CP Violation in the Higgs Sector of the MSSM at Finite Temeprature

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I. Introduction

3 requirements to generate BAU : $\dfrac{n_B}{s} \simeq 10^{-11}$

(1) baryon number violation

(2) C and CP violation

(3) departure from equilibrium

- EW theory satisfies these if the EWPT is first order.

$\Longrightarrow$ **Electroweak Baryogenesis**

- For sufficient CP violation, some extension of the Minimal SM is needed.

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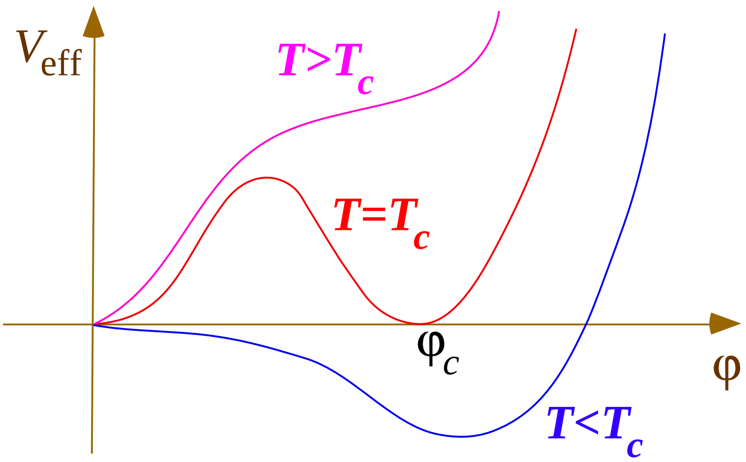
for a review,

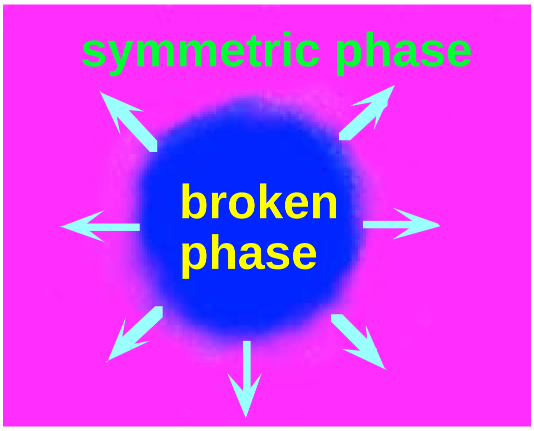
K.F., Prog.Theor.Phys.**96** ('96) 475.

V.A. Rubakov and M.E. Shaposhnikov, hep-ph/9603208.

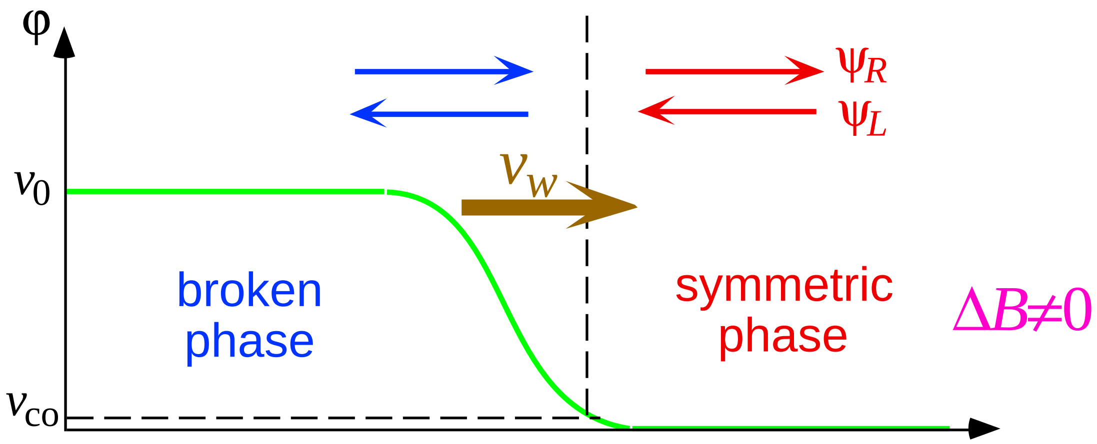
A.Cohen, et al., Ann.Rev.Nucl.Part.Sci. **33** ('93) 27.

first-order EWPT





at an expanding bubble wall ...



generated BAU (by the charge transport scenario)

$$\frac{n_B}{s} \simeq \mathcal{N}\frac{100}{\pi^2 g_*}\cdot\kappa\alpha_W^4\cdot\tau T\cdot\frac{F_Y}{v_w T^3}$$

$$\simeq 10^{-3}\times\frac{F_Y}{v_w T^3} \qquad \text{for an optimal case}$$

F_Y : chiral charge flux $\Leftarrow$ CP violation at EW bubble wall
$\Longleftrightarrow$ ΔR : reflection prob. of chiral fermions

CP violation effective for F_Y at the lowest order

- relative phases of μ, M_2, M_1, A_t in the MSSM
  chargino, neutralino, stop transport
  [Huet and Nelson, PRD53('96); Aoki, et al. PTP98('97)]
- relative phase θ of the two Higgs doublets

$$\langle \Phi_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho_i e^{i\theta_i} \end{pmatrix}$$

quarks and leptons $\Longleftarrow$ Yukawa coupl. $\propto \rho_i e^{i\theta}$
chargino, neutralino, stop mass matrix

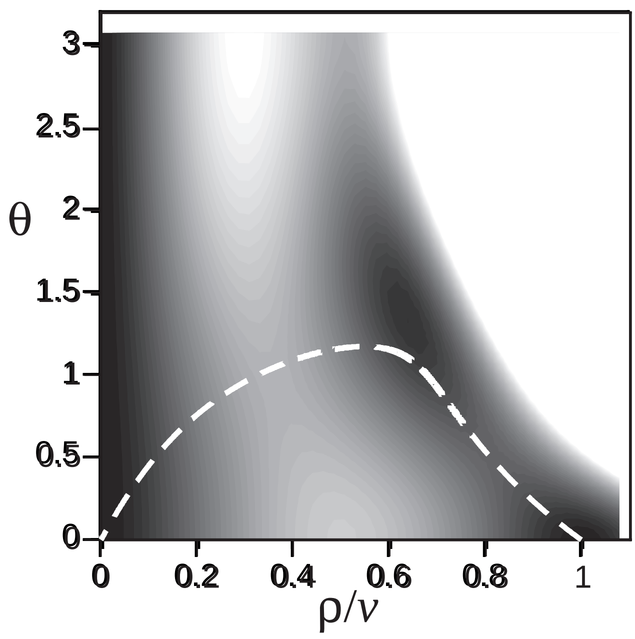
N.B. $\theta = \theta(x)$ $\therefore$ it cannot be rotated away
[Nelson et al. NPB373('92); FKOTT, PRD50('94), PTP95('96)]

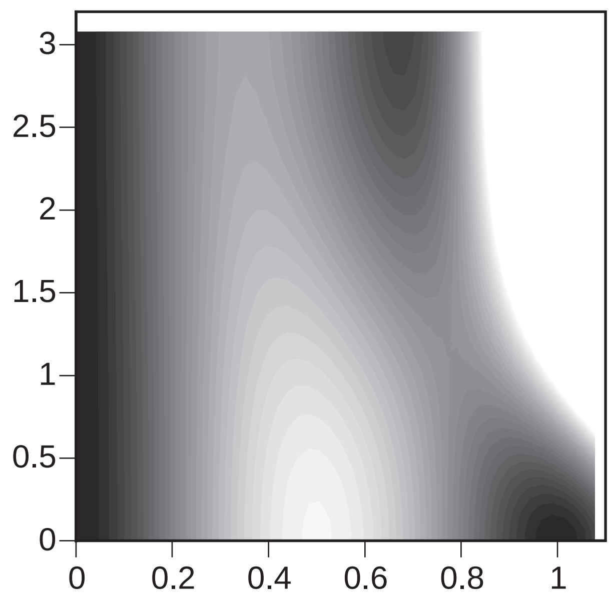
CP violating θ at the EWPT [FKOT, PTP98('97)]

$\Uparrow$

EOM for $\rho_1 = \rho_2$ and $\theta = \theta_1 - \theta_2$ with $V_{\rm eff}(\rho_i, \theta; T_C)$

examples of $V_{\rm eff}(\rho_i, \theta)$ with a fixed $\tan\beta$





a scenario to have large θ near the bubble wall

$=$ spontaneous CP violation in the transient region

$+$ small explicit CP violation

to resolve degeneracy between CP conjugate bubbles

net BAU

$$\frac{n_B}{s} = \frac{\sum_j \left(\frac{n_B}{s}\right)_j N_j}{\sum_j N_j}$$

with

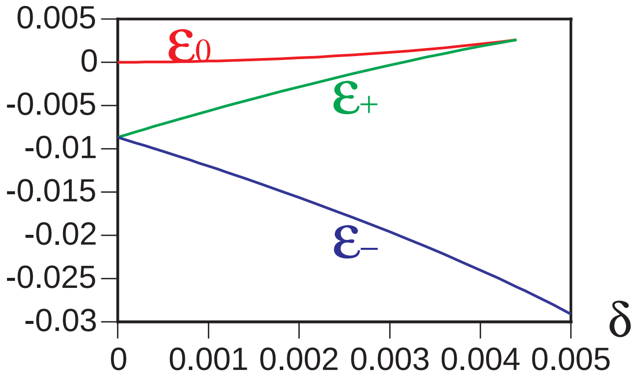
$$N_j = \exp(-4\pi R_C^2 \mathcal{E}_j / T_C) \quad \text{nucleation rate}$$

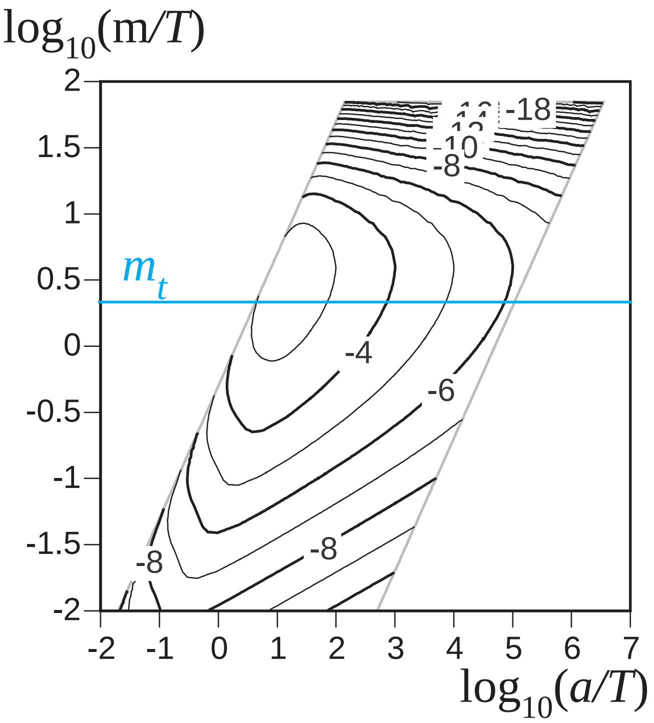
$$\mathcal{E}_j = \text{energy density of the type-}j\text{ bubble}$$

an example:

Higgs mixing term:

$$m_3^2 \longrightarrow m_3^2 e^{-\delta}$$





for $\delta = 1.0 \times 10^{-3}$

chiral charge flux

$$\log_{10} \frac{F_Q}{uT^3(Q_L - Q_R)}$$

$T = 100$GeV

$u = 0.58$

$a = 1/$wall thickness

II. Transitional CP Violation

Suppose that at $T \simeq T_C$,

$$
\begin{aligned}
& V_{\rm eff}(\rho_i, \theta = \theta_1 - \theta_2) \\
= \;& \frac{1}{2} m_1^2 \rho_1^2 + \frac{1}{2} m_2^2 \rho_2^2 - m_3^2 \rho_1 \rho_2 \cos\theta + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\
+ \;& \frac{\lambda_3 + \lambda_4}{4}\rho_1^2\rho_2^2 + \frac{\lambda_5}{4}\rho_1^2\rho_2^2 \cos 2\theta - \frac{1}{2}(\lambda_6 \rho_1^2 + \lambda_7 \rho_2^2)\rho_1\rho_2 \cos\theta \\
- \;& [A\rho_1^3 + \rho_1^2\rho_2(B_0 + B_1 \cos\theta + B_2 \cos 2\theta) \\
& \quad + \rho_1\rho_2^2(C_0 + C_1\cos\theta + C_2 \cos 2\theta) + D\rho_2^3] \\
= \;& \left[\frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2)\right] \\
& \quad \times \left[\cos\theta - \frac{2m_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)}\right]^2 \\
& + \theta\text{-independent terms}
\end{aligned}
$$

where all the parameters are real

conditions for spontaneous CP violation for a given (ρ_1, ρ_2)

$$
F(\rho_1, \rho_2) \equiv \frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2) > 0,
$$

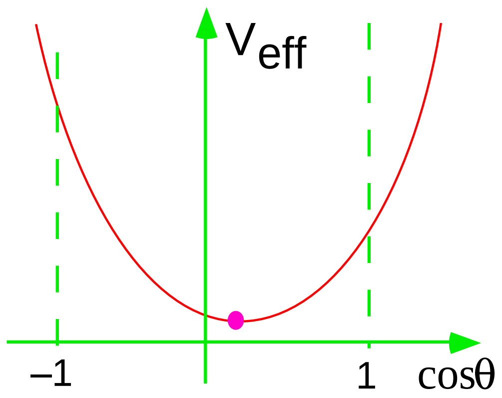
$$
-1 < G(\rho_1, \rho_2) \equiv \frac{2m_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)} < 1
$$

At $T \simeq T_C$, around the EW bubble wall

$$
(\rho_1, \rho_2) \; : \; (0, 0) \longrightarrow (v_C \cos\beta_C, v_C \sin\beta_C)
$$

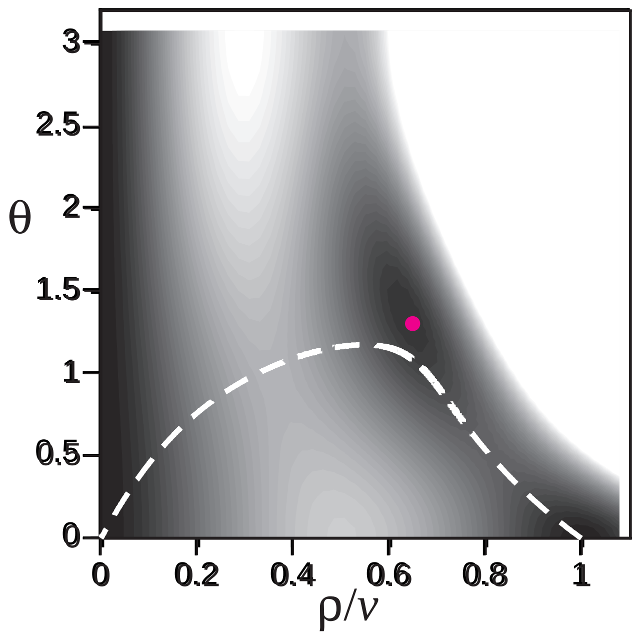
There may be a chance to satisfy the conditions in the transient region.

$F(\rho_1, \rho_2) > 0$

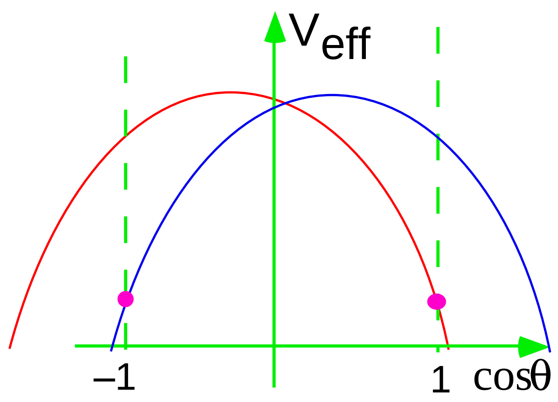


⇓

CP-violating
local minimum

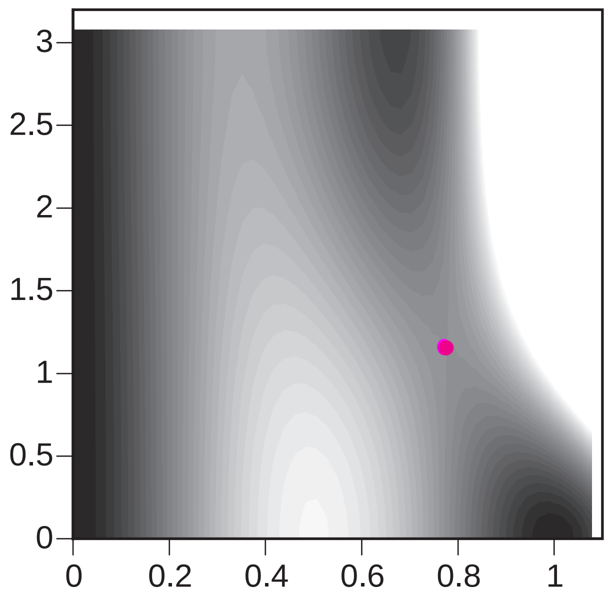


$F(\rho_1, \rho_2) < 0$



⇓

CP-violating
saddle point



III. Effective parameters of the MSSM

MSSM at the tree level:

$$\begin{aligned}\lambda_1 &= \lambda_2 = \frac{1}{4}(g_2^2 + g_1^2), \quad \lambda_3 = \frac{1}{4}(g_2^2 - g_1^2), \quad \lambda_4 = -\frac{1}{2}g_2^2, \\ \lambda_5 &= \lambda_6 = \lambda_7 = 0,\end{aligned}$$

the relevant parameters in F and G are induced by radiative and finite-temperature corrections

effective potential at the one-loop level

$$V_{\text{eff}} = V_0 + V_1(\rho_i, \theta) + \bar{V}_1(\rho_i, \theta; T),$$

where

$$\begin{aligned}V_1(\rho_i, \theta) &= \sum_j n_j \frac{m_j^4}{64\pi^2}\left[\log\left(\frac{m_j^2}{M_{\text{ren}}^2}\right) - \frac{3}{2}\right], \\ \bar{V}_1(\rho_i, \theta; T) &= \frac{T^4}{2\pi^2}\sum_j n_j \int_0^\infty dx\, x^2 \\ &\quad \times \log\left[1 - \text{sgn}(n_j)\exp\left(-\sqrt{x^2 + m_j^2/T^2}\right)\right].\end{aligned}$$

N.B. $m_j^2 = m_j^2(\rho_i, \theta)$

with

$$\begin{aligned}\varphi_d &= \frac{1}{\sqrt{2}}\begin{pmatrix}\rho_1 \\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}v_1 \\ 0\end{pmatrix}, \\ \varphi_u &= \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ \rho_2 e^{i\theta}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v_2 + iv_3\end{pmatrix}.\end{aligned}$$

We consider the contributions from the charginos and neutralinos (χ), stops ($\tilde{t}$) and charged Higgs bosons ($\phi^{\pm}$).

relevant effective parameters:

$$\left(m_3^2\right)_{\rm eff} = -\left.\frac{\partial^2 V_{\rm eff}}{\partial v_1 \partial v_2}\right|_0 = m_3^2 + \Delta_\chi m_3^2 + \Delta_{\tilde{t}} m_3^2 + \Delta_{\phi^\pm} m_3^2,$$

$$\lambda_5 = \frac{1}{2}\left(\left.\frac{\partial^4 V_{\rm eff}}{\partial v_1^2 \partial v_2^2}\right|_0 - \left.\frac{\partial^4 V_{\rm eff}}{\partial v_1^2 \partial v_3^2}\right|_0\right),$$

$$\lambda_6 = -\frac{1}{3}\left.\frac{\partial^4 V_{\rm eff}}{\partial v_1^3 \partial v_2}\right|_0 = \Delta_\chi \lambda_6 + \Delta_{\tilde{t}} \lambda_6 + \Delta_{\phi^\pm} \lambda_6,$$

$$\lambda_7 = -\frac{1}{3}\left.\frac{\partial^4 V_{\rm eff}}{\partial v_1 \partial v_2^3}\right|_0 = \Delta_\chi \lambda_7 + \Delta_{\tilde{t}} \lambda_7 + \Delta_{\phi^\pm} \lambda_7$$

ρ^3-terms in $V_{\rm eff}$ $\Longleftarrow$ zero modes of the bosons at $T \neq 0$

$W^\pm$, Z, Higgs, squarks, sleptons

θ-dependent ρ^3-terms (B_i and C_i)

$\Longleftarrow m_{\tilde{t}} \simeq 0$

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{q}}^2 + m_t^2 - \frac{3g_2^2 - g_1^2}{8}(\rho_1^2 - \rho_2^2) & \frac{y_t}{\sqrt{2}}(\mu\rho_1 + A_t \rho_2 e^{-i\theta}) \\ \frac{y_t}{\sqrt{2}}(\mu\rho_1 + A_t \rho_2 e^{i\theta}) & m_{\tilde{t}}^2 + m_t^2 + \frac{g_1^2}{6}(\rho_1^2 - \rho_2^2) \end{pmatrix}$$

and

$$\bar{V}_{\tilde{t}}(\rho_i, \theta; T) = 3\frac{T^4}{2\pi^2}\left[2I_B(a_+^2) + 2I_B(a_-^2)\right],$$

with

$$I_B(a_-^2) = \int_0^\infty dx\, x^2 \log\left(1 - e^{-\sqrt{x^2 + a_-^2}}\right)$$

$$= -\frac{\pi^4}{45} + \frac{\pi^2}{12}a_-^2 - \frac{\pi}{6}a_-^3 + \lambda_- a_-^4 + \dots$$

$a_-^2 \equiv m_-^2/T^2$; $m_-^2(\rho) =$ smaller eigenvalue of $M_{\tilde{t}}^2$

• charginos and neutralinos

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & -\frac{ig_2}{\sqrt{2}}\rho_2 e^{-i\theta} \\ -\frac{ig_2}{\sqrt{2}}\rho_1 & -\mu \end{pmatrix},$$

$$M_{\chi^0} = \begin{pmatrix} M_2 & 0 & -\frac{i}{2}g_2\rho_1 & \frac{i}{2}g_2\rho_2 e^{-i\theta} \\ 0 & M_1 & \frac{i}{2}g_1\rho_1 & -\frac{i}{2}g_1\rho_2 e^{-i\theta} \\ -\frac{i}{2}g_2\rho_1 & \frac{i}{2}g_1\rho_1 & 0 & \mu \\ \frac{i}{2}g_2\rho_2 e^{-i\theta} & -\frac{i}{2}g_1\rho_2 e^{-i\theta} & \mu & 0 \end{pmatrix}$$

chargino and neutralino contributions $(M_2 = M_1)$
$[M_2, \mu \in \mathbf{C}]$

$$\Delta_\chi m_3^2 = -2g_2^2\left(1 + \frac{1}{\cos^2\theta_W}\right)\mu M_2\, L(M_2,\mu) + \frac{g_2^2}{\pi^2}\left(1 + \frac{1}{\cos^2\theta_W}\right)\mu M_2 f_2^{(+)}\left(\frac{M_2}{T},\frac{\mu}{T}\right),$$

$$\Delta_\chi\lambda_5 = \frac{g_2^4}{8\pi^2}\left(1 + \frac{2}{\cos^4\theta_W}\right)\frac{\mu^2 M_2^2}{|\mu M_2|^2}K\left(\frac{M_2^2}{\mu^2}\right) - \frac{g_2^4}{\pi^2 T^4}\left(1 + \frac{2}{\cos^4\theta_W}\right)\mu^2 M_2^2 f_4^{(+)}\left(\frac{M_2}{T},\frac{\mu}{T}\right),$$

$$\Delta_\chi\lambda_6 = -\frac{g_2^4}{8\pi^2}\left(1 + \frac{2}{\cos^4\theta_W}\right)\frac{\mu}{M_2}\left[-H\left(\frac{M_2^2}{\mu^2}\right) + K\left(\frac{M_2^2}{\mu^2}\right)\right] + \frac{g_2^4}{\pi^2}\left(1 + \frac{2}{\cos^4\theta_W}\right)\left[\frac{\mu M_2}{T^2}f_3^{(+)}\left(\frac{M_2}{T},\frac{\mu}{T}\right) + \frac{\mu^3 M_2}{T^4}f_4^{(+)}\left(\frac{M_2}{T},\frac{\mu}{T}\right)\right] = \Delta_\chi\lambda_7$$

where

$$
\begin{aligned}
L(m_1, m_2) &= \frac{1}{16\pi^2}\left[1 - \frac{m_1^2}{m_1^2 - m_2^2}\log\frac{m_1^2}{M_{\rm ren}^2} + \frac{m_2^2}{m_1^2 - m_2^2}\log\frac{m_2^2}{M_{\rm ren}^2}\right] \\
&> 0, \\
H(\alpha) &= \frac{\alpha}{\alpha - 1}\left(\frac{1}{\alpha - 1}\log\alpha - 1\right) < 0, \\
K(\alpha) &= \frac{\alpha}{(\alpha - 1)^2}\left(\frac{\alpha + 1}{\alpha - 1}\log\alpha - 2\right) > 0,
\end{aligned}
$$

and

$$
\begin{aligned}
f_2^{(\mp)}(a, b) &= -\frac{1}{a^2 - b^2}\int_0^\infty dx\Big(\frac{x^2}{\sqrt{x^2 + a^2}}\frac{1}{e^{\sqrt{x^2+a^2}} \mp 1} \\
&\qquad - \frac{x^2}{\sqrt{x^2 + b^2}}\frac{1}{e^{\sqrt{x^2+b^2}} \mp 1}\Big) > 0, \\
f_3^{(\mp)}(a, b) &= \frac{1}{2(a^2 - b^2)}\int_0^\infty \frac{dx}{\sqrt{x^2 + a^2}}\frac{1}{e^{\sqrt{x^2+a^2}} \mp 1} \\
&\qquad - \frac{1}{a^2 - b^2} f_2^{(\mp)}(a, b) < 0, \\
f_4^{(\mp)}(a, b) &= \frac{1}{2(a^2 - b^2)^2}\int_0^\infty dx\Big(\frac{1}{\sqrt{x^2 + a^2}}\frac{1}{e^{\sqrt{x^2+a^2}} \mp 1} \\
&\qquad + \frac{1}{\sqrt{x^2 + b^2}}\frac{1}{e^{\sqrt{x^2+b^2}} \mp 1}\Big) \\
&\qquad - \frac{2}{(a^2 - b^2)^2} f_2^{(\mp)}(a, b) > 0,
\end{aligned}
$$

• stops with $m_{\tilde t} \simeq 0$ $\quad [\mu, A_t \in \mathbf{C}]$

$$\Delta_{\tilde t} m_3^2 = N_c y_t^2 \mu A_t^* \, L(m_{\tilde q}, 0) + \frac{3T^2}{\pi^2} \frac{y_t^2 \mu A_t^*}{m_{\tilde q}^2} \left[I_B'(a_{\tilde q}^2) - \frac{\pi^2}{12} \right] ,$$

$$\Delta_{\tilde t}\lambda_5 = -\frac{N_c y_t^4}{16\pi^2} \frac{\mu^2 A_t^{*2}}{m_{\tilde q}^2 M_{IR}^2} K\left(\frac{m_{\tilde q}^2}{M_{IR}^2} \right)$$
$$+ \frac{N_c y_t^4 \mu^2 A_t^{*2}}{\pi^2 (m_{\tilde q}^2)^2} \left[\frac{2T^2}{m_{\tilde q}^2} \left(-I_B'(a_{\tilde q}^2) + \frac{\pi^2}{12} \right) + I_B''(a_{\tilde q}^2) + 2\lambda_- \right]$$

$$\Delta_{\tilde t}\lambda_6 = \frac{N_c y_t^2}{16\pi^2} \frac{\mu A_t^*}{m_{\tilde q}^2} \Big[\frac{1}{4} \left(\frac{g_1^2}{3} - g_2^2 \right) - \frac{g_1^2 m_{\tilde q}^2}{3 M_{IR}^2} H\left(\frac{M_{IR}^2}{m_{\tilde q}^2} \right)$$
$$+ \frac{y_t^2 \mu^2}{M_{IR}^2} K\left(\frac{m_{\tilde q}^2}{M_{IR}^2} \right) \Big]$$
$$+ \frac{N_c y_t^2 \mu A_t^*}{\pi^2 m_{\tilde q}^2} \Big\{ \frac{2T^2}{m_{\tilde q}^2} \left(\frac{y_t^2 \mu^2}{m_{\tilde q}^2} + (-\frac{5}{3} g_1^2 + g_2^2) \right) \left[I_B'(a_{\tilde q}^2) - \frac{\pi^2}{12} \right]$$
$$- \left(\frac{y_t^2 \mu^2}{m_{\tilde q}^2} + \frac{3g_2^2 - g_1^2}{12} \right) I_B''(a_{\tilde q}^2) + 2 \left(\frac{g_1^2}{3} - \frac{y_t^2 \mu^2}{m_{\tilde q}^2} \right) \lambda_- \Big\} ,$$

$$\Delta_{\tilde t}\lambda_7 = \frac{N_c y_t^2}{16\pi^2} \frac{\mu A_t^*}{m_{\tilde q}^2} \Big[- \left(y_t^2 + \frac{1}{4} \left(\frac{g_1^2}{3} - g_2^2 \right) \right)$$
$$- \left(y_t^2 - \frac{g_1^2}{3} \right) \frac{m_{\tilde q}^2}{M_{IR}^2} H\left(\frac{M_{IR}^2}{m_{\tilde q}^2} \right) + \frac{y_t^2 A_t^2}{M_{IR}^2} K\left(\frac{m_{\tilde q}^2}{M_{IR}^2} \right) \Big]$$
$$+ \frac{N_c y_t^2 \mu A_t^*}{\pi^2 m_{\tilde q}^2} \Big\{ \frac{2T^2}{m_{\tilde q}^2} \left(\frac{y_t^2 \mu^2}{m_{\tilde q}^2} - (-\frac{5}{3} g_1^2 + g_2^2) \right) \left[I_B'(a_{\tilde q}^2) - \frac{\pi^2}{12} \right]$$
$$- \left(y_t^2 + \frac{y_t^2 \mu^2}{m_{\tilde q}^2} - \frac{3g_2^2 - g_1^2}{12} \right) I_B''(a_{\tilde q}^2)$$

$$+2\left(y_t^2 - \frac{g_1^2}{3} - \frac{y_t^2\mu^2}{m_{\tilde{q}}^2}\right)\ \lambda_-\}$$

and for $A_t/m_{\tilde{q}}, \mu/m_{\tilde{q}}, g_1/y_t \ll 1$,

$$C_1 \simeq \frac{T}{4\sqrt{2}\pi}|y_t|^3\frac{3\mu A_t}{m_{\tilde{q}}^2}\left[-1+\frac{1}{2}\left(\frac{A_t}{m_{\tilde{q}}}\right)^2+\frac{1}{6}\left(\frac{g_1}{y_t}\right)^2\right],$$

$$B_2 \simeq \frac{T}{4\sqrt{2}\pi}|y_t|^3\frac{3}{2}\left(\frac{\mu A_t}{m_{\tilde{q}}{}^2}\right)^2.$$

- charged Higgs bosons

$$\Delta_{\phi^\pm}m_3^2 = \frac{1}{2}g_2^2m_3^2\,L(\mu_1,\mu_2)+\frac{1}{4\pi^2}g_2^2m_3^2f_2^{(-)}\left(\frac{\mu_1}{T},\frac{\mu_2}{T}\right),$$

$$\Delta_{\phi^\pm}\lambda_5 = -\frac{g_2^4}{64\pi^2}\frac{m_3^4}{\mu_1^2\mu_2^2}K\left(\frac{\mu_1^2}{\mu_2^2}\right)-\frac{1}{8\pi^2T^4}g_2^4m_3^4f_4^{(-)}\left(\frac{\mu_1}{T},\frac{\mu_2}{T}\right),$$

$$\Delta_{\phi^\pm}\lambda_6 = \frac{g_2^4}{64\pi^2}\frac{m_3^2}{\mu_1^2}\{-H\left(\frac{\mu_1^2}{\mu_2^2}\right)+[1-\frac{m_1^2}{2\mu^2\cos^2\theta_W}$$
$$-\left(1-\frac{1}{2\cos^2\theta_W}\right)\frac{m_2^2}{\mu_2^2}]K\left(\frac{\mu_1^2}{\mu_2^2}\right)\}$$
$$+\frac{g_2^4m_3^2}{8\pi^2T^2}[f_3^{(-)}\left(\frac{\mu_1}{T},\frac{\mu_2}{T}\right)+(\frac{\mu_2^2}{T^2}-\frac{m_1^2}{2T^2\cos^2\theta_W}$$
$$-\left(1-\frac{1}{2\cos^2\theta_W}\right)\frac{m_2^2}{T^2})f_4^{(-)}\left(\frac{\mu_1}{T},\frac{\mu_2}{T}\right)],$$

$$\Delta_{\phi^\pm}\lambda_7 = \frac{g_2^4}{64\pi^2}\frac{m_3^2}{\mu_1^2}\{-H\left(\frac{\mu_1^2}{\mu_2^2}\right)+[1-\left(1-\frac{1}{2\cos^2\theta_W}\right)\frac{m_1^2}{\mu_2^2}$$
$$-\frac{m_2^2}{2\mu_2^2\cos^2\theta_W}]K\left(\frac{\mu_1^2}{\mu_2^2}\right)\}$$

$$+\frac{g_2^4 m_3^2}{8\pi^2 T^2}\Big[\, f_3^{(-)}\Big(\frac{\mu_1}{T}, \frac{\mu_2}{T}\Big) \;+ (\frac{\mu_2^2}{T^2} - \Big(1 - \frac{1}{2\cos^2\theta_W}\Big)\frac{m_1^2}{T^2}$$

$$-\frac{m_2^2}{2T^2\cos^2\theta_W}) f_4^{(-)}\Big(\frac{\mu_1}{T}, \frac{\mu_2}{T}\Big) \,\Big] \,,$$

where

$$\mu_{1,2}^2 = \frac{\bar{m}_1^2 + \bar{m}_2^2 \pm \sqrt{(\bar{m}_1^2 - \bar{m}_2^2)^2 + 4m_3^4}}{2}$$

and

$$\bar{m}_1^2 = m_1^2 + \frac{1}{16\pi^2}(3g_2^2 + g_1^2)T^2,$$

$$\bar{m}_2^2 = m_2^2 + \frac{1}{16\pi^2}(3g_2^2 + g_1^2 + 4y_t^2)T^2.$$

III. Numerical Results

input parameters

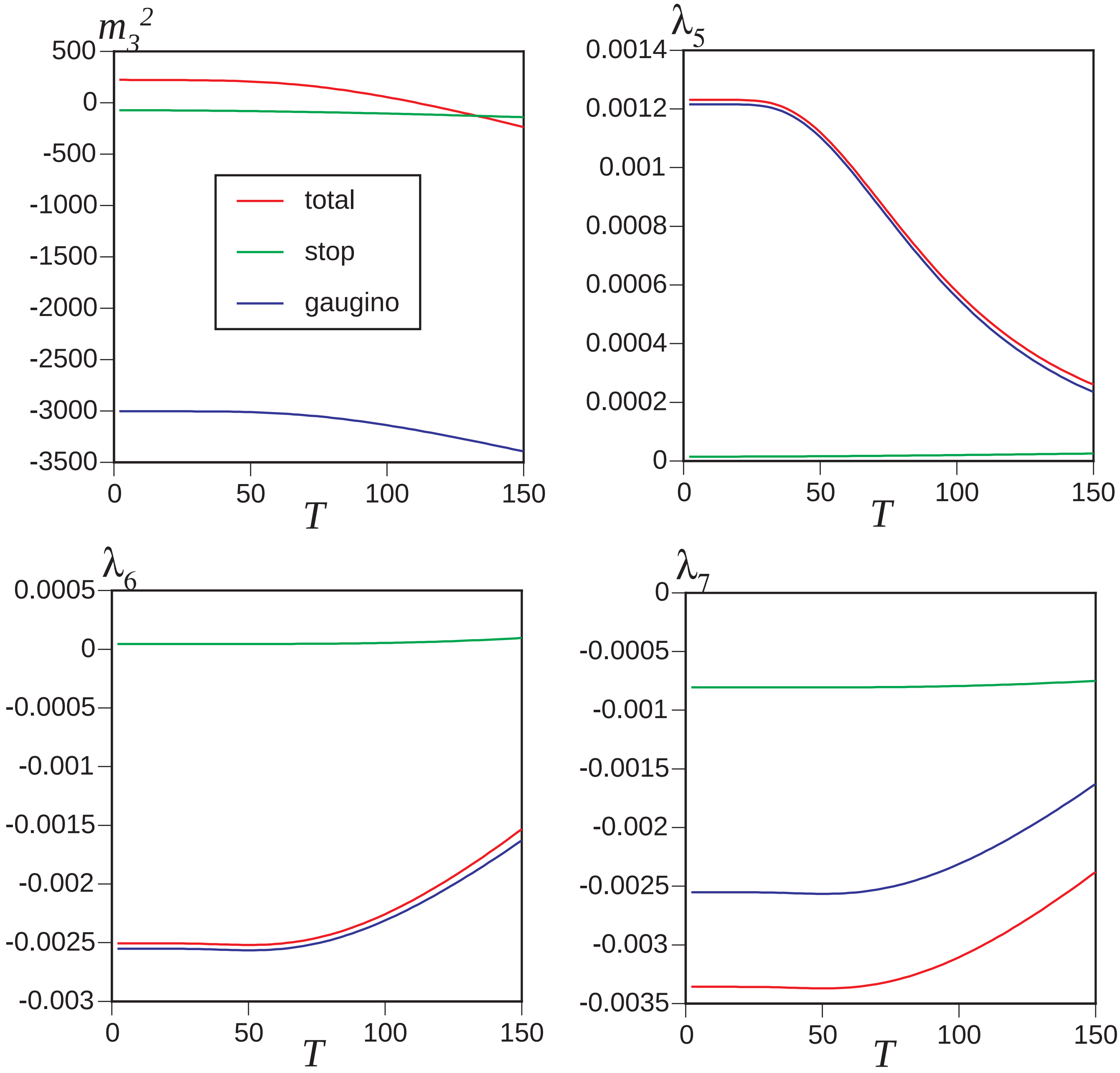
$v_0 = 246\text{GeV},\ m_t = 177\text{GeV},\ M_{\text{ren}} = 100\text{GeV},\ \tan\beta_0 = 5$

$$m_1^2 = m_3^2 \tan\beta_0 - \frac{1}{2} m_Z^2 \cos(2\beta_0),$$

$$m_2^2 = m_3^2 \cot\beta_0 + \frac{1}{2} m_Z^2 \cos(2\beta_0)$$

(A) $\mu A_t > 0$

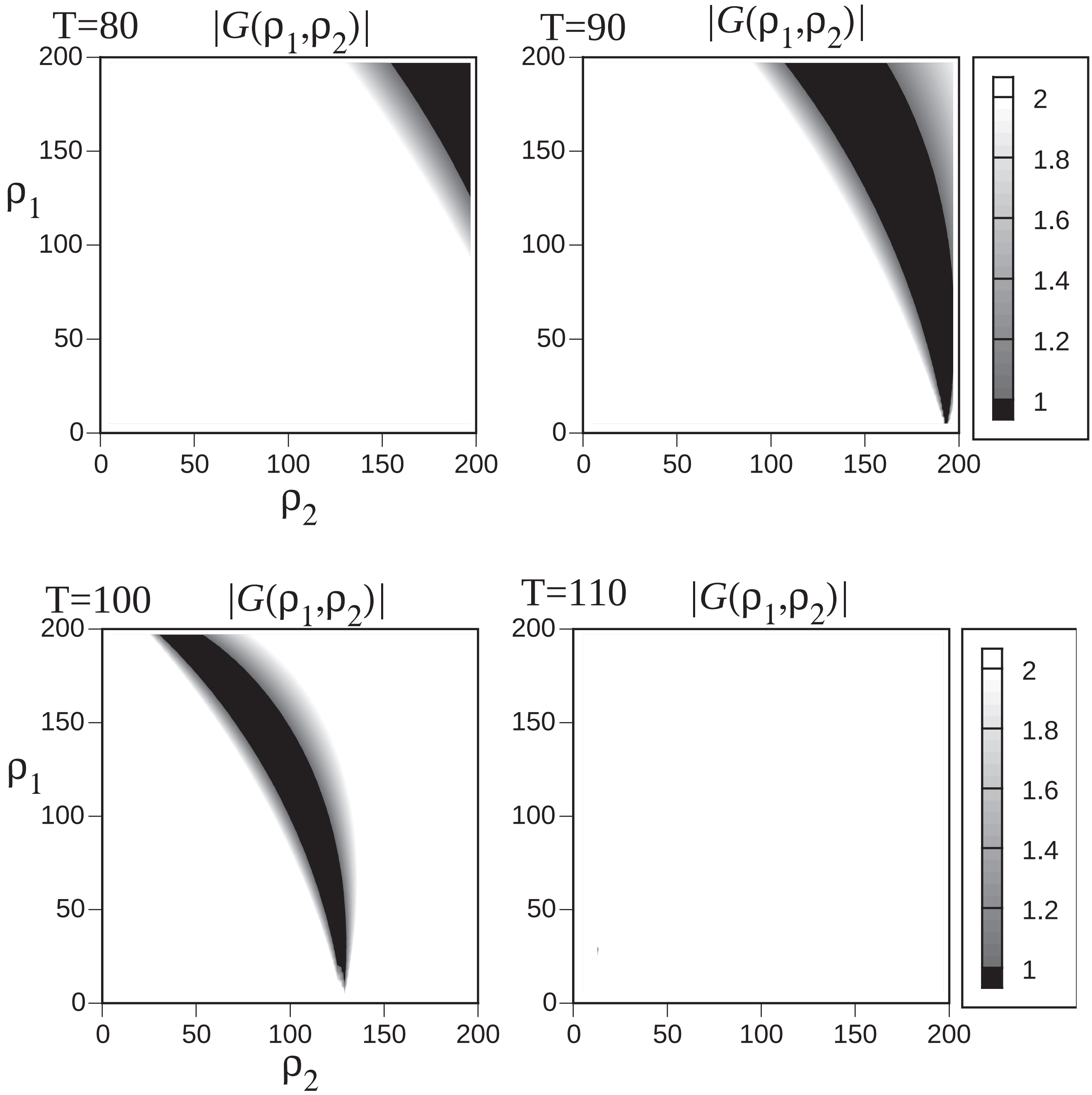
m_3^2	A_t	M_2	μ	$m_{\tilde{q}}$
3300	10	-400	200	400



$B_2/T = 7.368276 \times 10^{-6}, \qquad C_1/T = -2.313638 \times 10^{-3}$

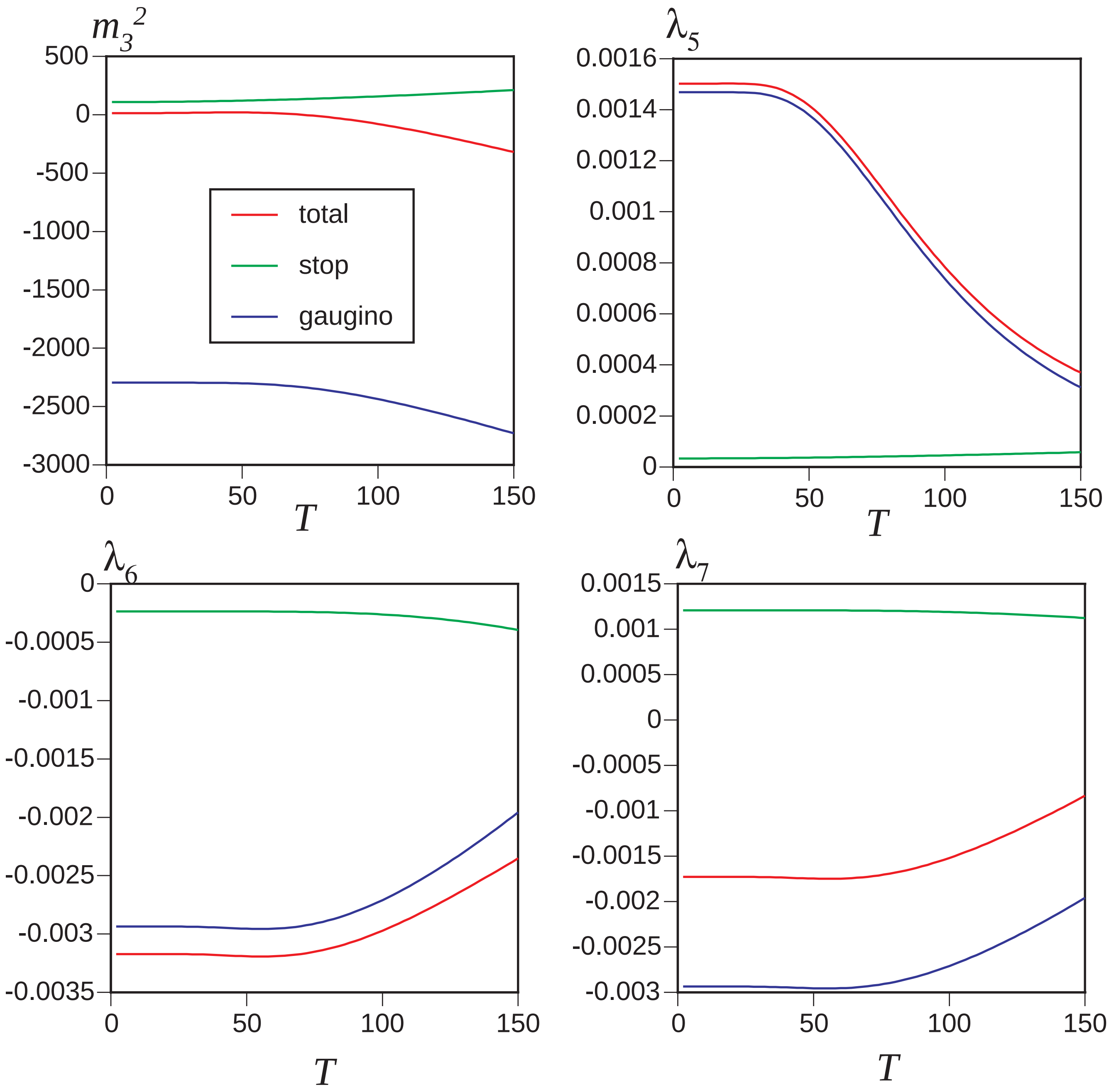
$\Longrightarrow$ $F(\rho_1, \rho_2) > 0$ for $\rho_2 > 5.104$ at $T = 100$

region in which $|G(\rho_1, \rho_2)| < 1$ is satisfied:



(B) $\mu A_t < 0$

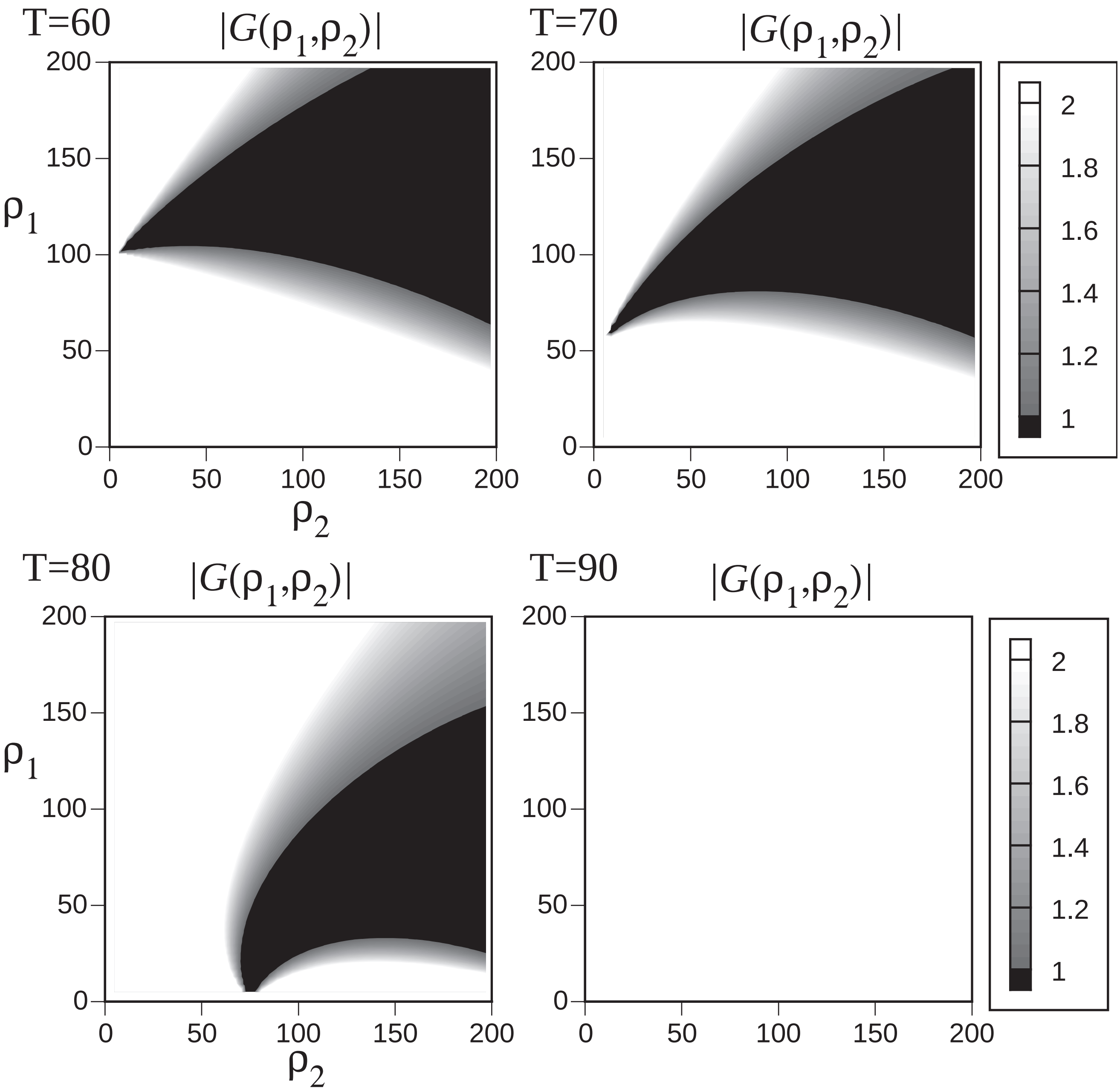
m_3^2	A_t	M_2	μ	$m_{\tilde{q}}$
2200	10	300	-300	400



$B_2/T = 1.657862 \times 10^{-5}, \qquad C_1/T = 3.470457 \times 10^{-3}$

$\Longrightarrow F(\rho_1, \rho_2) > 0$ for $\rho_2 > 8.463$ at $T = 100$

region in which $|G(\rho_1, \rho_2)| < 1$ is satisfied:



$$\left.\begin{array}{c} |\lambda_6|_{(A)} \simeq |\lambda_6|_{(B)} \\ |\lambda_7|_{(A)} > |\lambda_7|_{(B)} \\ C_1|_{(A)} < 0, \quad C_1|_{(B)} > 0 \end{array}\right\} \Rightarrow$$

weaker ρ-dependence of $G(\rho_1, \rho_2)$ in case (B)

$cf.\ G(\rho_1, \rho_2) = [2m_3^2 + \lambda_6 \rho_1^2 + \lambda_7 \rho_2^2 + 2C_1 \rho_2]/(\cdots)$

IV. Discussions

The condition for the transitional CP violation:

$$|G(\rho_1, \rho_2)| < 1$$

is satisfied at $T \simeq T_C$ if

1. $\mu M_2 < 0$ [to decrease $\left(m_3^2\right)_{\rm eff}$ by the χ-contributions]
2. m_3^2 is properly tuned at the tree-level [for $\left(m_3^2\right)_{\rm eff}$ to become as small as $\lambda_{6,7}\rho^2$]

Explicit CP Violation

$$\alpha = {\rm Arg}(\mu M_2) = {\rm Arg}(\mu M_1), \qquad \beta = {\rm Arg}(\mu A_t^*),$$

then

$$\begin{aligned} \left(m_3^2\right)_{\rm eff} &= m_3^2 + \Delta_{\phi^\pm}^{(0)} m_3^2 + e^{i\alpha}\Delta_\chi^{(0)} m_3^2 + e^{i\beta}\Delta_{\tilde t}^{(0)} m_3^2, \\ \lambda_5 &= \Delta_{\phi^\pm}^{(0)}\lambda_5 + e^{i2\alpha}\Delta_\chi^{(0)}\lambda_5 + e^{i2\beta}\Delta_{\tilde t}^{(0)}\lambda_5, \\ \lambda_{6,7} &= \Delta_{\phi^\pm}^{(0)}\lambda_{6,7} + e^{i\alpha}\Delta_\chi^{(0)}\lambda_{6,7} + e^{i\beta}\Delta_{\tilde t}^{(0)}\lambda_{6,7} \end{aligned}$$

$\Delta^{(0)} \equiv$ correction without explicit CP violation

If $\Delta_\chi^{(0)} \gg \Delta_{\tilde t}^{(0)}, \Delta_{\phi^\pm}^{(0)}$, by rephasing, $\lambda_{5,6,7} \in \mathbf{R}$ and

$$e^{-i\alpha}\left(m_3^2\right)_{\rm eff} = e^{-i\alpha} m_3^2 + \Delta_\chi^{(0)} m_3^2 \equiv e^{-i\delta}\left|\left(m_3^2\right)_{\rm eff}\right|$$

with

$$\tan\delta = -\frac{m_3^2 \sin\alpha}{m_3^2\cos\alpha + \Delta_\chi^{(0)} m_3^2}.$$

N.B $\left|m_3^2 + \Delta_\chi^{(0)} m_3^2\right| \ll m_3^2$ for transitional CP violation for the example (B) at $T = 80$

$$\Delta_{\tilde{t}}^{(0)} m_3^2 = 140.69, \qquad \Delta_{\chi}^{(0)} m_3^2 = -2356.73,$$

so that even for $\alpha = 10^{-3}$,

$$\tan\delta = -\frac{(m_3^2 + \Delta_{\tilde{t}}^{(0)} m_3^2)\sin\alpha}{(m_3^2 + \Delta_{\tilde{t}}^{(0)} m_3^2)\cos\alpha + \Delta_{\chi}^{(0)} m_3^2}$$

$$\simeq \frac{10^{-3}}{6.8 \times 10^{-3}} = 0.147$$

$\Longrightarrow$ only the lowest-energy bubble survives

This mechanism of CP violation

- is free from the constraints on CP violation at $T = 0$,
- can generate sufficient BAU,
- is free from the domain wall problem,
- is not bothered by the light scalar.

At $T = 0$, spontaneous CP viol. does not occur

$\Longrightarrow$ no pseudo-Goldstone boson

$$m_A^{\text{tree}} = \sqrt{m_3^2(\tan\beta_0 + \cot\beta_0)} = 107\text{GeV for case (B)}$$

N.B.

- CP violating minimum of V_{eff} in the transient region need not be the global minimum.
- We still need to know the global structure of V_{eff} to determine T_C and the profile of the EW bubble wall.

$\cdots$ numerical studies in progress