Spontaneous CP Violation in the MSSM at Finite Temperature

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I. Introduction

3 requirements to generate BAU :

(1) baryon number violation

(2) C and CP violation

(3) departure from equilibrium

EW theory satisfies these if the EWPT is first order.

$\Longrightarrow$ **Electroweak Baryogenesis**

For sufficient CP violation, some extension of the MSM is needed.

K.F., Prog.Theor.Phys.**96** ('96) 475.
V.A. Rubakov and M.E. Shaposhnikov, hep-ph/9603208.
A.Cohen, et al., Ann.Rev.Nucl.Part.Sci. **33** ('93) 27.

generated BAU (by the charge transport scenario)

$$\frac{n_B}{s} \simeq \mathcal{N}\frac{100}{\pi^2 g_*}\cdot\kappa\alpha_W^4\cdot\tau T\cdot\frac{F_Y}{v_w T^3}$$

$$\simeq 10^{-3}\times\frac{F_Y}{v_w T^3} \qquad \text{for an optimal case}$$

F_Y : chiral charge flux $\Leftarrow$ CP violation at EW bubble wall

CP viol. in Higgs sector $\Rightarrow$ propagation of quarks and leptons

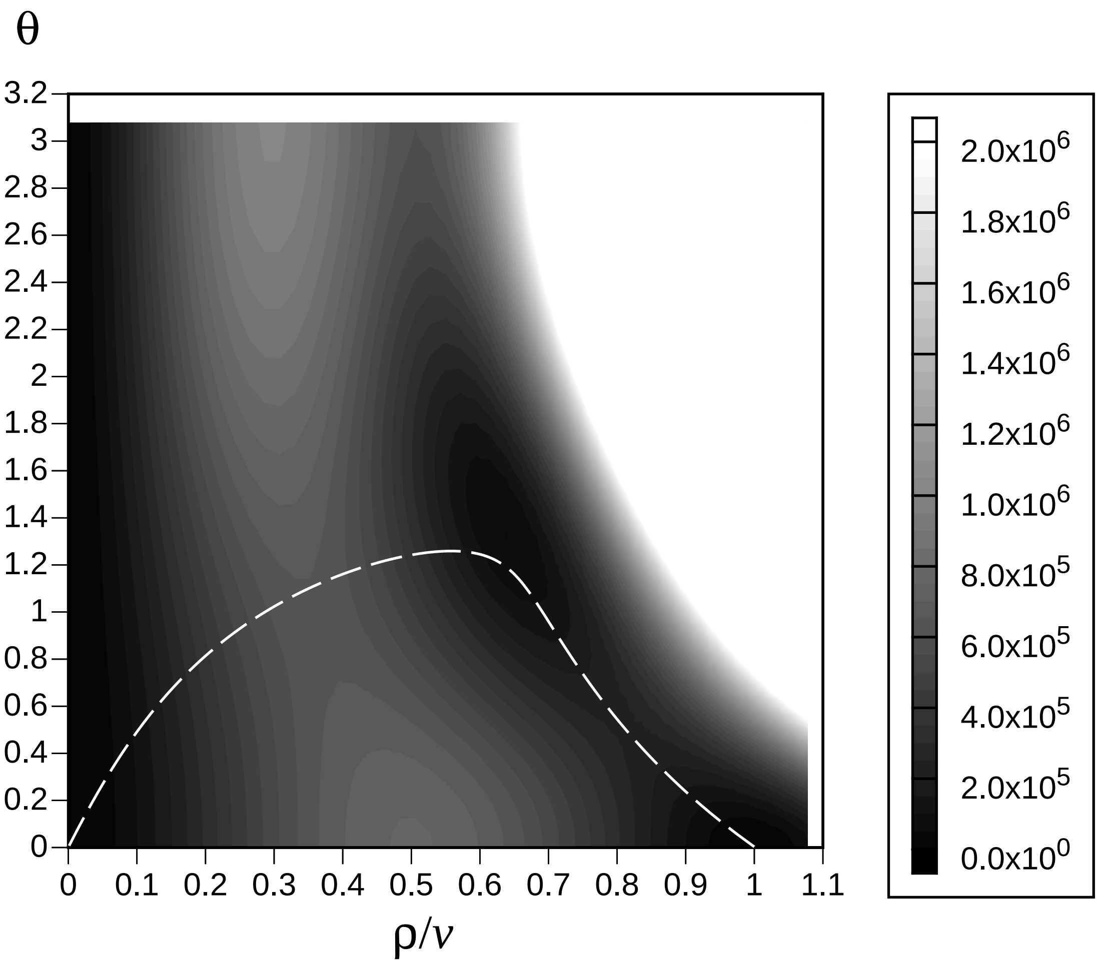
$\Uparrow$ Yukawa coupl. $\propto \rho_i e^{i\theta}$

more than two Higgs doublets [$\supset$ MSSM]

CP violation at EWPT $\Longleftarrow V_{\rm eff}(\rho_i, \theta; T_C)$

θ : relative phase $=$ CP violation

dynamically determined (ρ_i, θ) [FKOT, hep-ph/9704359]



a scenario to have large θ near the bubble wall

$=$ spontaneous CP viol. $+$ small explicit CP viol.

to resolve degeneracy between CP conjugate bubbles

net BAU

$$\frac{n_B}{s} = \frac{\sum_j \left(\frac{n_B}{s}\right)_j N_j}{\sum_j N_j}$$

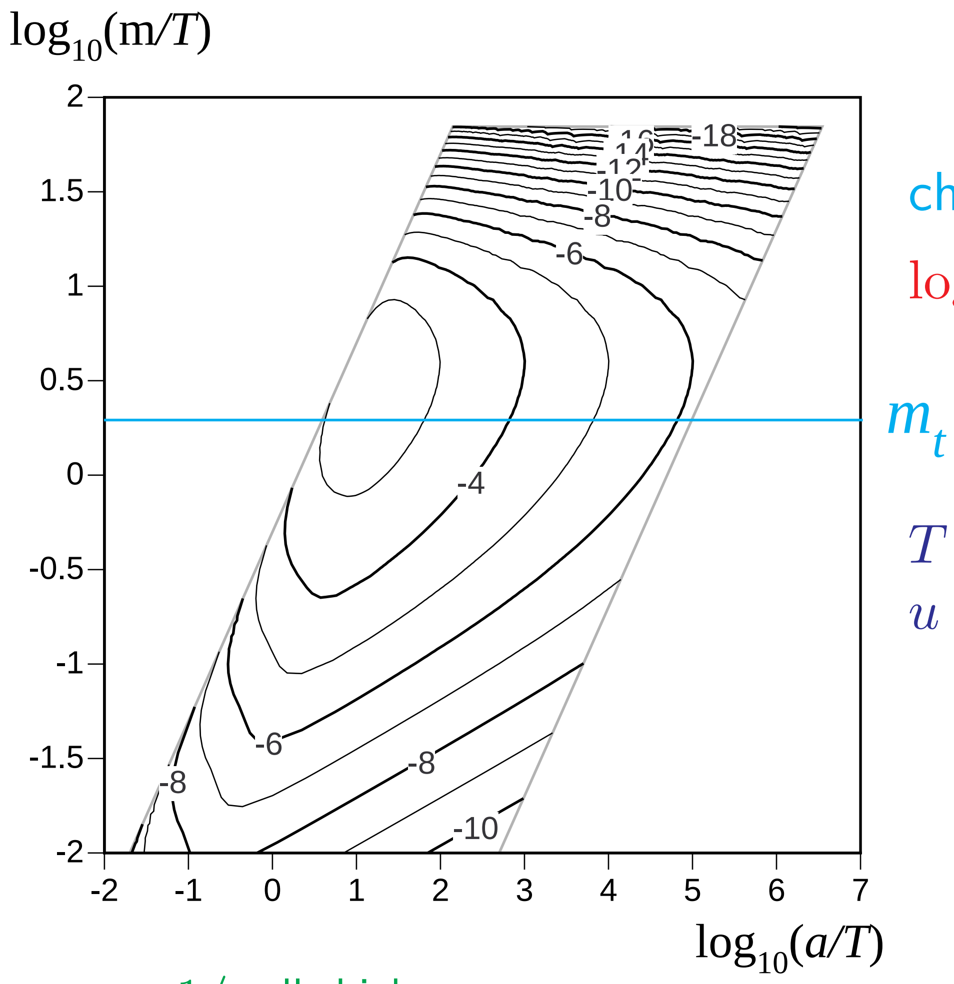
with

$$N_j = \exp(-4\pi R_C^2 \mathcal{E}_j / T_C)$$

$$\mathcal{E}_j = \text{energy density of the type-}j\text{ bubble}$$

an example:

$$m_3^2 \longrightarrow m_3^2 e^{-\delta} \text{ with } \delta = 10^{-3}$$



chiral charge flux

$$\log_{10} \frac{F_Q}{uT^3(Q_L - Q_R)}$$

$T = 100\text{GeV}$

$u = 0.58$

$a = 1/\text{wall thickness}$

II. Spontaneous CP Violation in the MSSM

Higgs potential

$$
\begin{aligned}
V_0 &= m_1^2 \varphi_d^\dagger \varphi_d + m_2^2 \varphi_u^\dagger \varphi_u + (m_3^2 \varphi_u \varphi_d + \text{h.c}) \\
&+ \frac{\lambda_1}{2}(\varphi_d^\dagger \varphi_d)^2 + \frac{\lambda_2}{2}(\varphi_u^\dagger \varphi_u)^2 + \lambda_3 (\varphi_u^\dagger \varphi_u)(\varphi_d^\dagger \varphi_d) \\
&+ \lambda_4 (\varphi_u \varphi_d)(\varphi_u \varphi_d)^* \\
&+ \left[\frac{\lambda_5}{2}(\varphi_u \varphi_d)^2 + (\lambda_6 \varphi_d^\dagger \varphi_d + \lambda_7 \varphi_u^\dagger \varphi_u)\varphi_u \varphi_d + \text{h.c} \right]
\end{aligned}
$$

for the MSSM

$$
\begin{aligned}
m_1^2 &= \tilde{m}_d^2 + |\mu|^2, \quad m_1^2 = \tilde{m}_u^2 + |\mu|^2, \quad m_3^2 = m_{3/2}\mu B, \\
\lambda_1 &= \lambda_2 = \frac{1}{4}(g_2^2 + g_1^2), \quad \lambda_3 = \frac{1}{4}(g_2^2 - g_1^2), \quad \lambda_4 = -\frac{1}{2}g_2^2, \\
\lambda_5 &= \lambda_6 = \lambda_7 = 0,
\end{aligned}
$$

$\Longrightarrow$ neither spontaneous nor explicit CP viol. in V_0

If we parametrize

$$
\varphi_d = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \varphi_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u e^{i\theta} \end{pmatrix},
$$

when all the parameters are real (no explicit CP viol.),

$$
V_0 = \frac{\lambda_5}{2} v_u^2 v_d^2 \left(\cos\theta + \frac{2m_3^2 - \lambda_6 v_d^2 - \lambda_7 v_u^2}{2\lambda_5 v_u v_d} \right)^2 + \ldots .
$$

In general, $\lambda_{5,6,7}$ are induced radiatively.

$$
\left.
\begin{array}{c}
\lambda_5 > 0 \\
\left| \frac{2m_3^2 - \lambda_6 v_d^2 - \lambda_7 v_u^2}{2\lambda_5 v_u v_d} \right| < 1
\end{array}
\right\} \Rightarrow \text{spontaneous } CP \text{ violion}
$$

At $T = 0$, bothered by the light scalar. [Maekawa]

Definition of effective parameters :

$$\bar{m}_3^2 = -\left.\frac{\partial^2 V_{\text{eff}}}{\partial v_1 \partial v_2}\right|_{v=0},$$

$$\bar{\lambda}_5 = \frac{1}{2}\left[\frac{\partial^4 V_{\text{eff}}}{\partial^2 v_1 \partial^2 v_2} - \frac{\partial^4 V_{\text{eff}}}{\partial^2 v_1 \partial^2 v_3}\right]_{v=0},$$

$$\bar{\lambda}_6 = -\frac{1}{3}\left.\frac{\partial^4 V_{\text{eff}}}{\partial^3 v_1 \partial v_2}\right|_{v=0},$$

$$\bar{\lambda}_7 = -\frac{1}{3}\left.\frac{\partial^4 V_{\text{eff}}}{\partial v_1 \partial^3 v_2}\right|_{v=0}$$

where $v_d = v_1, \quad v_u e^{i\theta} = v_2 + i v_3$

At one-loop level,
fermion loops yield positive contribution to $\bar{\lambda}_5$ at $T = 0$.
$\Uparrow$
chargino, neutralino

$$\left.\begin{array}{l}\text{bosonic contributions at } T = 0 \\ \textit{all} \text{ the contributions at } T > 0\end{array}\right\} \text{ to } \bar{\lambda}_5 \text{ are negative}$$

Here we present the results of one-loop approx. with contributions from
charged Higgs, stop, chargino and neutralino,

assuming that

- all the parameters are real and
- the gaugino mass parameters are the same: $M_2 = M_1$.

$$
\begin{aligned}
\bar{m}_3^2 &= (T\text{-independent renormalized value}) \\
&+ \frac{1}{2\pi^2}\left[\frac{1}{2}g_2^2 m_3^2 f_2^{(-)}\left(\frac{\mu_1}{T},\frac{\mu_2}{T}\right) + N_C y_t^2 \mu m_{3/2} A f_2^{(-)}\left(\frac{m_{\tilde{q}}}{T},\frac{m_{\tilde{t}}}{T}\right)\right. \\
&\qquad \left. +2\left(1+\frac{1}{\cos^2\theta_W}\right) g_2^2 \mu M_2 f_2^{(+)}\left(\frac{M_2}{T},\frac{\mu}{T}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
\bar{\lambda}_5 &= -\frac{g_2^4}{64\pi^2}\frac{m_3^4}{\mu_1^2\mu_2^2}K\left(\frac{\mu_1^2}{\mu_2^2}\right) - \frac{N_C y_t^4}{16\pi^2}\left(\frac{\mu m_{3/2}A}{m_{\tilde{q}} m_{\tilde{t}}}\right)^2 K\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{t}}^2}\right) \\
&+\frac{g_2^4}{8\pi^2}\left(1+\frac{2}{\cos^4\theta_W}\right) K\left(\frac{M_2^2}{\mu^2}\right) \\
&-\frac{1}{2\pi^2 T^4}\left[\frac{1}{4}g_2^4 m_3^4 f_4^{(-)}\left(\frac{\mu_1}{T},\frac{\mu_2}{T}\right)\right. \\
&\qquad + N_C y_t^4 (\mu m_{3/2} A)^2 f_4^{(-)}\left(\frac{m_{\tilde{q}}}{T},\frac{m_{\tilde{t}}}{T}\right) \\
&\qquad \left. +2g_2^4\left(1+\frac{2}{\cos^4\theta_W}\right) \mu^2 M_2^2 f_4^{(+)}\left(\frac{M_2}{T},\frac{\mu}{T}\right)\right]
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_{1,2}^2 &= \frac{m_1^2+m_2^2 \pm \sqrt{(m_1^2-m_2^2)^2+4m_3^4}}{2} \\
K(\alpha) &= \frac{\alpha}{(\alpha-1)^2}\left(\frac{\alpha+1}{\alpha-1}\log\alpha - 2\right)
\end{aligned}
$$

and

$$
\begin{aligned}
f_2^{(\mp)}(a,b) &= -\frac{1}{a^2-b^2}\int_0^\infty dx \left(\frac{x^2}{\sqrt{x^2+a^2}}\frac{1}{e^{\sqrt{x^2+a^2}}\mp 1}\right. \\
&\qquad \left. -\frac{x^2}{\sqrt{x^2+b^2}}\frac{1}{e^{\sqrt{x^2+b^2}}\mp 1}\right) \\
&> 0
\end{aligned}
$$

$$
\begin{aligned}
f_4^{(\mp)}(a,b) &= \frac{1}{2(a^2-b^2)^2}\left\{\int_0^\infty \frac{dx}{\sqrt{x^2+a^2}}\right. \\
&\quad \times\left(1+\frac{4x^2}{a^2-b^2}\right)\frac{1}{e^{\sqrt{x^2+a^2}}\mp 1}+(a\leftrightarrow b)\Big\} \\
&> 0
\end{aligned}
$$

$$
\begin{aligned}
\bar{\lambda}_6 &= \frac{g_2^4}{64\pi^2}\left\{\frac{m_3^2}{\mu_1^2}H\left(\frac{\mu_1^2}{\mu_2^2}\right)+\frac{m_3^2}{\mu_1^2\mu_2^2}\left[\mu_2^2-\frac{m_1^2}{2\cos^2\theta_W}-\left(1-\frac{1}{2\cos^2\theta_W}\right)m_2^2\right]K\left(\frac{\mu_1^2}{\mu_2^2}\right)\right\} \\
&\quad +\frac{N_C y_t^2}{16\pi^2}\left\{-\frac{1}{4}\left(\frac{g_1^2}{3}-g_2^2\right)\frac{\mu m_{3/2}A}{m_{\tilde{q}}^2}H\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{t}}^2}\right)+\frac{1}{3}g_1^2\frac{\mu m_{3/2}A}{m_{\tilde{t}}^2}H\left(\frac{m_{\tilde{t}}^2}{m_{\tilde{q}}^2}\right)\right. \\
&\quad\quad \left.+y_t^2\frac{\mu^3 m_{3/2}A}{m_{\tilde{q}}^2 m_{\tilde{t}}^2}K\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{t}}^2}\right)\right\} \\
&\quad -\frac{g_2^4}{8\pi^2}\left(1+\frac{2}{\cos^4\theta_W}\right)\frac{\mu}{M_2}\left[H\left(\frac{M_2^2}{\mu^2}\right)+K\left(\frac{M_2^2}{\mu^2}\right)\right] \\
&\quad +\frac{1}{2\pi^2}\left\{\frac{g_2^4 m_3^2}{4T^2}f_3^{(-)}\left(\frac{\mu_1}{T},\frac{\mu_2}{T}\right)\right. \\
&\quad\quad +\frac{g_2^4 m_3^2}{4T^4}\left[\mu_2^2-\frac{m_1^2}{2\cos^2\theta_W}-\left(1-\frac{1}{2\cos^2\theta_W}\right)m_2^2\right]f_4^{(-)}\left(\frac{\mu_1}{T},\frac{\mu_2}{T}\right) \\
&\quad\quad +N_C y_t^2\left[-\frac{1}{4}\left(\frac{g_1^2}{3}-g_2^2\right)\frac{\mu m_{3/2}A}{m_{\tilde{q}}^2}f_3^{(-)}\left(\frac{m_{\tilde{q}}}{T},\frac{m_{\tilde{t}}}{T}\right)\right. \\
&\quad\quad\quad +\frac{1}{3}g_1^2\frac{\mu m_{3/2}A}{m_{\tilde{t}}^2}f_3^{(-)}\left(\frac{m_{\tilde{t}}}{T},\frac{m_{\tilde{q}}}{T}\right) \\
&\quad\quad\quad \left.+y_t^2\frac{\mu^3 m_{3/2}A}{m_{\tilde{q}}^2 m_{\tilde{t}}^2}f_4^{(-)}\left(\frac{m_{\tilde{q}}}{T},\frac{m_{\tilde{t}}}{T}\right)\right] \\
&\quad\quad \left.+2g_2^4\left(1+\frac{2}{\cos^4\theta_W}\right)\left[\frac{\mu M_2}{T^2}f_3^{(-)}\left(\frac{M_2}{T},\frac{\mu}{T}\right)+\frac{\mu^3 M_2}{T^4}f_4^{(-)}\left(\frac{M_2}{T},\frac{\mu}{T}\right)\right]\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\bar{\lambda}_7 - \bar{\lambda}_6 \\
= \quad & \frac{g_2^4}{8\pi^2} \tan^2\theta_W \,(m_1^2 - m_2^2) m_3^2 \left[\frac{1}{8\mu_1^2\mu_2^2} K\left(\frac{\mu_1^2}{\mu_2^2}\right) + \frac{1}{T^4} f_4^{(-)}\left(\frac{\mu_1}{T}, \frac{\mu_2}{T}\right) \right] \\
+ \quad & \frac{N_C y_t^2}{16\pi^2} \mu m_{3/2} A \left\{ \left[\frac{1}{2}\left(\frac{g_1^2}{3} - g_2^2\right) + y_t^2 \right] \frac{1}{m_{\tilde{q}}^2} H\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{t}}^2}\right) + \left(-\frac{2}{3} g_1^2 + y_t^2\right) \frac{1}{m_{\tilde{t}}^2} H\left(\frac{m_{\tilde{t}}^2}{m_{\tilde{q}}^2}\right) \right. \\
& \qquad \left. + y_t^2 \frac{(m_{3/2}A)^2 - \mu^2}{m_{\tilde{q}}^2 m_{\tilde{t}}^2} K\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{t}}^2}\right) \right\} \\
& + \frac{N_C y_t^2}{2\pi^2} \mu m_{3/2} A \left\{ \left[\frac{1}{2}\left(\frac{g_1^2}{3} - g_2^2\right) + y_t^2 \right] \frac{1}{m_{\tilde{q}}^2} f_3^{(-)}\left(\frac{m_{\tilde{q}}}{T}, \frac{m_{\tilde{t}}}{T}\right) \right. \\
& \qquad + \left(-\frac{2}{3} g_1^2 + y_t^2\right) \frac{1}{m_{\tilde{t}}^2} f_3^{(-)}\left(\frac{m_{\tilde{t}}}{T}, \frac{m_{\tilde{q}}}{T}\right) \\
& \qquad \left. + y_t^2 \frac{(m_{3/2}A)^2 - \mu^2}{m_{\tilde{q}}^2 m_{\tilde{t}}^2} f_4^{(-)}\left(\frac{m_{\tilde{q}}}{T}, \frac{m_{\tilde{t}}}{T}\right) \right\}
\end{aligned}
$$

where

$$
H(\alpha) = \frac{\alpha}{\alpha - 1}\left(\frac{1}{\alpha - 1}\log\alpha - 1\right)
$$

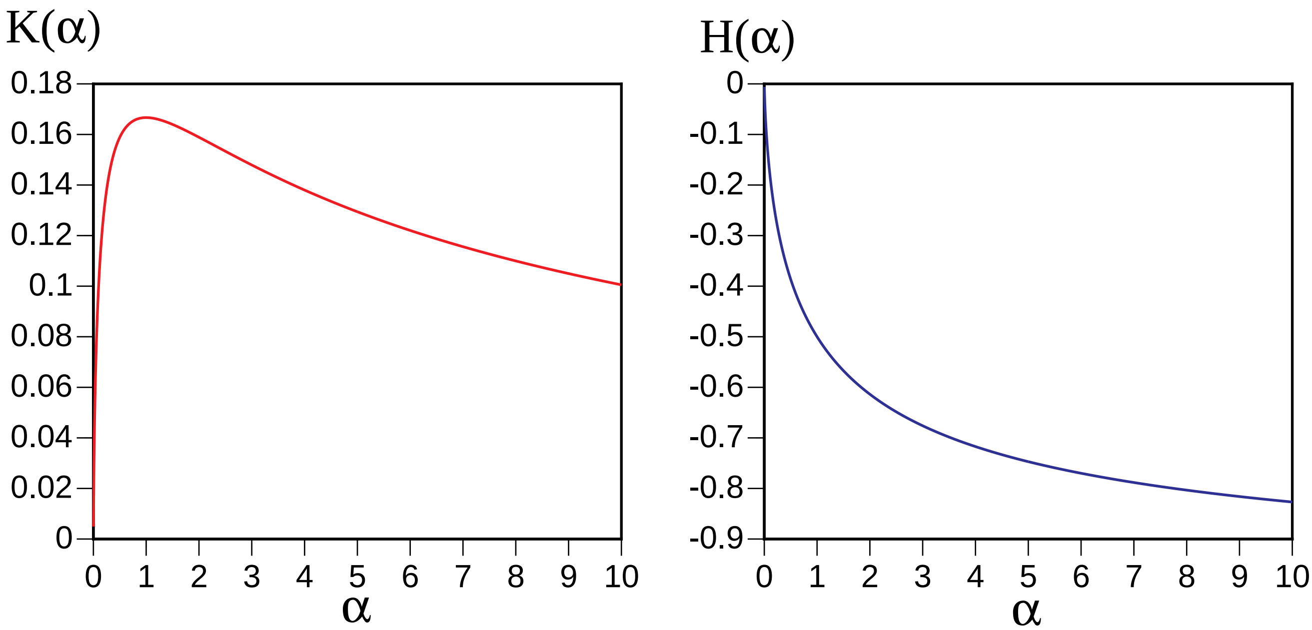
and

$$
\begin{aligned}
f_3^{(\mp)}(a, b) \quad = \quad & \frac{1}{2(a^2 - b^2)} \int_0^\infty \frac{dx}{\sqrt{x^2 + a^2}} \frac{1}{e^{\sqrt{x^2+a^2}} \mp 1} \\
& + \frac{1}{(a^2 - b^2)^2} \int_0^\infty dx \left(\frac{x^2}{\sqrt{x^2 + a^2}} \frac{1}{e^{\sqrt{x^2+a^2}} \mp 1} \right. \\
& \qquad \left. - \frac{x^2}{\sqrt{x^2 + b^2}} \frac{1}{e^{\sqrt{x^2+b^2}} \mp 1} \right) \\
< \quad & 0
\end{aligned}
$$

III. Numerical Analysis

(i) $\bar{\lambda}_5 > 0$?

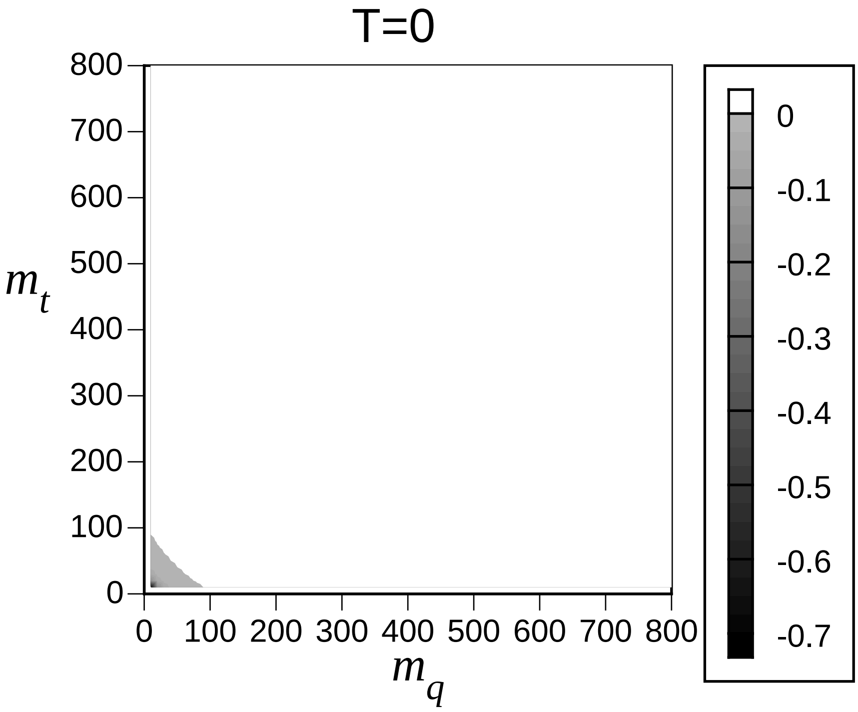
the postive contribution is maximum for $\mu = M_2$.

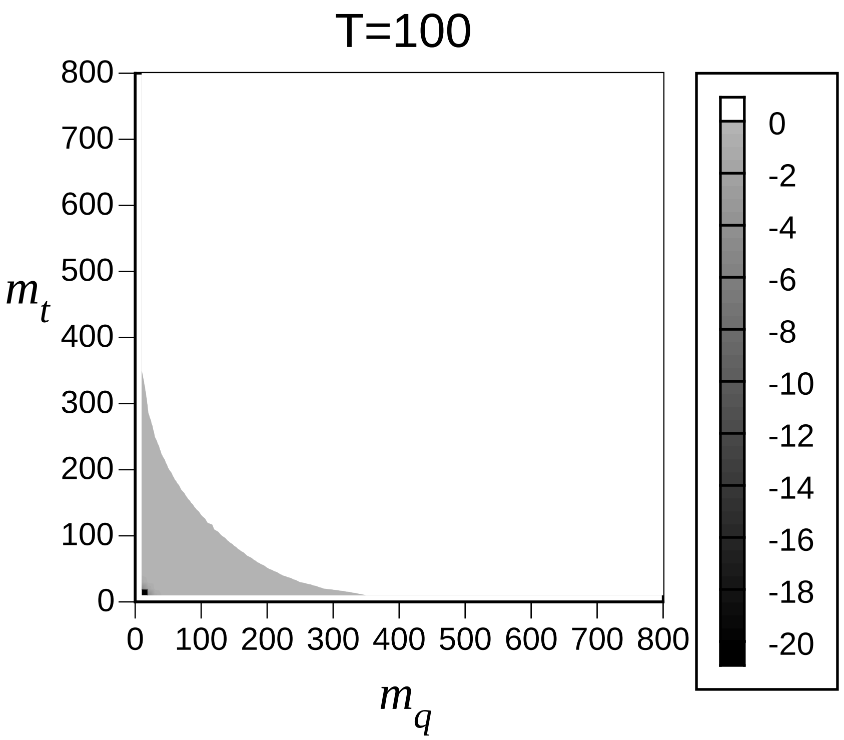


We calculated $\bar{\lambda}_5$ as a function of $(m_{\tilde{q}}, m_{\tilde{t}})$.

	m_1	m_2	m_3^2	$m_{3/2}A$	$\mu = M_2$	$\exists(m_{\tilde{q}}, m_{\tilde{t}})$ s.t. $\bar{\lambda}_5 > 0$?
set 1	200	150	2500	100	100	×
set 2	300	300	400	50	100	×
set 3	400	400	400	20	50	×
set 4	300	300	400	30	50	○
set 5	300	300	-100	30	50	○
set 6	300	300	-200	$10 \sim 30$	200	○
set 7	300	300	-300	$10 \sim 30$	300	○

e.g. for set 4, the value of $\bar{\lambda}_5$





For $\bar{\lambda}_5 > 0$, smaller $m_{3/2}A$ in the stop contribution is favored.

$[\because \quad \Delta_{\tilde{t}}\lambda_5 \propto N_C y_t^4]$

(ii) $-1 < c(v_u, v_d) \equiv \dfrac{2m_3^2 - \lambda_6 v_d^2 - \lambda_7 v_u^2}{2\lambda_5 v_u v_d} < 1$?

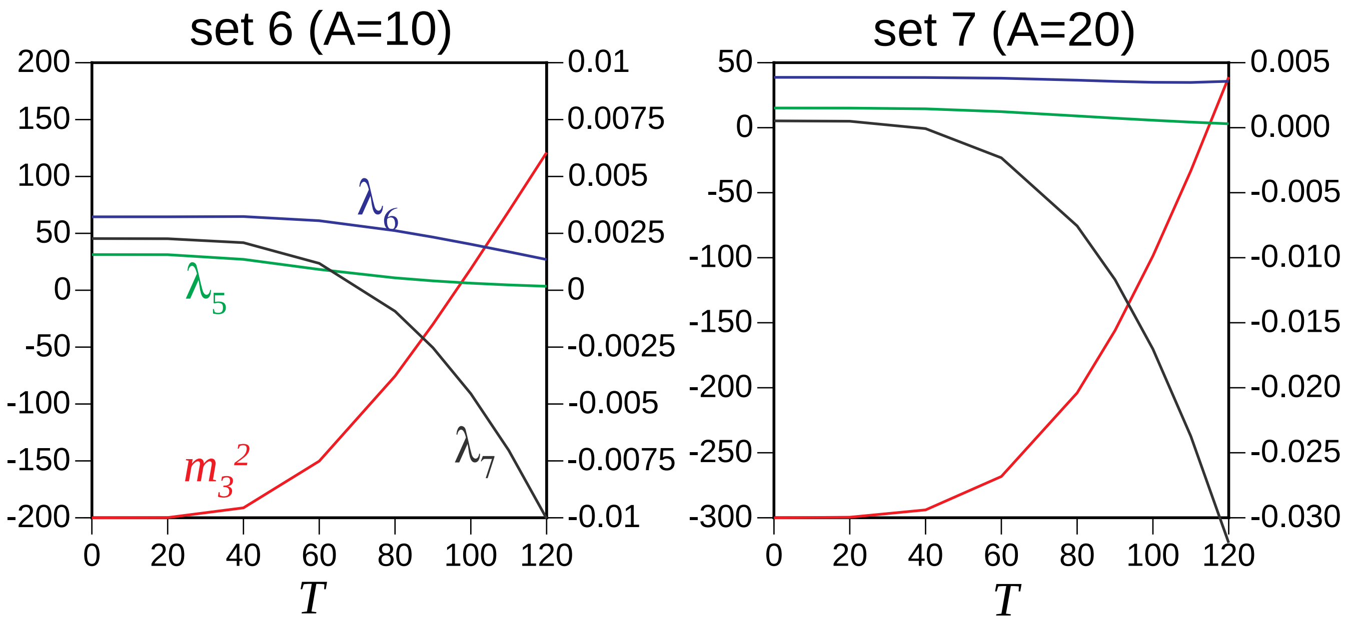
Since $\lambda_{5,6,7} \sim O(10^{-4\sim-3})$,

$$|\bar{m}_3^2| \sim O(\lambda_{5,6,7} \times v_{u,d}^2) \sim O(10^{2\sim3})$$

will be needed.

finite-T corrections to m_3^2 are always positve and nearly $\propto T^2$.

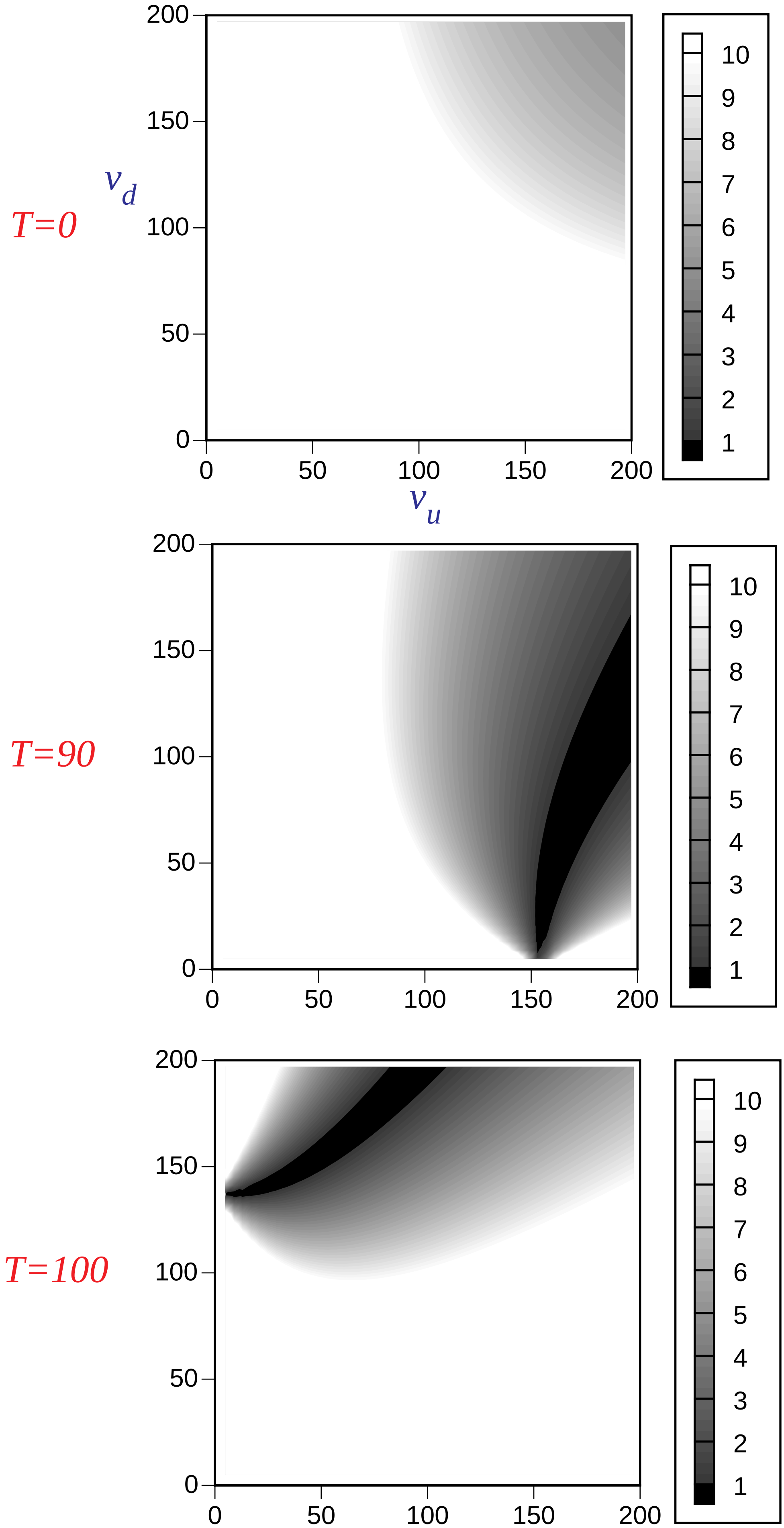
T-dependence of the parameters : $(m_{\tilde{q}}, m_{\tilde{t}}) = (400, 50)$



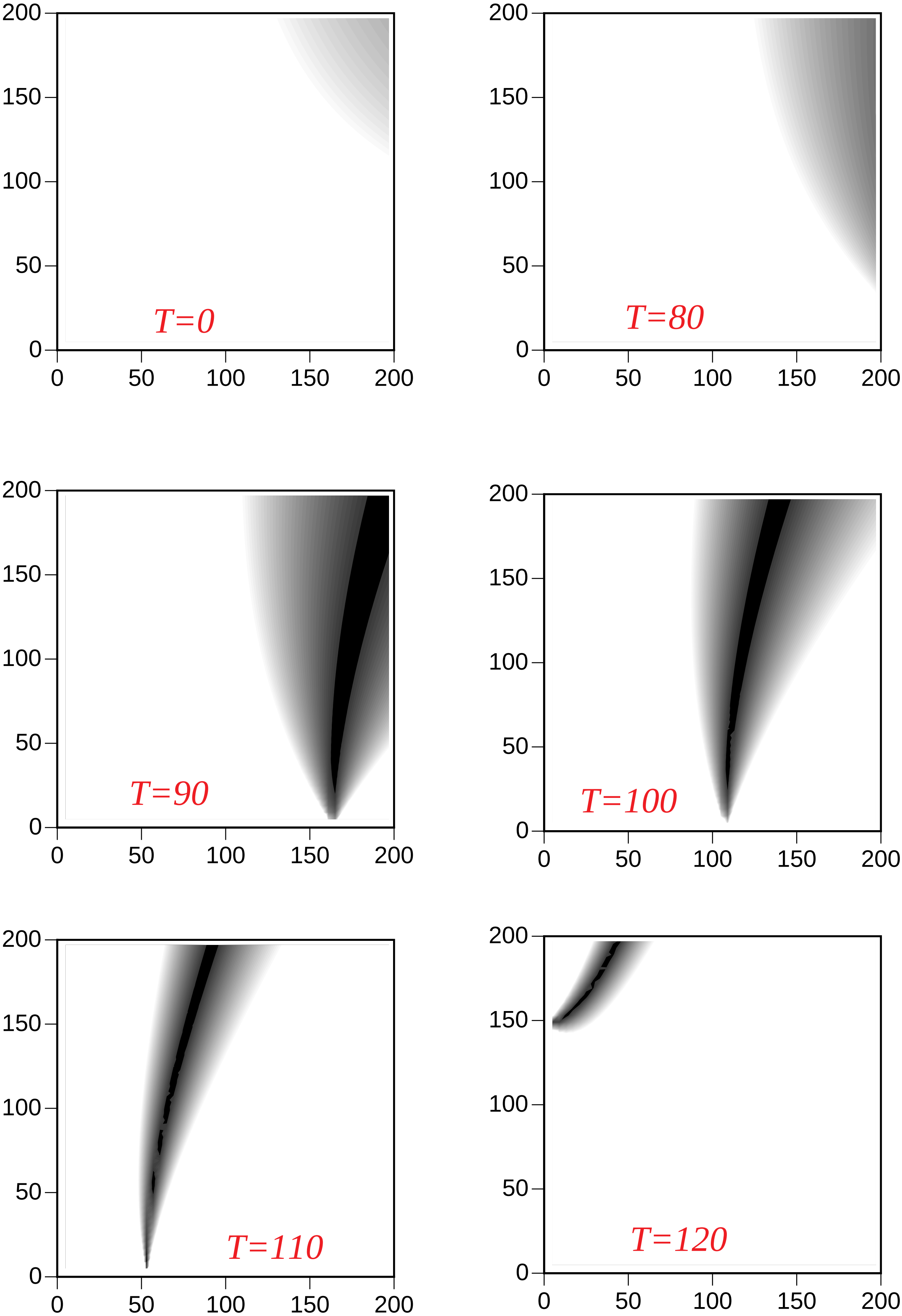
Is there (v_u, v_d) satisfying $|c(v_u, v_d)| < 1$?

$\Downarrow$

plot of $c(v_u, v_d)$ vs (v_u, v_d) for various temperatures for the parameter set 6 with $m_{3/2}A = 10$



for the parameter set 7 with $m_{3/2}A = 20$



IV. Discussions

To have large CP violation near the EW bubble wall by the spontaneous CPV mechanism,

1. for $\bar{\lambda}_5 > 0$,
   - smaller $m_{3/2}A$ is favored ($\Longleftrightarrow$ smaller stop contribution)
   - larger $m_{\tilde{q}} \cdot m_{\tilde{t}}$ is favored ($\Longleftrightarrow$ smaller stop contribution)
2. for $^{\exists}(v_u, v_d)$ s.t. $|c(v_u, v_d)| < 1$,

   $$|\bar{m}_3^2| \sim O(10^{2\sim 3}) \Longleftrightarrow m_3^2 = -O(10^{2\sim 3})$$

We found that
for some sets of the parameters, CP is spontaneosly violated at $T \neq 0$ but not at $T = 0$.

problems:

▷ The values of T and (v_u, v_d) near the EWPT are crucial.

$\Uparrow$

global structure of the effective potential $V_{\rm eff}(v_u, v_d, \theta; T)$

▷ finite-T effects on explicit CP violation