Family Asymmetry and Neutrino Matrix

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YITP workshop, Jan. 13/2005

- Seesaw model is an attractive model as a baryogenesis model which may explain neutrino mass.

- There are many CP violating phases which are not throughly studied. In the leptogenesis scenario, the total lepton number is given by a sum of a family number. $L = L_e + L_\mu + L_\tau$. The family asymmetries $L_{e,\mu,\tau}$ vary by changing CP phases and depend on the form of the Dirac matrix and Majorana mass of the model.

- If low energy CP violation Δ in neutrino oscillation is non-zero, what does it imply CP violation in the context of the seesaw ? In this work,

- A method which gives MNS matrix, $\Delta \sim$ Jarlskog invariant, light neutrino mass eigenvalues, for a given set of heavy Majorana masses and m_D.
  With the formulae, one can study the relation of CP

violation of low energy and leptogenesis (family asymmetries) in general. Also one can impliment the constraints from the lepton mixings and neutrino mass squared differences of experiments.

- As example which shows strong correlation, one can find one family dominant leptogenesis (eg. $L = L_\tau$). We identify the two zero textures which leads to the one family dominant leptogenesis scenario.

- As example which shows there is no correlation, we find the case that that even if all the lepton asymmetries are Null i.e., $L = L_e = L_\mu = L_\tau = 0$, the low energy CP violation of neutrino oscillation Δ is non-zero. We identify the texture which has such property and study the unitarity triangle of the leptonic matrix for this case.

Notation and Framework:

We apply the method to the minimal seesaw model with two heavy Majorana neutrino.(a massless light neutrino)

Dirac mass term:

$$\left(\mathbf{m_{D1}},\ \mathbf{m_{D2}} \right) = \begin{pmatrix} m_{De1} & m_{De2} \\ m_{D\mu 1} & m_{D\mu 2} \\ m_{D\tau 1} & m_{D\tau 2} \end{pmatrix} = \left(\mathbf{u_1}, \mathbf{u_2} \right) \begin{pmatrix} m_{D1} & 0 \\ 0 & m_{D2} \end{pmatrix}$$

Two unit vectors: $\mathbf{u_i} = \frac{\mathbf{m_{Di}}}{m_{Di}}$ and $m_{Di} = |\mathbf{m_{Di}}|$:

$$\mathbf{u_1} = \begin{pmatrix} u_{e1} \\ u_{\mu 1} \\ u_{\tau 1} \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} u_{e2} \\ u_{\mu 2} \\ u_{\tau 2} \end{pmatrix}$$

Heavy Majorana masses $M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$

Ex:

$$-m_{eff} = m_D \frac{1}{M} m_D^T = \mathbf{u_1 u_1^T} X_1 + \mathbf{u_2 u_2^T} X_2$$

with $X_1 = \frac{m_{D1}^2}{M_1}$ $X_2 = \frac{m_{D2}^2}{M_2}$ (two mass scales)

$$|(m_{eff})_{ee}| = |u_{e1}^2 X_1 + u_{e2}^2 X_2|$$

Light neutrino mass spectra (n_1, n_2, n_3) :

$$\det(m_{eff} m_{eff}^{\dagger} - n^2) = 0$$

Normal hierarchy $n_1 = 0$ $n_2^2 = m_{sol}^2, n_3^2 = m_{sol}^2 + m_{atm}^2$

$$\begin{aligned} n_2^2 + n_3^2 &= X_1^2 + X_2^2 + 2X_1 X_2 Re\left((\mathbf{u_1^{\dagger}} \cdot \mathbf{u_2})^{\mathbf{2}} \right) \\ n_2 n_3 &= X_1 X_2 (1 - |\mathbf{u_1^{\dagger}} \cdot \mathbf{u_2}|^2) \end{aligned}$$

$(\mathbf{u_1^{\dagger}} \cdot \mathbf{u_2}) = \cos(\beta) \exp(i\gamma_R)$

Two extreme cases:

(1) $\mathbf{u_1} \perp \mathbf{u_2} \quad \mathbf{u_1}^\dagger \cdot \mathbf{u_2} = 0 \rightarrow$

$$n_2 = Min(X_1, X_2) \quad n_3 = Max(X_1, X_2)$$

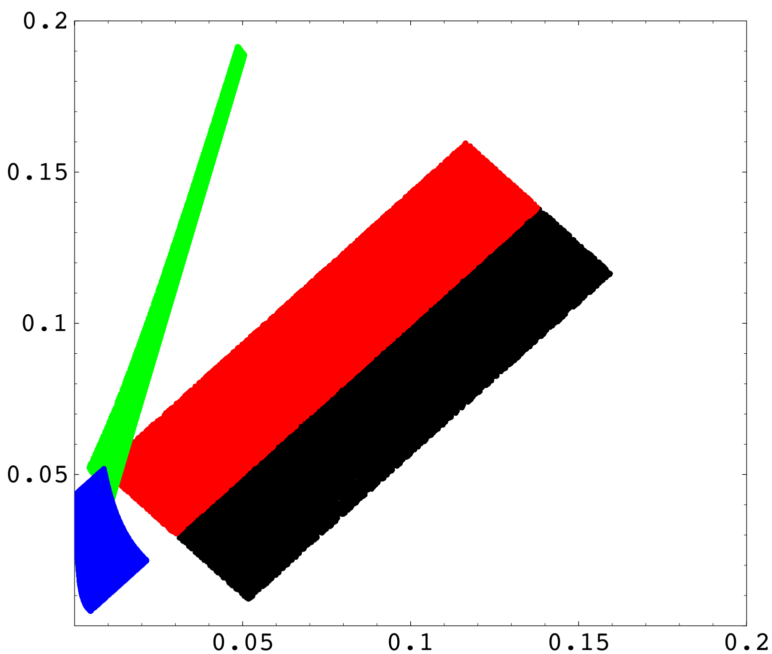
(2) $\mathbf{u_1} // \mathbf{u_2} \quad \mathbf{u_1}^\dagger \cdot \mathbf{u_2} = \exp(i\gamma_R)$

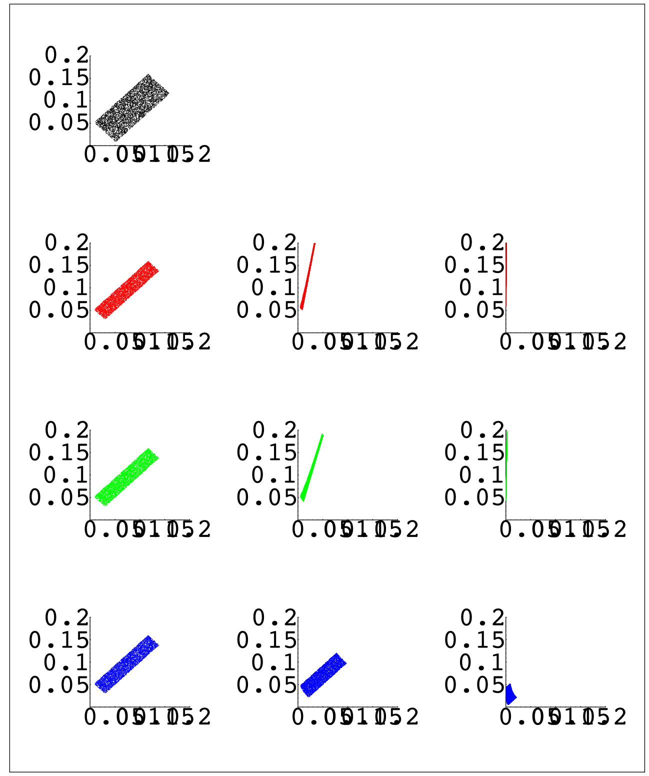
$$n_2 = 0$$

$$n_3 = \sqrt{X_1^2 + X_2^2 + 2X_1X_2\cos[2\gamma_R]}$$

$$= |X_1 - X_2| \quad \gamma_R = \frac{\pi}{2}$$

$$= X_1 + X_2 \quad \gamma_R = 0$$





Randomly generate (X_1, X_2) (black) :with a set of (γ_R, β) a pair of masses (n_2, n_3) are predicted. From left to right $\beta = \frac{\pi}{2} \sim 0$, From top to bottom $\gamma_R = 0 \sim \frac{\pi}{2}$.

CP violation:

Low energy CP violation in neutrino oscillation;

$$\begin{aligned}\Delta &= J(n_1^2 - n_2^2)(n_2^2 - n_3^2)(n_3^2 - n_1^2) \\ &= \mathrm{Im}\left((m_{eff} m_{eff}^\dagger)_{e\mu}(m_{eff} m_{eff}^\dagger)_{\mu\tau}(m_{eff} m_{eff}^\dagger)_{\tau e}\right)\end{aligned}$$

$$J = Im\left(U_{e1}^{MNS} U_{e2}^{MNS*} U_{\mu 1}^{MNS*} U_{\mu 2}^{MNS}\right)$$

Two ways to compute Δ

1) Construct U_{MNS} in terms of m_D, M and get J.

2) Compute the combination of the $m_{eff} = -m_D \frac{1}{M} m_D^T$

$$
\begin{aligned}
\Delta = \quad & \left(1-|\mathbf{u_1^\dagger}\cdot\mathbf{u_2}|^{\mathbf{2}}\right)\times \\
& \Big(X_1^4X_2^2\left(\mathrm{Im}[\left(u_{e1}^*u_{e2}u_{\mu1}u_{\mu2}^*\right)|u_{\tau1}|^2+\left(u_{\mu1}^*u_{\mu2}u_{\tau1}u_{\tau2}^*\right)|u_{e1}|^2+\left(u_{\tau1}^*u_{\tau2}u_{e1}u_{e2}^*\right)|u_{\mu1}|^2]\right) \\
& +X_1^3X_2^3\Big(\mathrm{Im}[(u_{e1}^*u_{e2})(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})(|u_{\tau1}u_{\mu2}|^2-|u_{\mu1}u_{\tau2}|^2) \\
& \qquad +(u_{\mu1}^*u_{\mu2})(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})(|u_{e1}u_{\tau2}|^2-|u_{\tau1}u_{e2}|^2) \\
& \qquad +(u_{\tau1}^*u_{\tau2})(\mathbf{u_1^\dagger}\cdot\mathbf{u_2})(|u_{\mu1}u_{e2}|^2-|u_{e1}u_{\mu2}|^2)]\Big) \\
& -X_1^2X_2^4\left(\mathrm{Im}[\left(u_{e1}^*u_{e2}u_{\mu1}u_{\mu2}^*\right)|u_{\tau2}|^2+\left(u_{\mu1}^*u_{\mu2}u_{\tau1}u_{\tau2}^*\right)|u_{e2}|^2+\left(u_{\tau1}^*u_{\tau2}u_{e1}u_{e2}^*\right)|u_{\mu2}|^2]\right)\Big)
\end{aligned}
$$

Two zero textures with $u_{e1}=0, u_{\mu2}=0$ or $u_{e2}=0, u_{\mu1}=0$ correspond to $\mathbf{u_1^\dagger}\cdot\mathbf{u_2}=u_{1\tau}^*u_{2\tau}$

$$
m_D=\begin{pmatrix} 0 & * \\ * & 0 \\ * & * \end{pmatrix} \; or \; \begin{pmatrix} * & 0 \\ 0 & * \\ * & * \end{pmatrix} \qquad \Delta=(1-|u_{\tau1}u_{\tau2}|^2)X_1^3X_2^3 Im[(u_{\tau1}^*u_{\tau2})^2](|u_{\mu1}u_{e2}|^2-|u_{e1}u_{\mu2}|^2).
$$

This case corresponds to τ leptogenesis $L=L_\tau$ and $\Delta\sim L_\tau$.

Two zero textures classification with $\mathbf{u_1}^\dagger \cdot \mathbf{u_2} = u_{a1}^* u_{a2}$ $(a = e, \mu, \tau)$. All the cases show one to one correlation bet. one family dominant leptogenesis and low energy CP. i.e., $L = L_a$ and $\Delta \sim L_a$.

(1) **e-leptogenesis**: $L = L_e$

$$\begin{pmatrix} * & * \\ * & 0 \\ 0 & * \end{pmatrix} \text{ or } \begin{pmatrix} * & * \\ 0 & * \\ * & 0 \end{pmatrix}$$

(2) μ **leptogenesis** : $L = L_\mu$

$$\begin{pmatrix} 0 & * \\ * & * \\ * & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & * \\ * & * \\ * & 0 \end{pmatrix}.$$

Neutrino mixings:U_{MNS} $\quad U^\dagger_{MNS} m_{eff} U^*_{MNS}$

1)Isolate zero mass state by unitary rotation U

2) solve two by two diagonalization $K \to U_{MNS} = UK$

$$U_{MNS} = \begin{pmatrix} \frac{\mathbf{u}_2^* \times \mathbf{u}_1^*}{\sqrt{1-|\mathbf{u}_1^\dagger \cdot \mathbf{u}_2|^2}} & \frac{\mathbf{u}_2 - (\mathbf{u}_1^\dagger \cdot \mathbf{u}_2)\mathbf{u}_1}{\sqrt{1-|\mathbf{u}_1^\dagger \cdot \mathbf{u}_2|^2}} & \mathbf{u}_1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & s_\theta e^{(-i\phi)} \\ 0 & -s_\theta e^{i\phi} & c_\theta \end{pmatrix} P$$

$$K^\dagger U^\dagger m_{eff} U^* K^* = -K^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix} K^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3 \end{pmatrix}$$

$$U_{MNS} = \begin{pmatrix} \frac{\mathbf{u_2^*} \times \mathbf{u_1^*}}{\sqrt{1 - |\mathbf{u_1^\dagger} \cdot \mathbf{u_2}|^2}} & \frac{\mathbf{u_2} - (\mathbf{u_1^\dagger} \cdot \mathbf{u_2})\mathbf{u_1}}{\sqrt{1 - |\mathbf{u_1^\dagger} \cdot \mathbf{u_2}|^2}} & \mathbf{u_1} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & s_\theta e^{(-i\phi)} \\ 0 & -s_\theta e^{i\phi} & c_\theta \end{pmatrix} P$$

$$\phi = arg.(X_1(\mathbf{u_1^\dagger} \cdot \mathbf{u_2})^* + X_2(\mathbf{u_1^\dagger} \cdot \mathbf{u_2}))$$

$$\tan 2\theta = \frac{2X_2\sqrt{1 - |\mathbf{u_1^\dagger} \cdot \mathbf{u_2}|^2}|X_1(\mathbf{u_1^\dagger} \cdot \mathbf{u_2})^* + X_2\mathbf{u_1^\dagger} \cdot \mathbf{u_2}|}{X_1^2 + X_2^2(2|\mathbf{u_1^\dagger} \cdot \mathbf{u_2}|^2 - 1) + 2X_1X_2\mathrm{Re.}(\mathbf{u_1^\dagger} \cdot \mathbf{u_2})^2}$$

Special case with $\mathbf{u_1^\dagger \cdot u_2 = 0}$

No leptogenesis $Im[(u_1^\dagger \cdot u_2)^2] = 0$.

$$(n_1, n_2, n_3) = (0, X_2, X_1)$$
$$U_{MNS} = \left(\mathbf{u_2^* \times u_1^*},\ \ \mathbf{u_2},\ \ \mathbf{u_1} \right)$$

By solving the constraint $\mathbf{u_1^\dagger \cdot u_2 = 0}$, which reduces the numbers of parameters in $\mathbf{u_1, u_2}$ from (4 real + 3 imaginary(CP)) to (3 real + 2 imaginary). We impose two mixing angles from experiments. Then, MNS is parametrized by (1 real + 2 imag.).

$$u_{e1} = s_{13}\ ,\ \mathrm{arg}(u_{e2}) = \theta_e,\ \mathrm{Im}(u_{\mu 2}\exp(-i\theta_e)) = x.$$

$$U_{MNS} =$$
$$\begin{pmatrix} (\mp\sqrt{\frac{2}{3} - 2x^2} - i\sqrt{2}x)c_{13} & \sqrt{\frac{1}{3}}e^{i\theta_e}c_{13} & s_{13} \\ * & (\mp\sqrt{\frac{1}{3} - x^2} - \sqrt{\frac{1}{6}}s_{13} + ix)e^{i\theta_e} & \sqrt{\frac{1}{2}}c_{13} \\ * & (\pm\sqrt{\frac{1}{3} - x^2} - \sqrt{\frac{1}{6}}s_{13} - ix)e^{i\theta_e} & \sqrt{\frac{1}{2}}c_{13} \end{pmatrix}$$

$$\begin{aligned} \Delta = &\quad X_1^2 X_2^2 (X_1^2 - X_2^2) Im\left(u_{\tau 1}^* u_{\tau 2} u_{e1} u_{e2}^*\right) \\ = &\quad n_2^2 n_3^2 (n_3^2 - n_2^2)\sqrt{\frac{1}{2}} s_{13} c_{13} \sqrt{\frac{1}{3}} \times (-x) \end{aligned}$$

$$|(m_{eff})_{ee}| = |s_{13}^2 X_1 + \frac{1}{3} c_{13}^2 \exp(2i\theta_e) X_2| \quad |U_{e3}^{MNS}| = s_{13}$$

Unitarity triangle of the model

$$U_{e2}^{MNS}U_{e3}^{MNS*}+U_{\mu 2}^{MNS}U_{\mu 3}^{MNS*}+U_{\tau 2}^{MNS}U_{\tau 3}^{MNS*}=0 \quad (1)$$

$$\sqrt{\frac{1}{3}}c_{13}s_{13}+ \quad \sqrt{\frac{1}{2}}c_{13}\left(\mp\sqrt{\frac{1}{3}-x^2}-\sqrt{\frac{1}{6}}s_{13}+ix\right)+$$

$$\sqrt{\frac{1}{2}}c_{13}\left(\pm\sqrt{\frac{1}{3}-x^2}-\sqrt{\frac{1}{6}}s_{13}-ix)\right).$$

Summary

- The method to compute the low energy CP violation Δ and MNS matrix in terms of m_D and M are presented. (c.f. Endoh, Kaneko, Kang, Tanimoto and T.M. PRL.89(2002))

- We have shown Δ for (3,2) seesaw model in the most general case.

- For family dominant leptogenesis $L = La$ model with two zero texture for m_D, it shows strong corrlation bet. low energy CP and leptogenesis. (c.f. Frampton,Glashow ,Yanagida PLB.548 (2002)) $L = L_a$ and $\Delta \sim La$

- Another model with no-leptogenesis and non-zero low energy CP is worked out.

- It turns out that there are many different scenarios for leptogenesis, if we looked into the lepton flavor composition of the total $L \sim -B$ numbers. (c.f. Endoh, Xiong, Morozumi PTP.1(2004).

- How to verify these differences is very challenging.