Neutrino Mixing as a Probe of Grand Unification

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(Dated: March 22, 2005)

The recent experimental measurements of the solar neutrino mixing angle θ_{sol} and the Cabibbo mixing angle θ_C reveal a surprising relation, $\theta_{sol} + \theta_C \simeq \frac{\pi}{4}$. We show that this empirical relation can be interpreted as a support of the idea of grand unification. In particular, we emphasis that the lepton mixing matrix derived from quark-lepton unification can lead to a shift of the complementarity relation at low energy. While the renormalization group effects generally lead to additive contribution on top of the shift, we discuss that the threshold corrections which may exist in some intermediate scale new physics such as supersymmetric standard model can diminish it, so we can achieve the complementarity relation at a low energy. Finally, we present a possibility to achieve the complementarity relation at a high energy by taking particular form of non-symmetric form of down Yukawa matrix.

PACS numbers:

Recently, it has been noted that the solar neutrino mixing angle θ_{sol} required for a solution of the solar neutrino problem and the Cabibbo angle θ_C reveal a surprising relation

$$\theta_{sol} + \theta_C \simeq \frac{\pi}{4}, \tag{1}$$

which is satisfied by the experimental results $\theta_{sol} + \theta_C = 45.4^\circ \pm 1.7^\circ$ to within a few percent accuracy [1–3]. This quark-lepton complementarity (QLC) relation (1) has been interpreted as an evidence for certain quark-lepton symmetry or quark-lepton unification as shown in Refs. [4–6]. Yet, it can be a coincidence in the sense that reproducing the exact QLC relation (1) at low energy scale in the framework of grand unification depends on the renormalization effects whose size can vary with the choice of parameter space [7]. But, we believe that establishing the origin of the QLC relation may be a challenge that underlying theory of flavor should address.

To effectively describe the deviation from maximal mixing of solar neutrino, small mixing element U_{e3} and possible deviation from maximal mixing of atmospheric neutrino, a parametrization of the PMNS mixing matrix in terms of a small parameter whose magnitude can be interestingly around $\sin\theta_C$ has been proposed as follows [8–11]:

$$U_{\rm PMNS} = U^\dagger(\lambda) U_{\rm bimax} \ . \tag{2}$$

Here $U(\lambda)$ is a mixing matrix parameterized in terms of a small parameter λ and $U_{\rm bimax}$ corresponds to the bi-maximal mixing matrix [12]. Among possible generic structures of the matrix $U(\lambda)$ which are compatible with experimental results on the neutrino oscillations, the "CKM-like" form of $U(\lambda)$ has rather profound implication in view of the connection between quarks and leptons.

In this talk, we first of all show that the lepton mixing matrix given in the form of Eq. (2) with the "CKM-like" $U(\lambda) \sim U_{\rm CKM}$ can be indeed realized in the framework of grand unification with symmetric Yukawa matrices when we incorporate seesaw mechanism, and examine whether or not $U_{\rm PMNS}$ reflecting quark-lepton unification given by (2) can predict the QLC relation (1) exactly. We see that the solar mixing angle derived from $U_{\rm PMNS}$ leads to correction to the QLC relation which can be more than 1σ deviation from the QLC relation as similarly shown in [5]. It is necessary to take into account the renormalization effects on the lepton mixing matrix when one compares the prediction at a high energy scale with the QLC relation observed at low energy scale. One can then expect that the deviation from the QLC relation is explained by renormalization effects. In the SM, the renormalization effect through the renormalization group (RG) running down to the weak scale is negligible because of small charged lepton Yukawa couplings. But, in some models such as the minimal supersymmetric standard model (MSSM), the renormalization effect on the leptonic mixing angle θ_{12} depends on the type of light neutrino mass spectrum as well as model parameters [13–15]. It is also known that in MSSM with large $\tan\beta$ and the quasi-degenerate neutrino mass spectrum the RG effects are generally large and can enhance the mixing angle θ_{12} at low energy [13–15]. Such an enhancement of θ_{12} is not suitable for achieving the QLC relation (1) at low energy because we need to diminish the deviation so as to get the exact QLC relation (1) at a low energy.

In this talk, we show that the sizeable *threshold correc-*

tions which may exist in the MSSM [16–18] can diminish the deviation from the QLC relation while keeping the mixing angle θ_{23} almost maximal and θ_{13} small, so that the QLC relation at low energy can be achieved in the case that the RG effects are suppressed. Some conditions on the parameters to realize the QLC relation will be discussed. Finally, we propose a possible way to achieve the QLC relation by taking a non-symmetric form of the down Yukawa matrix.

The flavor mixings stem from the mismatch between the left-handed rotations of the up-type and down-type quarks, and the charged leptons and neutrinos. For our purpose, it is useful to work in a basis where the quark and lepton Yukawa matrices are related. In general, the quark Yukawa matrices Y_u, Y_d are given by $Y_u = U_u Y_u^{diag} V_u^\dagger$, $Y_d = U_d Y_d^{diag} V_d^\dagger$, from which the observable quark mixing matrix is deduced as $U_{\rm CKM} = U_u^\dagger U_d$. For the charged lepton sector, the Yukawa matrix is given by $Y_l = U_l Y_l^{diag} V_l^\dagger$. For the neutrino sector, we introduce one right-handed singlet neutrino per family which leads to the seesaw mechanism, according to which the light neutrino mass matrix after the breakdown of the electroweak symmetry is given by

$$\begin{aligned} M_\nu &= M_{\rm Dirac}\frac{1}{M_R}M_{\rm Dirac}^T \\ &= \left(U_0 M_{\rm Dirac}^{diag} V_0^\dagger\right)\frac{1}{M_R}\left(V_0^* M_{\rm Dirac}^{diag} U_0^T\right), \end{aligned} \quad (3)$$

where U_0 and V_0 are the left-handed and right-handed mixing matrices of the Dirac neutrino mass matrix, respectively. We can then rewrite M_ν as follows

$$M_\nu = U_0 V_M M_\nu^{diag} V_M^T U_0^T, \quad (4)$$

where V_M represents the rotation of $M_{\rm Dirac}^{diag} V_0^\dagger \frac{1}{M_R} V_0^* M_{\rm Dirac}^{diag}$. The observable PMNS mixing matrix can then be written as [11]

$$U_{\rm PMNS} = U_l^\dagger U_\nu = U_l^\dagger U_0 V_M \ . \quad (5)$$

Now, let us consider how the PMNS mixing matrix given by Eq. (5) can be related with CKM mixing matrix in the context of quark-lepton unification.

(A) Minimal quark-lepton unification

Since the down-type quarks and the charged leptons are in general assigned into a multiplet in grand unification, we assume that the following simple relations hold in the minimal models of grand unification, $Y_e = Y_d^T$, $Y_u = Y_u^T$. Then, we deduce that $U_l = V_d^*$ from which

$$U_{\rm PMNS} = V_d^T U_0 V_M. \quad (6)$$

From this expression for $U_{\rm PMNS}$, we see that the contribution of $U_{\rm CKM}$ may appear in $U_{\rm PMNS}$ if we further assume $Y_\nu = Y_u$ which can be realized in some larger unified gauge group such as $SO(10)$. Then, one can obtain [11]

$$U_{\rm PMNS} = V_d^T U_d U_{\rm CKM}^\dagger V_M. \quad (7)$$

In addition, requiring symmetric form of the down-type quark Yukawa matrix, we finally obtain

$$U_{\rm PMNS} = U_{\rm CKM}^\dagger V_M, \quad (8)$$

where the mixing matrix V_M has bi-maximal mixing pattern as explained above. In this way, $U_{\rm PMNS}$ can be connected with $U_{\rm CKM}$.

To see whether the parametrization of $U_{\rm PMNS}$ given by (8) can lead to the QLC relation (1), it is convenient to present $U_{\rm PMNS}$ for the CP-conserving case as follows:

$$\begin{aligned} U_{\rm PMNS} &= U_{\rm CKM}^\dagger U_{23}^m U_{12}^m \\ &\equiv U_{23}(\theta_{23})U_{13}(\theta_{13})U_{12}(\frac{\pi}{4}-\theta_{12}) \ , \end{aligned} \quad (9)$$

where U_{12}^m and U_{23}^m correspond to the maximal mixing between (1,2) and (2,3) generations, respectively. Then, the mixing angles θ_{ij} in the second line of Eq. (9) can be presented in terms of Wolfenstein parameter λ as follows:

$$\begin{aligned} \sin\theta_{12} &\simeq \frac{1}{\sqrt{2}}\lambda + O(\lambda^3), \quad \sin\theta_{23} \simeq -\frac{1}{\sqrt{2}}\left(1-\frac{1}{2}\lambda^2\right), \\ \sin\theta_{13} &\simeq -\frac{1}{\sqrt{2}}\lambda \ . \end{aligned} \quad (10)$$

The solar neutrino mixing parameter $\sin\theta_{sol}$ in this parametrization becomes

$$\sin\theta_{sol} \simeq \sin\left(\frac{\pi}{4}-\theta_C\right) + \frac{\lambda}{2}(\sqrt{2}-1). \quad (11)$$

Thus, we see that the neutrino mixing matrix (9) originating from the quark-lepton unification obviously leads to a shift of the relation (1). Numerically, the shift amounts to $\delta\theta_{sol} \simeq 3^\circ$ and thus $\delta\sin^2\theta_{sol} \simeq 0.05$ which is more than 1σ deviation from the recent measurement of the solar neutrino experiment. While a dedicated experiment to measure θ_{12} with a sensitivity of a few % to $\sin^2\theta_{12}$ would be expected to confirm or rule out the deviation, we can expect that renormalization effects on the neutrino mixing matrix (9) may fill the gap between the QLC relation and the prediction (11)from high energy mixing matrix. Later, we will discuss the renormalization effect in detail.

(B) Realistic quark-lepton unification

Although the minimal quark-lepton unification can lead to an elegant relation between PMNS mixing and CKM mixing as shown in the above, it indicates undesirable mass relations between quarks and leptons at the GUT scale such as $m_d^{diag} = m_l^{diag}$. Recently, the following form for $U_l^\dagger U_0$ has been suggested based on a well known empirical relation $|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \simeq 3\sqrt{\frac{m_e}{m_\mu}}$ [19],

$$U_l^\dagger U_0 \simeq \begin{pmatrix} 1 & -\frac{\lambda}{3} & \frac{1}{3}\theta\lambda^2 \\ \frac{\lambda}{3} & 1 & 2\theta\lambda \\ -\theta\lambda^2 & -2\theta\lambda & 1 \end{pmatrix}, \quad (12)$$

where the deviation from unitarity is just of order $\theta\lambda^3$. This form of mixing matrix can be obtained by introducing the Higgs sector transforming under the representation **45** of $SU(5)$ or **126** of $SO(10)$ [20]. In this case, the solar neutrino mixing $\sin\theta_{sol}$ is then given by

$$\sin\theta_{sol} \simeq \sin\left(\frac{\pi}{4}-\theta_C\right)+\frac{\lambda}{2}\left(\sqrt{2}-\frac{1}{3}\right) \ . \qquad (13)$$

Numerically, the deviation from the QLC relation amounts to $\delta\theta_{sol} \simeq 7^\circ$, much more than in the observed QLC relation. One possible way to generate the correct prediction for $\sin\theta_{sol}$ based on the lepton mixing with U_l given by Eq. (12) is to abandon the exact bimaximal form of the neutrino mixing matrix V_M and to consider the generic corrections to the bimaximal neutrino mixing matrix that can account for the QLC relation [19]. But, we consider an alternative possibility that the threshold corrections can diminish the deviation from the QLC relation.

Now, let us examine how the renormalization effects can diminish the deviation from the QLC relation. In general, the radiative corrections to the effective neutrino mass matrix can be presented as follows:

$$\begin{aligned} M_\nu &= I\cdot M_\nu^0\cdot I \\ &= I\cdot U^T_{\rm CKM}U^*_{\rm bimax}M_D U^\dagger_{\rm bimax}U_{\rm CKM}\cdot I \\ &= I\cdot U^T_{\rm CKM}U^{m^*}_{23}M_{D12}U^{m\dagger}_{23}U_{\rm CKM}\cdot I \ , \end{aligned} \qquad (14)$$

where $M_D = Diag[m_1,m_2,m_3]$, $M_{D12} = U^{m^*}_{12}M_D U^{m\dagger}_{12}$, and the matrix $I \equiv I_A\delta_{AB}, (A,B=e,\mu,\tau)$ stands for the radiative corrections. The correction I generally consists of two parts $I = I^{RG}+I^{TH}$ where I^{RG} are the renormalization group corrections which are explicitly presented in Ref. [21] and I^{TH} are electroweak scale threshold corrections [16]. We note that the flavor blind interactions such as gauge interactions contribute to overall scale of neutrino masses whereas the charged lepton Yukawa interactions generate flavor dependent radiative corrections I in SM and in MSSM. The typical size of RG corrections I^{RG} is known to be about 10^{-6} in the SM and MSSM with small $\tan\beta$, and thus negligible. In addition, supersymmetry can induce flavor dependent threshold corrections related with slepton mass splitting which can dominate over the charged lepton Yukawa corrections [17]. Even if I_τ is the dominant contribution in SM, it is not guaranteed at all in MSSM due to the threshold corrections. We have numerically checked that RG evolution from the seesaw scale to the weak scale *enhances* the size of θ_{12} in the case that θ_{13} and θ_{23} are kept to be small and almost maximal mixing, respectively. Thus, the case of sizable RG effects is not suitable for our purpose. Instead, we examine whether the threshold corrections can be suitable for diminishing the deviation from the QLC relation while keeping θ_{23} nearly maximal and θ_{13} small in the case that RG effect is negligible. To achieve our goal, we note that the contribution I_e should be dominant over $I_{\mu,\tau}$ because only I_e can lead to the right amount of the shift of θ_{12} while keeping the changes of θ_{23} and θ_{13} small. Taking $|I_e| >> |I_{\mu,\tau}|$, the neutrino mass matrix corrected by the leading contributions is rewritten as follows:

	(A)	(B)
m_1 (eV)	I_e	I_e
0.15	-4.0×10^{-5}	-1.0×10^{-4}
0.1	-8.5×10^{-5}	-2.2×10^{-4}
0.05	-3.4×10^{-4}	-8.6×10^{-4}

TABLE I: Parameter set (I_e, m_1) leading to the QLC relation. The columns (A) and (B) correspond to the minimal unification and realistic case, respectively.

$$M_\nu \ \simeq \ U^T_{\rm CKM}U^{m^*}_{23}\left[I_D+I_e\Lambda_\lambda\right]M_{D12}\left[I_D+I_e\Lambda^\dagger_\lambda\right]U^{m\dagger}_{23}U_{\rm CKM} \qquad (15)$$

where I_D is 3×3 identity matrix, and the matrix Λ_λ for minimal unification is given up to λ^2 order by

$$\Lambda_\lambda \ = \ \begin{pmatrix} 1 & -\frac{\lambda}{\sqrt{2}} & -\frac{\lambda}{\sqrt{2}} \\ -\frac{\lambda}{\sqrt{2}} & \frac{\lambda^2}{2} & \frac{\lambda^2}{2} \\ -\frac{\lambda}{\sqrt{2}} & \frac{\lambda^2}{2} & \frac{\lambda^2}{2} \end{pmatrix}, \qquad (16)$$

and we can obtain Λ_λ for the realistic unification by replacing λ with $\lambda/3$ in Eq.(16).

Let us discuss how much the lepton mixing angles can be shifted by the renormalization effects. We first of all do numerical analysis in a model independent way based on the form given by Eqs. (25–26). For our numerical analysis, we take $\Delta m^2_{sol}\equiv m_2^2-m_1^2\simeq 7.1\times10^{-5}$ eV^2 and $\Delta m^2_{atm}\equiv m_3^2-m_2^2\simeq 2.5\times10^{-3}$ eV^2. Regarding M_ν as the light neutrino mass matrix at low energy scale and varying the parameter I_e and the smallest light neutrino mass m_1, we find which parameter set (I_e, m_1) can lead to the QLC relation exactly and the results are presented in Table I. We note that the required values for I_e are negative. The first and the second column in Table I correspond to the minimal unification and realistic case, respectively. In our analysis, we have also checked that θ_{23} is almost unchanged, whereas the shift of θ_{13} is about 1° for both cases (A) and (B). From Table I, we see that a larger value of I_e is required to achieve the relation (1) as m_1 goes down.

How can we obtain such a value of I_e while keeping $|I_e| >> |I_{\mu,\tau}|$? As shown in Ref. [18], it requires the existence of new states which gives a dominant contributions to I_e. In MSSM, it can easily be realized by taking into account the contribution of chargino (pure W-ino). In the case of a diagonal slepton mass matrix in the same basis where the charged lepton mass matrix is diagonal, the contribution is presented by [18]

$$I_f \simeq \frac{g^2}{32\pi^2}\left(-\frac{1}{x_f}+\frac{x_f^2-1}{x_f^2}\ln(1-x_f)-(f\to\mu)\right) \quad (17)$$

where $x_f = 1-(M_f/\tilde{m})^2$ and $M_f, \tilde{m}$ are the f-th charged slepton mass and W-ino mass, respectively. We see that the size of $|I_e|$ is about 10 times larger than that of $|I_{\mu,\tau}|$ for $M_e \sim 2M_{\mu,\tau}$, and the value of I_e becomes negative and of the order of $10^{-4} \sim 10^{-3}$ for $x_e \gtrsim 0.65$, which are required to achieve the exact QLC relation at low energy. We also note that the contribution of the tau Yukawa coupling Y_τ in MSSM to I_τ can be as much as I_e for large $\tan\beta$. Thus, our estimation is suitable for small $\tan\beta$ so that the contribution of Y_τ should be negligible compared to I_e.

In passing, we discuss a possible way to diminish the correction to QLC appeared in Eq. (11). While we have considered the case with symmetric form of fermion Yukawa matrices so far, non-symmetric types of Y_d and Y_l are generally allowed. Then, the term $V_d^T U_d$ in Eq. (13) is not trivial unity matrix in the non-symmetric Yukawa structure and it can generate additional correction to the solar neutrino mixing parameter $\sin\theta_{sol}$, which can diminish the correction to QLC. We have checked that $\sin\theta_{sol} \simeq \sin(\pi/4-\theta_C)$ for the case of the minimal unification (A) when $V_D^T U_d$ has the form in the leading order of λ,

$$V_d^T U_d = \begin{pmatrix} 1 & (1-\sqrt{2})\lambda & -(1-\sqrt{2})\lambda \\ -(1-\sqrt{2})\lambda & 1 & 0 \\ (1-\sqrt{2})\lambda & 0 & 1 \end{pmatrix} \quad (18)$$

For the realistic case (B), one can obtain the QLC relation by taking the matrix form (18) but replacing λ with $\lambda/3$. We note that in order to achieve the QLC relation at low energy scale in those cases, renormalization effects should be negligibly small. But, it is quite arbitrary to generate the form of $V_d^T U_d$ given above from some underlying symmetries or flavor structure.

In summary, we have discussed that the QLC relation, $\theta_{sol} + \theta_C = \frac{\pi}{4}$, can be interpreted as an evidence of the quark-lepton unification. In particular, we have found that the lepton mixing matrix derived from quark-lepton unification can lead to a shift of the complementarity relation at a high energy. While the renormalization group effects generally lead to additive contribution on top of the shift, we show that the threshold corrections which may exist in the supersymmetric standard model diminish it, so we can achieve the complementarity relation at a low energy. In addition, we commented on how to achieve the QLC relation by taking a non-symmetric form of the down Yukawa matrix.

The work of S.K.K was supported by BK21 program of the Ministry of Education in Korea. The work of C.S.K. was supported in part by CHEP-SRC Program, in part by Grant No. R02-2003-000-10050-0 from BRP of the KOSEF, in part by Grant No. F01-2004-000-10292-0 of KOSEF-NSFC International Collaborative Research Grant. The work of J.L. was supported by BK21 program of the Ministry of Education in Korea.

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