Induced CP violation in the Higgs sector of the MSSM

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I. Introduction

Higgs potential of the MSSM

$$V_0 = m_1^2\varphi_d^\dagger\varphi_d + m_2^2\varphi_u^\dagger\varphi_u + (m_3^2\varphi_u\varphi_d + \text{h.c.}) + \frac{g_2^2+g_1^2}{8}(\varphi_d^\dagger\varphi_d - \varphi_u^\dagger\varphi_u)^2 + \frac{g_2^2}{2}(\varphi_d^\dagger\varphi_d)(\varphi_u^\dagger\varphi_u)$$

m_3^2 can be made real and positive by rephasing φ_u or φ_d.

relative phase of φ_u and $\varphi_d \neq 0 \Longrightarrow$ CP violation

minimum of V_0:

$$\varphi_d = \begin{pmatrix} v_0\cos\beta \\ 0 \end{pmatrix}, \qquad \varphi_u = \begin{pmatrix} 0 \\ v_0\sin\beta \end{pmatrix}$$

with

$$m_1^2 = m_3^2\tan\beta - \frac{1}{2}m_Z^2\cos(2\beta), \quad m_2^2 = m_3^2\cot\beta + \frac{1}{2}m_Z^2\cos(2\beta)$$

$\therefore$ **No CP violation in the Higgs sector at the tree level** with radiative corrections,

CP can be spontaneously broken, but with too light boson

[Maekawa, N.P.Suppl.**37A** ('94)]

$$
\begin{aligned}
V_{\rm eff} &= \bar{m}_1^2\varphi_d^\dagger\varphi_d + \bar{m}_2^2\varphi_u^\dagger\varphi_u + (\bar{m}_3^2\varphi_u\varphi_d + \text{h.c}) \\
&\quad + \frac{\bar{\lambda}_1}{2}(\varphi_d^\dagger\varphi_d)^2 + \frac{\bar{\lambda}_2}{2}(\varphi_u^\dagger\varphi_u)^2 + \bar{\lambda}_3(\varphi_u^\dagger\varphi_u)(\varphi_d^\dagger\varphi_d) \\
&\quad + \bar{\lambda}_4(\varphi_u\varphi_d)(\varphi_u\varphi_d)^* \\
&\quad + \left[\frac{\bar{\lambda}_5}{2}(\varphi_u\varphi_d)^2 + (\bar{\lambda}_6\varphi_d^\dagger\varphi_d + \bar{\lambda}_7\varphi_u^\dagger\varphi_u)\varphi_u\varphi_d + \text{h.c}\right] \\
&= \frac{\bar{\lambda}_5}{2}\rho_1^2\rho_2^2\left[\cos\theta - \frac{2\bar{m}_3^2 + \bar{\lambda}_6\rho_1^2 + \bar{\lambda}_7\rho_2^2}{2\bar{\lambda}_5\rho_1\rho_2}\right]^2 + \theta\text{-indep.}
\end{aligned}
$$

if all the parameters are real, where

$$
\varphi_d = \frac{1}{\sqrt{2}}\begin{pmatrix}\rho_1 \\ 0\end{pmatrix}, \qquad \varphi_u = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ \rho_2 \mathrm{e}^{i\theta}\end{pmatrix}.
$$

$$
\left.\begin{array}{c} \bar{\lambda}_5 > 0 \\ \left|\frac{2\bar{m}_3^2+\bar{\lambda}_6\rho_1^2+\bar{\lambda}_7\rho_2^2}{2\bar{\lambda}_5\rho_1\rho_2}\right| < 1 \end{array}\right\} \Longrightarrow \text{CP is spontaneously violated}
$$

Since $\lambda_{5,6,7} = 0$ at the tree level and $\bar{\lambda}_{5,6,7} \sim O(10^{-3})$,

$\bar{m}_3^2 \sim O(10^{3-4})$GeV^2 for these conditions to be satisfied

$\Longrightarrow$ too light boson [cf. $m_A^{\rm tree} = \sqrt{2m_3^2/\sin(2\beta)}$]

At finite T, the parameters recieve T-dep. corrections.

For some set of the parameters in the MSSM, $\bar{m}_3^2(T)$ becomes very small at T near EWPT.

$\Longrightarrow \theta \sim O(1)$ for $\rho_1 \in (0, v_C \cos\beta_C)$, $\rho_2 \in (0, v_C \sin\beta_C)$

$\Longrightarrow$ EW baryogensis

KF, Kakuto, Otsuki & Toyoda, hep-ph/9903276

N.B.

- In general, the complex parameters μ, A, M_2 and M_1 in the MSSM, through radiative corrections, make $\bar{m}_3^2$ and $\bar{\lambda}_{5,6,7}$ *complex-valued*.
- Some combinations of $\delta_\mu = \mathrm{Arg}\mu$, $\delta_A = \mathrm{Arg}A$, $\delta_2 = \mathrm{Arg}M_2$, $\delta_1 = \mathrm{Arg}M_1$ and θ are constrained by n-EDM etc.

  e.g. chargino mass matrix

$$\begin{pmatrix} \tilde{W}^- & \tilde{\phi}_d^- \end{pmatrix} \begin{pmatrix} M_2 & -\frac{i}{\sqrt{2}} g_2 v_2 e^{-i\theta} \\ -\frac{i}{\sqrt{2}} g_2 v_1 & -\mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{\phi}_u^+ \end{pmatrix}$$

$$\xrightarrow{\text{rephasing}} \begin{pmatrix} |M_2| & -\frac{i}{\sqrt{2}} g_2 v_2 \\ -\frac{i}{\sqrt{2}} g_2 v_1 & -|\mu| \, e^{i(\theta+\delta_\mu+\delta_2)} \end{pmatrix}$$

$|d_n| < 10^{-25} e \cdot \mathrm{cm}$ [Kizukuri & Oshimo, P.R.**D46** ('92)]

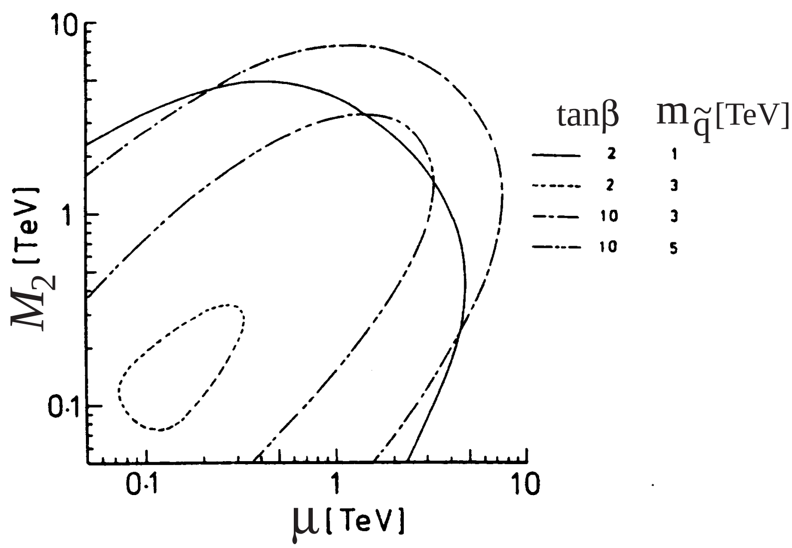


FIG. 4. The parameter regions compatible with the present experimental upper bound on the neutron EDM.

$\theta + \delta_\mu + \delta_2 = \pi/4$
$\mathrm{Arg}A = \pi/4$

inside is excluded

- (v_1, v_2, θ) must be determined dynamically from the minimum of V_{eff}.

II. Effective Potential

effective potential as a function of $\boldsymbol{v} \equiv (v_1, v_2, v_3)$ at the one-loop level

$$\varphi_d = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \varphi_u = \begin{pmatrix} 0 \\ v_2 + i v_3 \end{pmatrix}$$

$$V_{\rm eff}(\boldsymbol{v}) = V_0(\boldsymbol{v}) + \Delta V(\boldsymbol{v})$$

where

$$\Delta V(\boldsymbol{v}) = \Delta_g V(\boldsymbol{v}) + \Delta_t V(\boldsymbol{v}) + \Delta_{\tilde{t}} V(\boldsymbol{v}) + \Delta_{\chi^\pm} V(\boldsymbol{v}) + \Delta_{\chi^0} V(\boldsymbol{v})$$

with

$$\begin{aligned}
\Delta_g V(\boldsymbol{v}) &= 3 \cdot 2F\left(m_W^2(\boldsymbol{v})\right) + 3F\left(m_Z^2(\boldsymbol{v})\right), \\
\Delta_t V(\boldsymbol{v}) &= -4 \cdot 3 \cdot F(m_t^2(\boldsymbol{v})), \\
\Delta_{\tilde{t}} V(\boldsymbol{v}) &= 2 \cdot 3 \cdot \sum_{a=1,2} F\left(m_{\tilde{t}_a}^2(\boldsymbol{v})\right), \\
\Delta_{\chi^\pm} V(\boldsymbol{v}) &= -4 \sum_{a=1,2} F\left(m_{\chi_a^\pm}^2(\boldsymbol{v})\right), \\
\Delta_{\chi^0} V(\boldsymbol{v}) &= -2 \sum_{a=1,2,3,4} F\left(m_{\chi_a^0}^2(\boldsymbol{v})\right)
\end{aligned}$$

and

$$\begin{aligned}
F(m^2) &\equiv \frac{m^4}{64\pi^2}\left(\log\frac{m^2}{M_{\rm ren}^2} - \frac{3}{2}\right), \\
m_A^2(\boldsymbol{v}) &= \text{(mass)}^2\text{-eigenvalue}
\end{aligned}$$

$$m_W^2 = \frac{g_2^2}{4}(v_1^2+v_2^2+v_3^2), \quad m_Z^2 = \frac{g_2^2+g_1^2}{4}(v_1^2+v_2^2+v_3^2),$$

$$m_t^2 = \frac{y_t^2}{2}(v_2^2+v_3^2)$$

mass matrices with complex parameters:

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + \frac{3g_2^2-g_1^2}{12}(v_1^2-v_2^2-v_3^2) & \frac{y_t}{\sqrt{2}}\left[\mu v_1 + A_t(v_2 - iv_3)\right] \\ \frac{y_t}{\sqrt{2}}\left[\mu v_1 + A_t(v_2 + iv_3)\right] & m_{\tilde{t}_R}^2 + m_t^2 + \frac{g_1^2}{6}(v_1^2-v_2^2-v_3^2) \end{pmatrix},$$

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & -\frac{i}{\sqrt{2}}g_2(v_2 - iv_3) \\ -\frac{i}{\sqrt{2}}g_2 v_1 & -\mu \end{pmatrix},$$

$$M_{\chi^0} = \begin{pmatrix} M_2 & 0 & -\frac{i}{2}g_2 v_1 & \frac{i}{2}g_2(v_2 - iv_3) \\ 0 & M_1 & \frac{i}{2}g_1 v_1 & -\frac{i}{2}g_1(v_2 - iv_3) \\ -\frac{i}{2}g_2 v_1 & \frac{i}{2}g_1 v_1 & 0 & \mu \\ \frac{i}{2}g_2(v_2 - iv_3) & -\frac{i}{2}g_1(v_2 - iv_3) & \mu & 0 \end{pmatrix}$$

At finite temperature,

$$F(m^2) \longrightarrow F(m^2) \pm \frac{T^4}{2\pi^2}\int_0^\infty dx\, x^2 \, \log\left(1 \mp e^{-\sqrt{x^2+m^2/T^2}}\right)$$

input

$m_W = 80.4\text{GeV},\ m_Z = 91.2\text{GeV},\ m_t = 175\text{GeV},$

$v_0 = 246\text{GeV},\ \tan\beta$

$m_3^2,\ m_{\tilde{t}_L},\ m_{\tilde{t}_R} \in \mathbf{R};\qquad \mu,\ M_2 = M_1,\ A_t \in \mathbf{C}$

m_1^2 and m_2^2 in V_0; $\quad\longleftarrow\quad \frac{\partial V_{\text{eff}}}{\partial v_1} = \frac{\partial V_{\text{eff}}}{\partial v_2} = 0$

$$\begin{cases} m_1^2 = m_3^2 \tan\beta - \dfrac{1}{2} m_Z^2 \cos(2\beta) - \dfrac{1}{v_1}\dfrac{\partial \Delta V(\boldsymbol{v})}{\partial v_1} \\ m_2^2 = m_3^2 \cot\beta + \dfrac{1}{2} m_Z^2 \cos(2\beta) + \dfrac{1}{v_2}\dfrac{\partial \Delta V(\boldsymbol{v})}{\partial v_2} \end{cases}$$

$\delta_\mu,\ \delta_2 = \delta_1,\ \delta_A = 0$ or 10^{-3}

output

- location $\boldsymbol{v}_{\text{min}}$ of the minimum of $V_{\text{eff}}(\boldsymbol{v}) \longrightarrow \theta$
  $\Longleftarrow$ numerical search in the order-parameter space $\boldsymbol{v}$
- masses of the neutral Higgs bosons
  $\Longleftarrow$ eigenvalues of $\left(\left.\dfrac{\partial^2 V_{\text{eff}}(\boldsymbol{v})}{\partial v_i \partial v_j}\right|_{\boldsymbol{v}_{\text{min}}} \right)$

• Effective parameters — polynomial approx. to $V_{\rm eff}$

Among $\bar{m}^2_{1,2,3}$ and $\bar{\lambda}_{1,2,3,4,5,6,7}$, those relevant to CP violation are

$$\bar{m}_3^2 = -\frac{\partial^2 V_{\rm eff}}{\partial v_1 \partial v_2}\bigg|_0 = m_3^2 + \Delta_\chi m_3^2 + \Delta_{\tilde{t}} m_3^2,$$

$$\bar{\lambda}_5 = \frac{1}{2}\left(\frac{\partial^4 V_{\rm eff}}{\partial v_1^2 \partial v_2^2}\bigg|_0 - \frac{\partial^4 V_{\rm eff}}{\partial v_1^2 \partial v_3^2}\bigg|_0\right) = \Delta_\chi \lambda_5 + \Delta_{\tilde{t}} \lambda_5,$$

$$\bar{\lambda}_6 = -\frac{1}{3}\frac{\partial^4 V_{\rm eff}}{\partial v_1^3 \partial v_2}\bigg|_0 = \Delta_\chi \lambda_6 + \Delta_{\tilde{t}} \lambda_6,$$

$$\bar{\lambda}_7 = -\frac{1}{3}\frac{\partial^4 V_{\rm eff}}{\partial v_1 \partial v_2^3}\bigg|_0 = \Delta_\chi \lambda_7 + \Delta_{\tilde{t}} \lambda_7,$$

◇ chargino and neutralino contributions $(M_2 = M_1)$

$$\Delta_\chi m_3^2 = 2g_2^2\left(1 + \frac{1}{\cos^2\theta_W}\right)\mu M_2 \cdot i\int_k \Delta_1(k)\Delta_\mu(k),$$

$$\Delta_\chi \lambda_5 = -2g_2^4\left(1 + \frac{1}{\cos^4\theta_W}\right)(\mu M_2)^2 \cdot i\int_k \Delta_1^2(k)\Delta_\mu^2(k),$$

$$\Delta_\chi \lambda_6 = 2g_2^4\left(1 + \frac{1}{\cos^4\theta_W}\right)\mu M_2 \cdot i\int_k k^2\Delta_1^2(k)\Delta_\mu^2(k),$$

$$= \Delta_\chi \lambda_7,$$

where

$$\Delta_1(k) = \frac{1}{k^2 - |M_2|^2}, \qquad \Delta_\mu(k) = \frac{1}{k^2 - |\mu|^2}$$

◇ stop contributions

$$\Delta_{\tilde{t}} m_3^2 = -3y_t^2 (\mu A_t^*) \cdot i \int_k \Delta_L(k) \Delta_R(k),$$

$$\Delta_{\tilde{t}} \lambda_5 = 3y_t^4 (\mu A_t^*)^2 \cdot i \int_k \Delta_L^2(k) \Delta_R^2(k),$$

$$\Delta_{\tilde{t}} \lambda_6 = -3y_t^2 (\mu A_t^*) \cdot i \int_k \Big[\left(\frac{g_2^2}{4} - \frac{g_1^2}{12} \right) \Delta_L^2(k) \Delta_R(k) + \frac{g_1^2}{3} \Delta_L(k) \Delta_R^2(k) + y_t^2 |\mu|^2 \Delta_L^2(k) \Delta_R^2(k) \Big],$$

$$\Delta_{\tilde{t}} \lambda_7 = -3y_t^2 (\mu A_t^*) \cdot i \int_k \Big\{ \left[y_t^2 - \left(\frac{g_2^2}{4} - \frac{g_1^2}{12} \right) \right] \Delta_L^2(k) \Delta_R(k) + \left(y_t^2 - \frac{g_1^2}{3} \right) \Delta_L(k) \Delta_R^2(k) + y_t^2 |A_t|^2 \Delta_L^2(k) \Delta_R^2(k) \Big\},$$

where

$$\Delta_L(k) = \frac{1}{k^2 - m_{\tilde{t}_L}^2}, \qquad \Delta_R(k) = \frac{1}{k^2 - m_{\tilde{t}_R}^2}.$$

Hence,

$$\begin{aligned}
\bar{m}_3^2 &= m_3^2 + e^{i(\delta_\mu+\delta_2)}\left|\Delta_\chi m_3^2\right| + e^{i(\delta_\mu-\delta_A)}\left|\Delta_{\tilde{t}} m_3^2\right|, \\
\bar{\lambda}_5 &= e^{2i(\delta_\mu+\delta_2)}\left|\Delta_\chi \lambda_5\right| + e^{2i(\delta_\mu-\delta_A)}\left|\Delta_{\tilde{t}} \lambda_5\right|, \\
\bar{\lambda}_6 &= e^{i(\delta_\mu+\delta_2)}\left|\Delta_\chi \lambda_6\right| + e^{i(\delta_\mu-\delta_A)}\left|\Delta_{\tilde{t}} \lambda_6\right|, \\
\bar{\lambda}_7 &= e^{i(\delta_\mu+\delta_2)}\left|\Delta_\chi \lambda_7\right| + e^{i(\delta_\mu-\delta_A)}\left|\Delta_{\tilde{t}} \lambda_7\right|
\end{aligned}$$

N.B.

- $V_{\text{eff}} \sim \bar{m}_3^2 \varphi_u \varphi_d, \quad \bar{\lambda}_5 (\varphi_u \varphi_d)^2, \quad (\bar{\lambda}_6 \varphi_d^\dagger \varphi_d + \bar{\lambda}_7 \varphi_u^\dagger \varphi_u) \varphi_u \varphi_d$

  $\Longrightarrow$ In general, only one of these complex parameters can be made real.

  Then the remaining phases are naively $O(\delta_\mu + \delta_2)$ or $O(\delta_\mu - \delta_A)$

- If $|\Delta_\chi| \gg |\Delta_{\tilde{t}}|$, by rephasing $\varphi_{u,d}$,

$$\begin{aligned}
\lambda_{5,6,7} &\longmapsto \text{real}, \\
\bar{m}_3^2 &\longmapsto m_3^2 e^{-i(\delta_\mu+\delta_2)} + \left|\Delta_\chi m_3^2\right| \equiv e^{-i\delta}\left|\bar{m}_3^2\right|,
\end{aligned}$$

  where

$$\tan\delta = -\frac{m_3^2 \sin(\delta_\mu + \delta_2)}{m_3^2 \cos(\delta_\mu + \delta_2) + |\Delta_\chi m_3^2|}$$

  If $\left|m_3^2 \cos(\delta_\mu + \delta_2) + \left|\Delta_\chi m_3^2\right|\right| \ll 1$, δ becomes O(1).

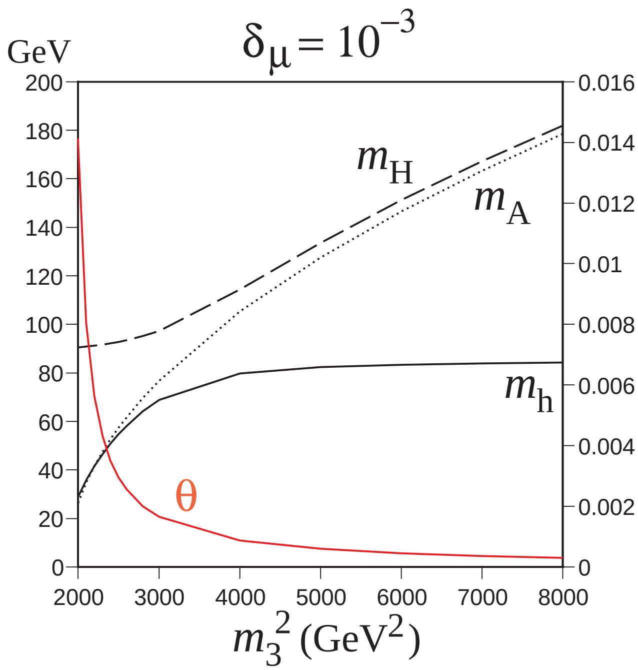
$$\Longleftarrow \left\{ \begin{array}{l} \text{light (pseudo)scalar} \\ \text{near the EWPT} \end{array} \right.$$

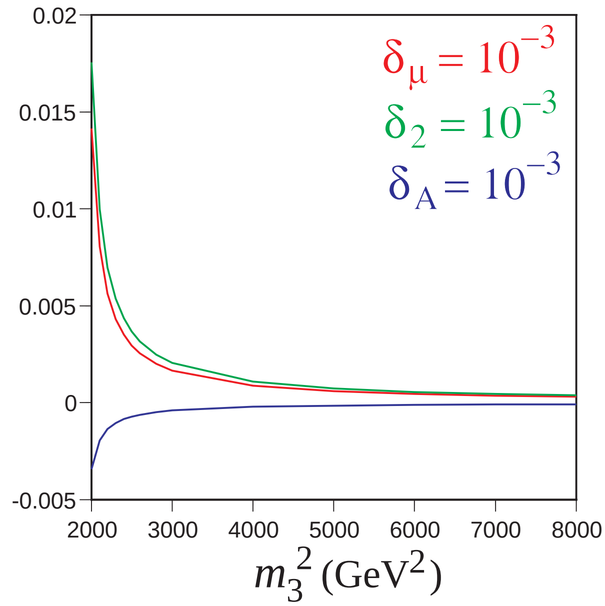
III. Numerical Results

$\tan\beta = 5,\ \mu = -500\text{GeV},\ m_{\tilde{t}_R} = 10\text{GeV},\ A_t = 20\text{GeV}$

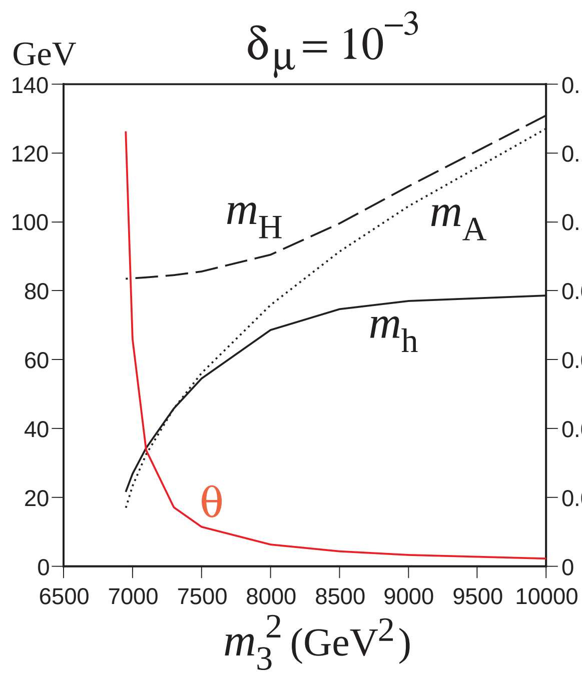
$T = 0$

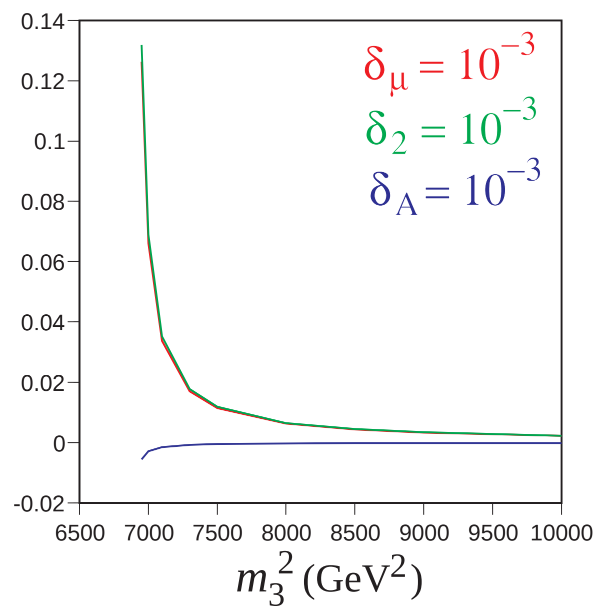
(1) $M_2 = M_1 = 200\text{GeV},\ m_{\tilde{t}_L} = 400\text{GeV}$





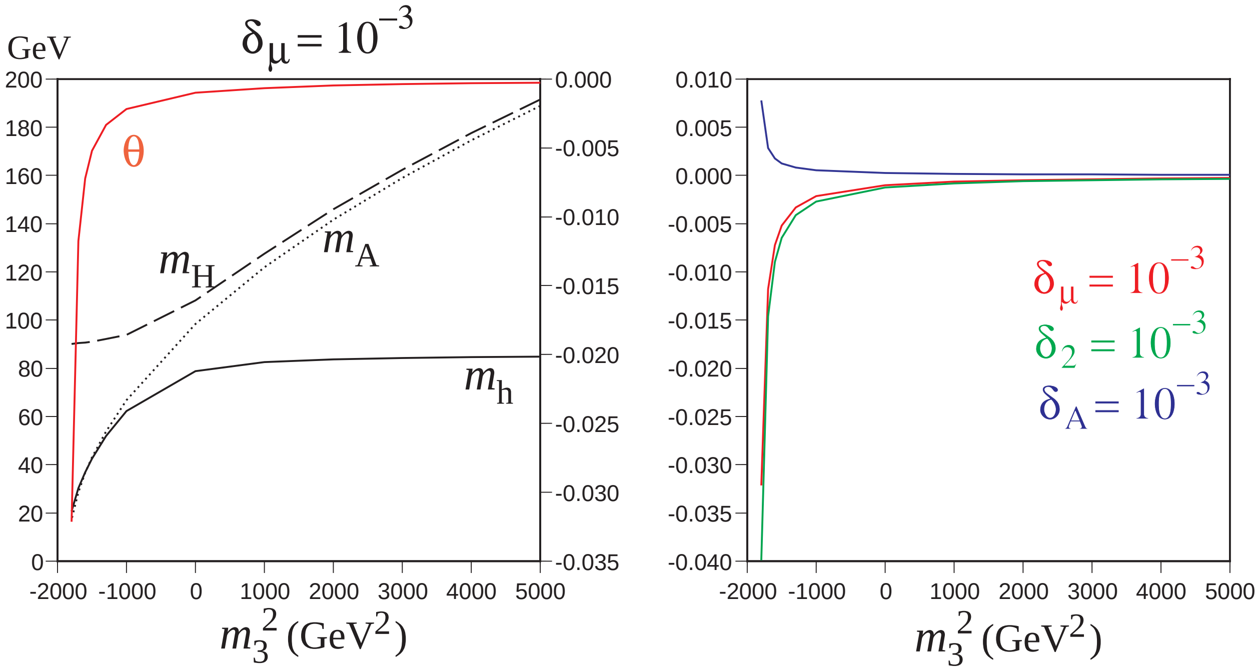
(2) $M_2 = M_1 = 500\text{GeV},\ m_{\tilde{t}_L} = 200\text{GeV}$





$\mu = -500\text{GeV} \longrightarrow \mu = 500\text{GeV}$

(1) $M_2 = M_1 = 200\text{GeV}$, $m_{\tilde{t}_L} = 400\text{GeV}$



Since $\bar{m}_3^2, \bar{\lambda}_5, \bar{\lambda}_6, \bar{\lambda}_7 \ \propto \ \mu$, $\mu \mapsto -\mu$ changes the sign of the imaginary part of V_{eff}.

$T \neq 0$

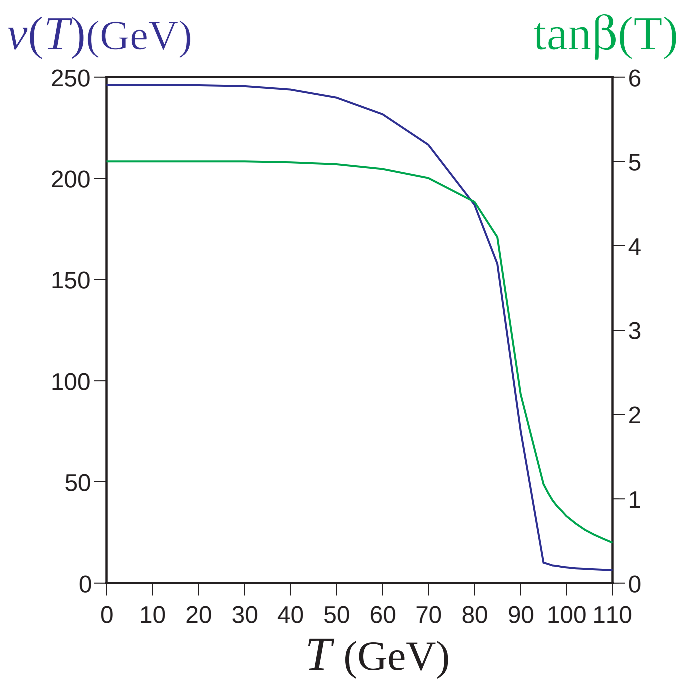
(2) $\mu = -500$GeV, $M_2 = M_1 = 500$GeV, $m_{\tilde{t}_L} = 200$GeV

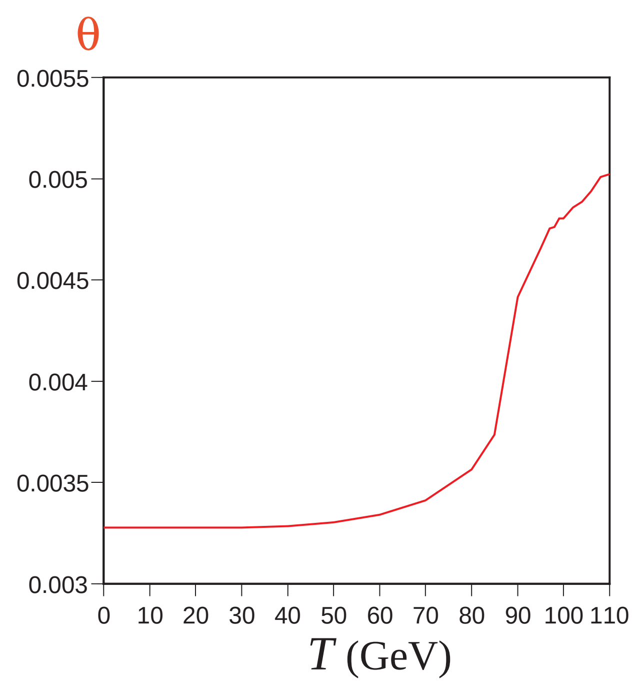
$\delta_\mu = 10^{-3}$, $\delta_2 = \delta_A = 0$.

For these parameters,

$$m_h = 77.0\text{GeV}, \quad m_A = 104.6\text{GeV}, \quad m_H = 110.3\text{GeV}$$

T-dependence of $|\boldsymbol{v}|$, $\tan\beta$ and θ





VI. Discussions

- In general, $\mathrm{Arg}\bar{\lambda}_{5,6,7} \sim O(\delta_\mu, \delta_2, \delta_A)$.

  The induced CP phase θ is not always suppressed.

- n-EDM constrains $\theta + \delta_\mu + \delta_2$.

  $\delta_\mu > 0,\ \delta_2 > 0$ with $\left\{ \begin{array}{c} \mu < 0 \\ \mu > 0 \end{array} \right\}$ induces $\left\{ \begin{array}{c} \theta > 0 \\ \theta < 0 \end{array} \right\}$

  $\Downarrow$

  $\left\{ \begin{array}{c} \text{Stronger} \\ \text{Weaker} \end{array} \right\}$ constraints on δ_μ (δ_2) when the mass parameters $M_2,\ \mu \lesssim O(100\mathrm{GeV})$.

- $\left. \begin{array}{c} \text{For (unacceptably) light Higgs} \\ \text{At finite } T \text{ near the EWPT} \end{array} \right\} \Longrightarrow$ large θ induced

- If our analyses on φ_u and φ_d are applied to the system of $\tilde{q}_L$ and $\tilde{q}_R$, CP violation responsible for the new B-genesis mechanism can be studied

  B-ball, B-string, etc.