, when we assume that three rotation angles in the CKM matrix are fixed by the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. Phase conventions which lead to successful prediction under the maximal CP violation hypothesis are only two: the original Kobayashi-Maskawa phase convention and the Fritzsch-Xing phase convention. Thereby, possible structures of the quark mass matrices are speculated.

1 Introduction

Recent remarkable progress of the experimental B physics has put the shape of the unitary triangle in the quark sector within our reach. The world average value of the angle β [1] which has been obtained from B_d decays is $\sin 2\beta = 0.736 \pm 0.049$ $\left(\beta = 23.7^{\circ}{}^{+2.2^\circ}_{-2.0^\circ}\right)$, and the best fit [1] for the Cabibbo-Kobayashi-Maskawa (CKM) matrix V also gives $\gamma = 60^\circ \pm 14^\circ$, and $\beta = 23.4^\circ \pm 2^\circ$, where the angles α, β and γ are defined by

$$\alpha = \text{Arg}\left[-\frac{V_{31}V_{33}^*}{V_{11}V_{13}^*}\right], \quad \beta = \text{Arg}\left[-\frac{V_{21}V_{23}^*}{V_{31}V_{33}^*}\right], \quad \gamma = \text{Arg}\left[-\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*}\right]. \tag{1.1}$$

We are interested what logic can give the observed magnitude of the CP violation.

Usually, we assume a peculiar form of the quark mass matrices at the start, and thereby, we predict a magnitude of the CP violation and a shape of the unitary triangle. However, at the present talk, I would like to investigate a quark mass matrix model on the basis of an inverse procedure [2]: by noticing that predictions based on the maximal CP violation hypothesis [3] depend on the phase convention, I will, at the start, investigate what phase conventions can give favorable predictions of the unitary triangle under the maximal CP violation hypothesis, and then I will investigate what quark mass matrices can give such a phase convention of the CKM matrix. Here, we have assumed that the three rotation angles in the CKM matrix V are fixed by the observed values [1] of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$, and only the CP violating phase parameter δ is free.

There are, in general, 9 cases for the phase convention of the CKM matrix. When we define the expression of the CKM matrix V as

$$V = V(i,k) \equiv R_i^T P_j R_j R_k \qquad (i \neq j \neq k), \tag{1.2}$$

where

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \quad R_3(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.3}$$

($s = \sin\theta$ and $c = \cos\theta$) and $P_1 = \mathrm{diag}(e^{i\delta},\ 1,\ 1)$, $P_2 = \mathrm{diag}(1,\ e^{i\delta},\ 1)$, and $P_3 = \mathrm{diag}(1,\ 1,\ e^{i\delta})$.

For example, the standard phase conventions of the CKM matrix, which corresponds to the case $V(1,3)$, is given by

$$V_{SD} = R_1(\theta_{23})P_3(\delta_{13})R_2(\theta_{13})P_3^\dagger(\delta_{13})R_3(\theta_{12}). \tag{1.4}$$

The rephasing invariant quantities is given by $J = c_{13}^2 s_{13} c_{12} s_{12} c_{23} s_{23} \sin\delta_{13}$. The maximal CP violation hypothesis asserts that the CP violation phase is chosen so that J is maximal, i.e. $\delta_{13} = \pi/2$. The choice $\delta_{13} = \pi/2$ leads to the prediction

$$\alpha = 68.5^\circ, \quad \beta = 21.5^\circ, \quad \gamma = 89.96^\circ. \tag{1.5}$$

The predicted value of β is favorable to the observed value, but the value of γ is in disagreement with the observed value.

On the contrary, in the original Kobayashi-Maskawa phase convention $V(1,1)$

$$V_{KM} = R_1^T(\theta_2)P_3(\delta_{KM}+\pi)R_3(\theta_1)R_1(\theta_3), \tag{1.6}$$

the rephasing invariant J is given by $J = c_1 s_1^2 c_2 s_2 c_3 s_3 \sin\delta_{KM}$, so that the maximal CP violation hypothesis $\delta_{KM} = \pi/2$ predicts

$$\alpha = 89.96^\circ, \quad \beta = 23.2^\circ, \quad \gamma = 66.8^\circ, \tag{1.7}$$

which are in good agreement with experiments.

Thus, the predictions under the maximal CP violation hypothesis are dependent on the phase conventions.

2 Why does the shape of the unitary triangle depend on the phase convention?

Note that in the present maximal CP violation hypothesis we have assumed that only free parameter is a CP violation phase δ and the rotation angles θ_i are fixed.

Our assumption is as follows: The phase factors in the quark mass matrices M_f ($f = u, d$) are factorized by the phase matrices P_f as

$$M_f = P_{fL}^\dagger \widetilde{M}_f P_{fR} \ , \tag{2.1}$$

where $\widetilde{M}_f$ are real matrices and they are diagonalized by the orthogonal matrices R_f as

$$R_f^\dagger \widetilde{M}_f R_f = D_f \ , \tag{2.2}$$

so that the CKM matrix V is given by

$$V = R_u^T P R_d \ , \tag{2.3}$$

where $P = P_u^\dagger P_d$. The quark masses m_{fi} are only determined by $\widetilde{M}_f$. In other words, the rotation parameters are given only in terms of the quark mass ratios, and they are independent of the phase parameter δ. In such a scenario, the maximal CP violation hypothesis means that the CP violation phase δ takes its maximum value without changing the quark mass values.

3 General expressions of the CKM matrix and the related formulae

We define the general forms of the CKM matrix $V(i,k)$ by Eq. (1.2). Then, for the 9 cases of $V(i,k)$, the rephasing invariant quantity J is given by

$$J = \frac{|V_{i1}||V_{i2}||V_{i3}||V_{1k}||V_{2k}||V_{3k}|}{(1-|V_{ik}|^2)|V_{ik}|} \sin\delta \ . \tag{3.1}$$

The angles ϕ_l $(l=1,2,3)$ in the unitary triangle are also given by

$$\sin\phi_\ell = \frac{|V_{i1}||V_{i2}||V_{i3}||V_{1k}||V_{2k}||V_{3k}|\sin\delta}{|V_{m1}||V_{m3}||V_{n1}||V_{n3}|(1-|V_{ik}|^2)|V_{ik}|}, \tag{3.2}$$

where (l,m,n) is a cyclic permutation of $(1,2,3)$ and $(\phi_1,\phi_2,\phi_3)=(\beta,\alpha,\gamma)$. Note that the magnitudes $|V_{i1}|$, $|V_{i2}|$, $|V_{i3}|$, $|V_{1k}|$, $|V_{2k}|$, and $|V_{3k}|$ are independent of the phase δ.

Under the approximation $1 \gg |V_{us}|^2 \simeq |V_{cd}|^2 \gg |V_{cd}|^2 \simeq |V_{ts}|^2 \gg |V_{ub}|^2$, we obtain the following 4 types of J: Case (a) $J \simeq |V_{us}||V_{cd}||V_{ub}| \sim \delta$ for $V(1,2)$, $V(1,3)$, $V(2,1)$ and $V(2,3)$; Case (b) $J \simeq |V_{ub}||V_{tb}|\sin\delta$ for $V(1,1)$ and $V(3,3)$; Case (c) $J \simeq |V_{us}||V_{cb}||V_{td}|\sin\delta$ for $V(3,1)$ and $V(3,2)$; Case (d) $J \simeq |V_{cb}|^2\sin\delta$ for $V(2,2)$.

Under the maximal CP violation hypothesis, only two cases can give the observed shape of the CKM matrix and value of J: one is $V(1,1)$, i.e. the original Kobayashi-Maskawa phase convention; another one is $V(3,3)$, i.e. the Fritzsch-Xing phase convention [4].

Table 1 Predictions from the $V(1,1)$ and $V(3,3)$ models

	α	β	γ
$V(1,1)$	90.0°	23.2°	66.8°
$V(3,3)$	89.0°	23.2°	67.8°
Experiment		$\beta = 23.7^\circ{}^{+2.2^\circ}_{-2.0^\circ}$	$\gamma = 60^\circ \pm 14^\circ$

4 Quark mass matrices speculated from the Fritzsch-Xing phase convention

The successful case $V(3,3) = R_3^T P_1 R_1 R_3$ suggests the following quark mass matrix structure [4, 5]:

$$\widetilde{M}_f = R_1(\theta_{23}^f) R_3(\theta_{12}^f) D_f R_3^T(\theta_{12}^f) R_1^T(\theta_{23}^f) \ (f=u,d). \tag{4.1}$$

The explicit forms of M_f are given by

$$\widetilde{M}_f = \begin{pmatrix} m_1 c_{12}^2 + m_2 s_{12}^2 & (m_2-m_1)c_{12}s_{12}c_{23} & -(m_2-m_1)c_{12}s_{12}s_{23} \\ (m_2-m_1)c_{12}s_{12}c_{23} & (m_1 s_{12}^2 + m_2 c_{12}^2)c_{23}^2 + m_3 s_{23}^2 & (m_3 - m_2 c_{12}^2 - m_1 s_{12}^2)c_{23}s_{23} \\ -(m_2-m_1)c_{12}s_{12}s_{23} & (m_3 - m_2 c_{12}^2 - m_1 s_{12}^2)c_{23}s_{23} & (m_1 s_{12}^2 + m_2 c_{12}^2)s_{23}^2 + m_3 c_{23}^2 \end{pmatrix} \tag{4.2}$$

If we assume $M_{11}^d = 0$, we obtain the well–known relation

$$|V_{us}| \simeq s_{12}^d \simeq \sqrt{m_d/m_s} \simeq 0.22 \ . \tag{4.3}$$

Also if we assume $M^u_{11} = 0$, we obtain

$$|V_{ub}|/|V_{cb}| = s^u_{12}/c^u_{12} \simeq \sqrt{m_u/m_c} = 0.059 \ , \tag{4.4}$$

which is roughly consistent with the observed value [1] $|V_{ub}|/|V_{cb}| = 0.089^{+0.015}_{-0.014}$. If we assume $M^u_{22} = 0$ together with s^d_{23}, we obtain

$$|V_{cb}| \simeq s_{23} = s^u_{23} \simeq \sqrt{m_c/m_t} \simeq 0.061 \ . \tag{4.5}$$

For a further detailed phenomenological study of the V(3,3) model with the renormalization group effects, see Ref. [5].

5 Application to the lepton sector

From the analogy to the quark sector, we consider that the lepton mixing matrix U is also given by $V(3,3)$. Then, the maximal CP violation hypothesis predicts $J \simeq (1/4)\sin 2\theta_{solar}|U_{13}|$. The requirement $M^e_{11} = 0$ predicts $|U_{13}| \simeq \sqrt{m_e/m_\mu} = 0.049$, where we have assume $s_{23} = \pi/4$.

6 Summary

Under the maximal CP violation hypothesis, only two expressions $V(1,1)$ and $V(3,3)$ can give the successful predictions for the unitary triangle: $\alpha = 90^\circ$, $\beta = 23^\circ$ and $\gamma = 67^\circ$. The Fritzsch-Xing expression $V(3,3)$ suggests a quark mass matrix structure $M_q = P_1 R_1 R_3 D_q R_3^T R_1^T P_1^\dagger$, which leads to $|V_{us}| \simeq \sqrt{m_d/m_s}$ under $M^d_{11} = 0$ and to $|V_{us}|/|V_{cb}| \simeq \sqrt{m_u/m_c}$ under $M^u_{11} = 0$.

However, we have some open questions:

(i) What mechanism can cause such a maximal CP violation?

(ii) What mechanism can give the successful quark mass matrix structure, $M_q = P_1 R_1 R_3 D_q R_3^T R_1^T P_1^\dagger$?

(iii) Is there a simple ansatz for the mixing angle θ_{23} (especially, for $\theta_{23} = \pi/4$ in the lepton sector)?

Anyhow, phenomenology on the unitary triangle will provide a promising clue to the unified understanding of the quark and lepton mass matrices.

References

[1] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. **B592**, 1 (2004).

[2] Y. Koide, Phys. Lett. **B607**, 123 (2005).

[3] D. Hochberg and R. Sachs, Phys. Rev. **D27**, 606 (1983); B. Stech, Phys. Lett. **B130**, 189 (1983); L. Wolfenstein, Phys. Lett. **B144**, 425 (1984); M. Shin, Phys. Lett. **B145**, 285 (1984); H. Fritzsch, Phys. Rev. **D32**, 3058 (1985); M. Gronau and J. Schechter, Phys. Rev. Lett. **54**, 385 (1985); M. Gronau, R. Jonson and J. Schechter, Phys. Rev. Lett. **54**, 2176 (1985); F. J. Botella and L. -L. Chau, Phys. Lett. **B168**, 97 (1984). And also see, C. Jarlskog, in *Introduction to CP Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1989), p.3, and references therein.

[4] H. Fritzsch and Z. -z. Xing, Phys. Lett. **B413**, 396 (1997).

[5] Z. -z. Xing, Phys. Rev. **D68**, 073008 (2003).