Neutrino Mass Matrix in the model with Non-Abelian Discrete Symmetry [1]

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1 Introduction

In recent years, the observation of solar [2, 3] and atmospheric [4] neutrino oscillations has dramatically improved our knowledge of neutrino masses and lepton mixing. The neutrino mass-squared differences $\Delta_{\rm sun}$ and $\Delta_{\rm atm}$, and the mixing angles $\tan^2\theta_{\rm sun}$ and $\sin^2 2\theta_{\rm atm}$, are now quite well determined. The third mixing angle, represented by the matrix element U_{e3} of the lepton mixing matrix U (MNS matrix [5]), is constrained to be small by the non-observation of neutrino oscillations at the CHOOZ experiment [6]. In spite of all this progress, the available information on neutrino masses and lepton mixing is not sufficient to uncover the mechanism of neutrino mass generation. In particular, we do not yet know whether the observed features of lepton mixing are due to some underlying flavour symmetry, or they are mere mathematical coincidences [7] of the seesaw mechanism [8]. This talk is based on a study of a special class of symmetries and of the consequences of their perturbative violation. We show the concrete model with D_4 symmetry that U_{e3} and $\cos 2\theta_{23}$ vanishes in section 2, and departures of these values from the symmetric limit will be discussed in section 3.

2 The model with D_4 symmetry

The D_4 is one of non-Abelian discrete symetries. The D_4 model [9] has three lepton families, including three right-handed neutrinos which enable the seesaw mechanism [8]. We generically denote e, μ, and τ by α; we have then three left-handed lepton doublets $D_{\alpha L}$, three right-handed charged-lepton singlets α_R, and three right-handed neutrino singlets $\nu_{\alpha R}$. The scalar sector consists of three Higgs doublets ϕ_1, ϕ_2, and ϕ_3, together with two *real* gauge singlets χ_1 and χ_2. The charge assignment of D_4 and extra Z_2 to these particles are presented in the table 1. In this table, $\mathbf{1}^{++}, \mathbf{1}^{+-}, \mathbf{1}^{-+}, \mathbf{1}^{--}$ and $\mathbf{2}$ are irreducible representations of D_4 where $\mathbf{1}^{++}$ is the trivial one dimensional reps. and $\mathbf{2}$ is the two dimensional reps., respectively. The above multiplets and charge assignment determines Yukawa Lagrangian. Corresponding neutrino Dirac and charged lepton mass matrix are given by

$$M_D = \begin{pmatrix} y_1\phi_1 & & \\ & y_2\phi_1 & \\ & & y_2\phi_1 \end{pmatrix}, \quad M_\ell = \begin{pmatrix} y_3\phi_1 & & \\ & y_4\phi_2 + y_5\phi_3 & \\ & & y_4\phi_2 - y_5\phi_3 \end{pmatrix} \tag{1}$$

and right-handed Majorana neutrino mass matrix is also given by

$$M_R = \begin{pmatrix} M & y_\chi\chi_1 & y_\chi\chi_2 \\ y_\chi\chi_1 & M' & \\ y_\chi\chi_2 & & M' \end{pmatrix}, \tag{2}$$

where y_1, y_2, y_3, y_4, y_5 and y_χ are the Yukawa couplings. Therefore, lepton mixings must result exclusively from a non-diagonal M_R, and its arise by virtue of the non-zero VEVs of $\chi_{1,2}$. Under the

[1] This work is based on the collaboration with Walter Grimus, Anjan S. Joshipura, Luís Lavoura, Hideyuki Sawanaka and Morimitsu Tanimoto [1].

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group	D_{eL}	$(D_{\mu L}, D_{\tau L})$	e_R	(μ_R, τ_R)	ν_{eR}	$(\nu_{\mu R}, \nu_{\tau R})$	Φ_1	Φ_2	Φ_3	(χ_1, χ_2)
D_4	$\mathbf{1^{++}}$	$\mathbf{2}$	$\mathbf{1^{++}}$	$\mathbf{2}$	$\mathbf{1^{++}}$	$\mathbf{2}$	$\mathbf{1^{++}}$	$\mathbf{1^{++}}$	$\mathbf{1^{-+}}$	$\mathbf{2}$
extra Z_2	$\mathbf{1}$	$\mathbf{1,1}$	$\mathbf{-1}$	$\mathbf{1,1}$	$\mathbf{-1}$	$\mathbf{-1,-1}$	$\mathbf{-1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1,1}$

Table 1: The charge assignment of D_4 and the extra Z_2 symmetry are presented.

assumptions of

$$W \sim |M|, |M'| \gg v \quad \text{where} \quad \langle 0|\chi_1|0\rangle = \langle 0|\chi_2|0\rangle \equiv \frac{W}{\sqrt{2}} \ , \tag{3}$$

[3] light Majorana neutrino mass matrix has the following form obtained by seesaw mechanism :

$$M_\nu = M_D M_R^{-1} M_D^T = \begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix} \ , \tag{4}$$

where X, A, B and C are the complex numbers. This forms of neutrino mass matrix in the basis where the charged lepton mass matrix is diagonal predicts vanishing U_{e3} and $\cos 2\theta_{23}$.

3 Non-zero U_{e3} and $\cos 2\theta_{23}$

Let us consider a general perturbation δM_ν to equation (4). The matrix δM_ν is a general complex symmetric matrix, but part of it can be absorbed through a redefinition of the parameters in equation (4). The remaining part can be written, without loss of generality, as

$$\delta M_\nu = \begin{pmatrix} 0 & \epsilon_1 & -\epsilon_1 \\ \epsilon_1 & \epsilon_2 & 0 \\ -\epsilon_1 & 0 & -\epsilon_2 \end{pmatrix} \ , \quad \epsilon \equiv \frac{\epsilon_1}{A} \ , \quad \epsilon' \equiv \frac{\epsilon_2}{B} \ , \tag{5}$$

The perturbation is controlled by two parameters, ϵ_1 and ϵ_2, which are complex and model-dependent. We want to study their effects perturbatively, i.e. we want to assume ϵ_1 and ϵ_2 to be small. One finds that, to first order in ϵ and ϵ', the only effect of the δM_ν in equation (5) is to generate non-zero U_{e3} and $\cos 2\theta_{23}$. The neutrino masses, as well as the solar angle, do not receive any corrections. The induced values of $|U_{e3}|$ and $|\cos 2\theta_{23}|$ are strongly correlated to neutrino mass hierarchies. This makes it possible to draw some general conclusions even if we do not know the magnitudes of ϵ, ϵ'. In Table 2, we give expressions and values for $|U_{e3}|$ and $|\cos 2\theta_{23}|$ in case of the hierarchical ($m_1 < m_2 < m_3$), inverted ($m_1 \approx m_2 \sim \sqrt{\Delta_{\rm atm}} \gg m_3$) and quasi-degenerate neutrino spectrum. CP conservation is assumed but we distinguish two different cases (a) the Dirac solar pair corresponding to $\sigma = \rho = 0$ and the Pseudo-Dirac solar pair with[4] $\rho = \pi/2, \sigma = 0$, where ρ and σ are the Majorana phases in neutrino mass matrix. We have also given approximate values in some cases assuming the common degenerate mass $m \sim 0.3$ eV.

Scattered plots of the predicted values for $|\cos 2\theta_{23}|$ and $|U_{e3}|$ are given in Figure 1 in the case of normal neutrino mass hierarchy. [5] CP conservation ($\rho = \sigma = 0$, real ϵ, ϵ') is assumed. Neutrino masses and θ_{12} do not receive any corrections at $O(\epsilon, \epsilon')$ and hence do not appreicablly change by perturbations. We therefore randomly varied these input parameters in the experimentally allowed regions. m_1 was varied upto m_2. On the other hand, ϵ, ϵ' are unkown unless the symmetry breaking

[3] These assumptions are easily realized in the context of seesaw models.
[4] The physically different case with $\rho = 0, \sigma = \pi/2$ has similar results.
[5] Scattered plots for the other types of hierarchies and CP violating phases are presented in [1].

Normal Hierarchy	$\|U_{e3}\| \approx c_{12}s_{12}\sqrt{\frac{\Delta_\odot}{\Delta_{\rm atm}}}(\epsilon + \frac{\epsilon'}{2}) \approx 0.09(\epsilon + \frac{\epsilon'}{2})$
$m_1 \ll m_2; m_2^2 \approx \Delta_{\rm sun}; m_3^2 \approx \Delta_{\rm atm}$	$\|\cos 2\theta_{23}\| \approx \epsilon'$
Inverted Hierarchy	$\|U_{e3}\| \approx \frac{\Delta_{\rm sun}}{2\Delta_{\rm atm}} s_{12}c_{12}(\epsilon - \frac{\epsilon'}{2}) \approx 0.009(\epsilon - \frac{\epsilon'}{2})$
$\sigma = 0; \rho = 0$	$\|\cos 2\theta_{23}\| \approx \epsilon'$
$\sigma = 0; \rho = \pi/2$	$\|U_{e3}\| \approx \frac{1}{2}\sin 4\theta_{12}(\epsilon - \frac{\epsilon'}{2}) \approx 0.4(\epsilon - \frac{\epsilon'}{2})$
	$\|\cos 2\theta_{23}\| \approx 2(\epsilon \sin^2 2\theta_{12} + \frac{\epsilon'}{2}\cos^2 2\theta_{12})$
Quasi Degenerate	$\|U_{e3}\| \approx 2\epsilon' c_{12}s_{12}\frac{m^2}{\Delta_{\rm atm}}\frac{\Delta_\odot}{\Delta_{\rm atm}} \approx 1.6\epsilon$
$\sigma = 0; \rho = 0$	$\|\cos 2\theta_{23}\| \approx 4\frac{m^2}{\Delta_{\rm atm}}\epsilon' \approx 180\epsilon'$
$\sigma = 0; \rho = \pi/2$	$U_{e3} \approx 4\frac{m^2}{\Delta_{\rm atm}}c_{12}s_{12}(\epsilon s_{12}^2 + \frac{\epsilon'}{2}c_{12}^2) \approx 81(\epsilon s_{12}^2 + \frac{\epsilon'}{2}c_{12}^2)$
	$\|\cos 2\theta_{23}\| \approx 8\frac{m^2}{\Delta_{\rm atm}}c_{12}^2(\epsilon s_{12}^2 + \frac{\epsilon'}{2}c_{12}^2) \approx 259(\epsilon s_{12}^2 + \frac{\epsilon'}{2}c_{12}^2)$

Table 2: Leading order predictions for $|U_{e3}|$, $|\cos 2\theta_{23}|$ in case of different neutrino mass hierarchies with CP conservation. The numerical estimates are based on the best fit values of neutrino parameters and the quasi degenerate mass $m = 0.3$ eV.

is specified, so these are varied randomly in the range $-0.3 \sim 0.3$ with the condition that the output parameters should lie in the 90% CL limit [3, 10]:

$$\begin{aligned} 0.33 \leq \tan^2\theta_{\rm sun} \leq 0.49\ , \qquad & 7.7\times10^{-5} \leq \Delta_{\rm sun} \leq 8.8\times10^{-5}\ {\rm eV}^2, \quad 90\%{\rm C.L.}\ , \\ 0.92 \leq \sin^2 2\theta_{\rm atm}\ , \qquad & 1.5\times10^{-3} \leq \Delta_{\rm atm} \leq 3.4\times10^{-3}\ {\rm eV}^2, \quad 90\%{\rm C.L.}\ . \end{aligned} \tag{6}$$

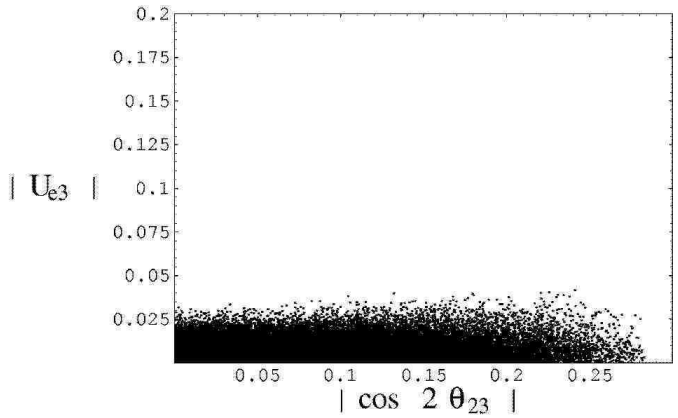

Figure 1: The scattered plots showing the allowed values of $|\cos 2\theta_{23}|$ and $|U_{e3}|$ in the case of the normal neutrino mass hierarchy. ϵ, ϵ' are randomly varied in the range $-0.3 \sim 0.3$ The Majorana phases are chosen as $\rho = 0,\ \sigma = 0$.

4 Conclusions

This talk was devoted to study of these parameters within a specific theoretical framework of D_4 symmetry. The vanishing of $|U_{e3}|$ and maximal atmospheric neutrino mixing angle was shown to follow from D_4 and extra Z_2 symmetry of the model. We showed that breaking of this can be characterized by two dimensionless parameters ϵ, ϵ' and we studied their effects perturbatively and numerically.

It was found that the magnitudes of $|U_{e3}|$ and $|\cos 2\theta_{23}|$ are strongly dependent upon the neutrino mass hierarchies and CP violating phases. The $|U_{e3}|$ gets strongly suppressed in case of the inverted

or quasi degenerate neutrino spectrum if $\rho = \sigma$ while similar suppression occurs in the case of normal hierarchy independent of this phase choice. The choice $\rho \neq \sigma$ can lead to a larger values ~ 0.1 for $|U_{e3}|$ which could be close to the experimental value in some cases with inverted or quasi degenerate spectrum, see Figures (5) and (12). In contrast, the $|\cos 2\theta_{23}|$ could be large, near its present experimental limit in most cases studied. For the normal and inverted mass spectrum, the magnitude of $|\cos 2\theta_{23}|$ is similar to the magnitudes of the perturbations ϵ, ϵ' while it can get enhanced compared to them if the neutrino spectrum is quasi-degenerate.

Neutrino models with other non-Abelian discrete symmetry are also possible. Recently, we proposed a neutrino model that based on quaternion group [11].

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References

[1] W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka and M. Tanimoto, hep-ph/0408123, to be published in Nucl. Phys. **B**.

[2] Super-Kamiokande Collaboration (S. Fukuda et al.), Phys. Rev. Lett. **86** (2001) 5651, 5656; SNO Collaboration (Q. R. Ahmad et al.), Phys. Rev. Lett. **87** (2001) 071301; ibid. **89** (2002) 011301; ibid. **89** (2002) 011302; nucl-ex/0309004.

[3] KamLAND Collaboration (K. Eguchi et al.), Phys. Rev. Lett. **90** (2003) 0212021; KamLAND Collaboration (T. Araki et al.), hep-ex/0406035.

[4] Super-Kamiokande Collaboration (Y. Fukuda et al), Phys. Rev. Lett. **81** (1998) 1562; ibid. **82** (1999) 2644; ibid. **82** (1999) 5194.

[5] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.

[6] CHOOZ Collaboration (M. Apollonio et al.), Phys. Lett. **B466** (1999) 415.

[7] A. Yu. Smirnov, Phys. Rev. **D48** (1993) 3264; G. Dutta and A. S. Joshipura, Phys. Rev. **D51** (1995) 3838; M. Honda, S. Kaneko, and M. Tanimoto, Phys. Lett. **B593** (2004) 165; I. Dorsner and A. Yu. Smirnov, Nucl. Phys. **B698** (2004) 386.

[8] T. Yanagida, in *Proceedings of the workshop on unified theories and baryon number in the universe (Tsukuba, Japan, 1979)*, eds. O. Sawada and A. Sugamoto (Tsukuba: KEK report 79-18, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, eds. D.Z. Freedman and F. van Nieuwenhuizen (Amsterdam: North Holland, 1979).

[9] W. Grimus and L. Lavoura, Phys. Lett. B 572 (2003) 189.

[10] M. Maltoni, T. Schwetz, M. Tórtola, and J.W.F. Valle, Phys. Rev. **D68** (2003) 113010.

[11] M. Frigerio, S. Satoru, E. Ma and M. Tanimoto, Phys. Rev. **D71** (2005) 011901.