Status of the Electroweak Baryogenesis

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our goal

Baryon Asymmetry of the Universe

$$\frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} \simeq (0.37 - 0.88) \times 10^{-10}$$

key words

- sphaleron — leptogenesis: $L \longrightarrow B$
- electroweak phase transition (EWPT) — related to Higgs physics

CP violation, $\cdots$

content

- Introduction
  - Saharov's conditions
  - Baryogenesis in GUTs and the others
- Sphaleron Process
- Electroweak Baryogenesis
  - EW Phase Transition vs Higgs mass
- Summary

3 requirements for generation of the BAU [Sakharov, '67]

(1) baryon number violation

(2) C and CP violation

(3) departure from equilibrium

$\because$ (2) If C *or* CP is conserved, no B is generated: $\Leftarrow$ B is odd under C and CP.

indeed . . .

ρ_0 : baryon-symmetric initial state of the universe $s.t.$

$$\langle n_B \rangle_0 = \mathrm{Tr}[\rho_0 n_B] = 0$$

time evolution of $\rho \Longleftrightarrow$ Liouville eq.: $i\hbar \dfrac{\partial \rho}{\partial t} + [\rho, H] = 0$

If H is C- *or* CP-invariant, $[\rho, \mathcal{C}] = 0$ *or* $[\rho, \mathcal{CP}] = 0$ [spont. CP viol. $\Rightarrow [\rho, \mathcal{CP}] \neq 0$]

Since $\mathcal{C}B\mathcal{C}^{-1} = -B$ and $\mathcal{CP}B(\mathcal{CP})^{-1} = -B$ [*i.e.*, B is vectorlike, odd under C.]

$$\langle n_B \rangle = \mathrm{Tr}[\rho n_B] = \mathrm{Tr}[\rho \mathcal{C} n_B \mathcal{C}^{-1}] = -\mathrm{Tr}[\rho n_B]$$

or

$$\langle n_B \rangle = \mathrm{Tr}[\rho \mathcal{CP} n_B (\mathcal{CP})^{-1}] = -\mathrm{Tr}[\rho n_B]$$

$\therefore$ Both C and CP must be violated to have $\langle n_B \rangle \neq 0$, starting from $\langle n_B \rangle_0 = 0$.

possiblities ?

- B violation
  - explicit violation — GUTs
  - spontaneous viol. — $\langle\text{squark}\rangle \neq 0$
  - chiral anomaly — sphaleron process

  It must be suppressed at present for protons not to decay.

- C violation $\Longleftarrow$ chiral gauge interactions (EW, GUTs)

- CP violation
  - KM phase in the MSM
  - beyond the SM — SUSY, extended Higgs sector

- out of equilibrium
  - expansion of the universe — $\Gamma_{\Delta B \neq 0} \simeq H(T)$
  - first-order phase transition
  - reheating (or preheating) after inflation

the first example — GUTs [Yoshimura, PRL '78]

$SU(5)$ model:

matter: $\left\{ \begin{array}{lll} \mathbf{5}^* : \psi^i_L & \ni & d^c_R, l_L \\ \mathbf{10} : \chi_{[ij]L} & \ni & q_L, u^c_R, e^c_R \end{array} \right.$ gauge: $A_\mu = \begin{pmatrix} G_\mu, B_\mu & X^{a\alpha}_\mu \\ X^{a\alpha}_\mu & W_\mu, B_\mu \end{pmatrix}$

$i = 1-5 \;\longrightarrow\; (\alpha = 1-3, a = 1, 2)$

$$\mathcal{L}_{\rm int} \ni g\bar{\psi}\gamma^\mu A_\mu \psi + g{\rm Tr}\left[\bar{\chi}\gamma^\mu\{A_\mu, \chi\}\right]$$
$$\ni gX^a_{\alpha\mu}\left[\varepsilon^{\alpha\beta\gamma}\bar{u}^c_{R\gamma}\gamma^\mu q_{L\beta a} + \epsilon_{ab}\left(\bar{q}^\alpha_{Lb}\gamma^\mu e^c_R + \bar{l}_{Lb}\gamma^\mu d^{c\alpha}_R\right)\right]$$

in the decay of X-$\bar{X}$ pairs

process	br. ratio	ΔB
$X \longrightarrow qq$	r	$2/3$
$X \longrightarrow \bar{q}\bar{l}$	$1-r$	$-1/3$
$\bar{X} \longrightarrow \bar{q}\bar{q}$	$\bar{r}$	$-2/3$
$\bar{X} \longrightarrow q, l$	$1-\bar{r}$	$1/3$

$$\langle \Delta B \rangle = \frac{2}{3}r - \frac{1}{3}(1-r) - \frac{2}{3}\bar{r} + \frac{1}{3}(1-\bar{r}) = r - \bar{r}$$

$\therefore$ C *or* CP is conserved($r = \bar{r}$)

$\Longrightarrow \Delta B = 0$

If the inverse process is suppressed, $B \propto r - \bar{r}$ is generated.

At $T \simeq m_X$, decay rate of X $= \Gamma_D \simeq \alpha m_X$ $\alpha \sim 1/40$ for gauge boson,

$\Gamma_D \simeq H(T \simeq m_X) \Longrightarrow$ decay and production of $X\bar{X}$ are out of equilibrium

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The $SU(5)$ GUT model conserves $B - L$. *i.e.* $(B + L)$-genesis

$\Downarrow$

Any B is washed-out by the sphaleron process, as we shall see later

$\downarrow$

new varieties of baryogenesis

e.g. Leptogenesis $\Rightarrow$ BAU $B = -L$

- $^\exists$Majorana neutrino $\Rightarrow$ L-violating interaction [Fukugita & Yanagida, PL174B]

  $\left.\begin{array}{c}\text{decoupling of heavy-}\nu\text{ decay}\\ CP\text{ violation in the heavy -}\nu\text{ sector}\end{array}\right\} \Rightarrow \text{Leptogenesis} \overset{\text{sphaleron}}{\Longrightarrow} \text{BAU}$

  [review: Buchmüller et al., hep-ph/0401240]

- Affleck-Dine mechanism in a SUSY model [A-D, NPB249; Dine, et al., NPB458]

  $\langle\tilde{q}\rangle \neq 0$ or $\langle\tilde{l}\rangle \neq 0$ along (nearly) flat directions, at high temperature

  coherent motion of complex $\langle\tilde{q}\rangle$, $\langle\tilde{l}\rangle \neq 0$ $B(L)$, C, CP viol.

  $\Longrightarrow$ B- and/or L-genesis

- Electroweak Baryogenesis

(1) $\Delta(B+L) \neq 0$ $\begin{cases}\text{enhanced by sphaleron at } T > T_C \\ \text{suppressed by instanton at } T = 0\end{cases}$

(2) C-violation (chiral gauge); CP-violation: KM phase or extension of the MSM

(3) first-order EWPT with expanding bubble walls

- topological defects

  EW string, domain wall $\sim$ EW baryogenesis effective volume is too small

Sphaleron Process

⋆ Anomalous fermion number nonconservation ⟸ axial anomaly in the SM

$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}]$$

$$\partial_\mu j^\mu_{B-L} = 0$$

N_f = number of the generations

$\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

integrating these equations,

$$\begin{aligned} B(t_f) - B(t_i) &= \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x \left[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right] \\ &= N_f\,[N_{CS}(t_f) - N_{CS}(t_i)] \end{aligned}$$

where N_{CS} is the Chern-Simons number:
in the $A_0 = 0$ gauge,

$$N_{CS}(t) = \frac{1}{32\pi^2}\int d^3x\, \epsilon_{ijk}\left[g^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g'^2 B_{ij}B_k\right]_t$$

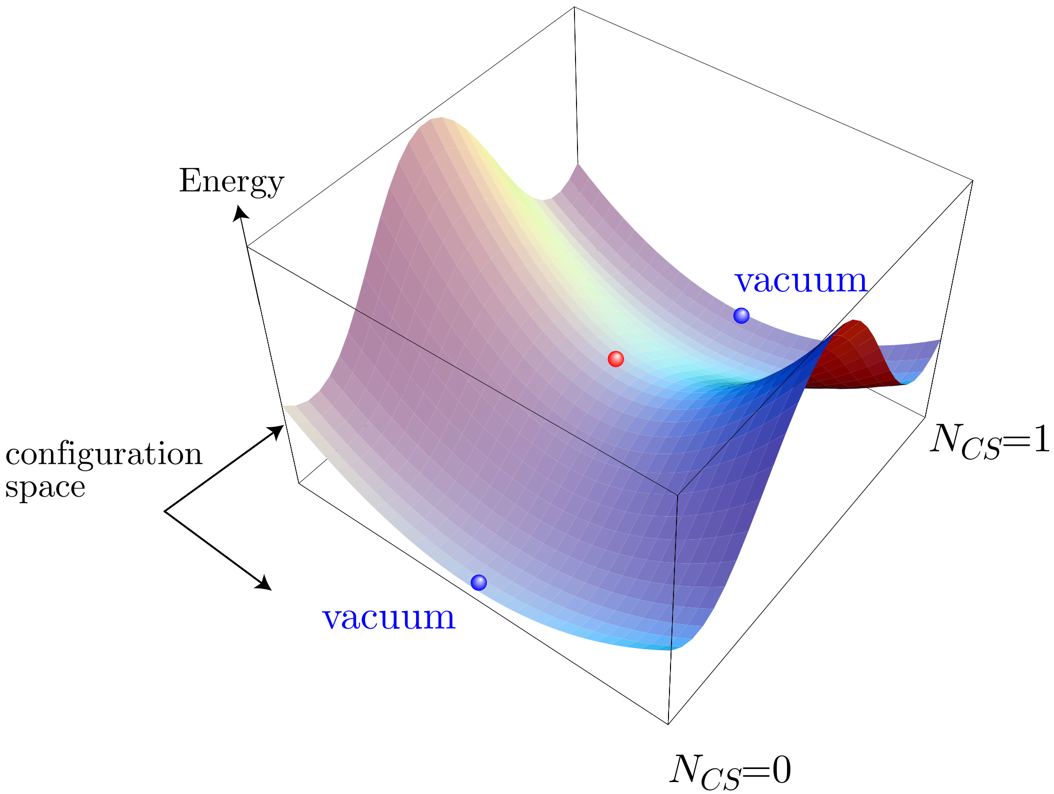
classical vacua of the gauge sector: $\mathcal{E} = \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2) = 0$

$\Longleftrightarrow F_{\mu\nu} = B_{\mu\nu} = 0$

$\Longleftrightarrow A = iU^{-1}dU$ and $B = dv$ with $U \in SU(2)$

$\therefore \; U(\boldsymbol{x}) : S^3 \ni \boldsymbol{x} \longrightarrow U \in SU(2) \simeq S^3$

$\pi_3(S^3) \simeq \mathbf{Z} \Rightarrow$ $U(\boldsymbol{x})$ is classified by an integer N_{CS}

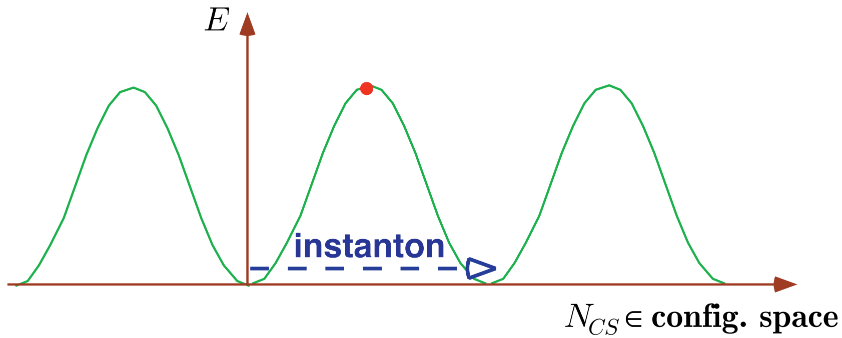


background U changes with $\Delta N_{CS} = 1$
$\Longrightarrow \Delta B = 1$ $(\Delta L = 1)$ in each (left-) generation

$\Longleftrightarrow$ fermion:
- level crossing
- index theorem

Transition of the field config. with $\Delta B \neq 0$

- ▷ quantum tunneling — low temperature
- ▷ thermal activation — high temperature



transition rate with $N_{CS} = 1$ $\Longleftarrow$ WKB approx.

At $T = 0$, tunneling amplitude $\simeq e^{-S_{\text{instanton}}} = e^{-4\pi^2/g^2}$

instanton
- ⋆ 4d solution with finite euclidean action
- ⋆ integer Pontrjagin index $\sim \int d^4x_E\, F_{\mu\nu}\tilde{F}^{\mu\nu}$

What is Sphaleron ?

sphaleros : $\sigma\varphi\alpha\lambda\epsilon\rho o\sigma$ = 'ready to fall'

a saddle-point solution of 4d $SU(2)$ gauge-Higgs system

[Klinkhammer & Manton, PRD30 ('84)]

$E_{\rm sph} = 8 - 14$ TeV

- ⋆ unstable
- ⋆ static (3d) solution with finite energy
- ⋆ Chern-Simons No. = "1/2"

$\Longrightarrow$ over-barrier transition at finite temperature

$$\Gamma_{\rm sph} \sim e^{-E_{\rm sph}/T}$$

cf. for EW theory

$$\Gamma_{\rm tunneling} \sim e^{-2S_{\rm instanton}} = 10^{-164}$$

⋆ Transition rate

[Arnold and McLerran, P.R.D36('87)]

♣ $\frac{\omega_-}{(2\pi)} \lesssim T \lesssim T_C$ ω_-:negative-mode freq. around the sphaleron

$$\Gamma^{(b)}_{\rm sph} \simeq k\, \mathcal{N}_{\rm tr}\, \mathcal{N}_{\rm rot}\, \frac{\omega_-}{2\pi} \left(\frac{v^2}{T}\right)^3 e^{-E_{\rm sph}/T}$$

$\mathcal{N}_{\rm tr} = 26$, $\mathcal{N}_{\rm rot} = 5.3 \times 10^3 \leftarrow$ zero modes

$\omega_-^2 \simeq (1.8 \sim 6.6) m_W^2$ for $10^{-2} \leq \lambda/g^2 \leq 10$, $k \simeq O(1)$

♣ $T \gtrsim T_C$ symmetric phase — no mass scale

$$\Gamma^{(s)}_{\rm sph} \simeq \kappa(\alpha_W T)^4$$

▷ Monte Carlo simulation $\langle N_{CS}(t) N_{CS}(0)\rangle = \langle N_{CS}\rangle^2 + A e^{-2\Gamma V t}$ as $t \to \infty$

$\kappa > 0.4$	$SU(2)$ gauge-Higgs system	[Ambjørn, et al. N.P.B353('91)]
$\kappa = 1.09 \pm 0.04$	$SU(2)$ pure gauge system	[Ambjørn and Krasnitz, P.L.B362('95)]

'sphaleron transition' even in the symmetric phase.

B and L in the Hot Universe

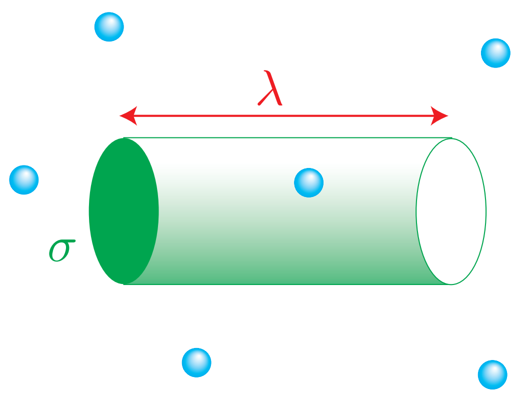
reaction rate: $\Gamma(T) > H(t) \Longleftrightarrow$ the process is in chemical equilibrium

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} \simeq \sqrt{\frac{8\pi G}{3}\rho} \simeq 1.66\sqrt{g_*}\frac{T^2}{m_{Pl}}$$

$\Gamma(T) \longrightarrow$ time scale of interactions

mean free path : $\lambda \cdot \sigma = \dfrac{1}{n}$

$m \ll T \Rightarrow \lambda \simeq \bar{t} =$ mean free time



$$n = g\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{e^{\omega_k/T} \mp 1} \overset{m \ll T}{\simeq} g\begin{cases}\dfrac{\zeta(3)}{\pi^2}T^3 \\ \dfrac{3}{4}\dfrac{\zeta(3)}{\pi^2}T^3\end{cases}$$

$$\overset{m \gg T}{\simeq} g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$\zeta(3) = 1.2020569\cdots$

For relativistic particles at T, $\sigma \simeq \frac{\alpha^2}{s} \simeq \frac{\alpha^2}{T^2}$, we have $\lambda \simeq \frac{10}{gT^3}\left(\frac{\alpha^2}{T^2}\right)^{-1} = \frac{10}{g\alpha^2 T}$.

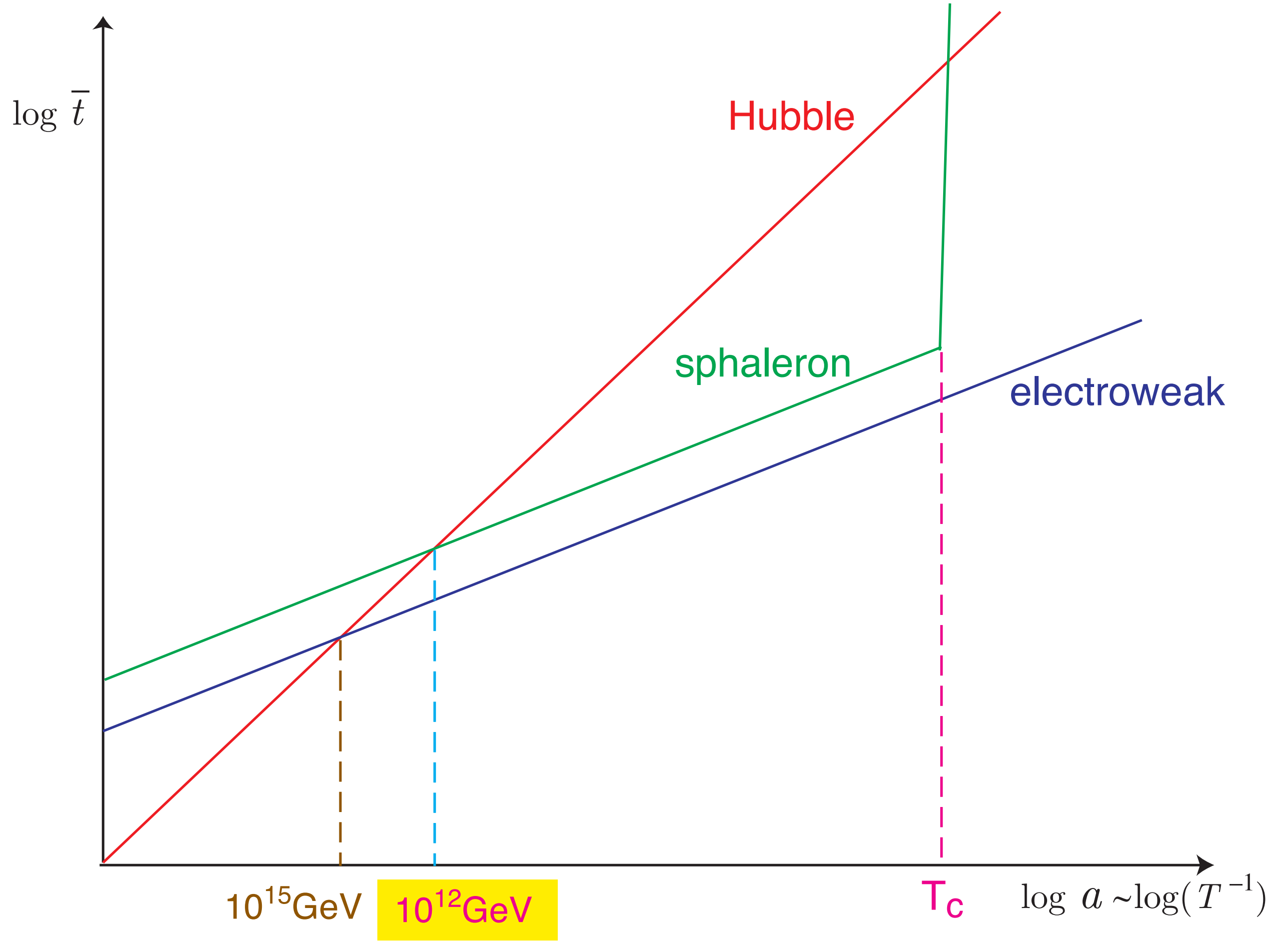
For $T = 100\text{GeV}$, $H^{-1} \simeq 10^{14}\text{GeV}^{-1}$,

$\lambda_s \simeq \frac{1}{\alpha_s{}^2 T} \sim 1\,\text{GeV}^{-1}$	for strong interactions
$\lambda_{EW} \simeq \frac{1}{\alpha_W{}^2 T} \sim 10\,\text{GeV}^{-1}$	for EW interactions
$\lambda_Y \simeq \left(\frac{m_W}{m_f}\right)^4 \lambda_{EW}$	for Yukawa interactions

time scale of sphaleron process

$$\bar{t}_{\text{sph}} = (\Gamma_{\text{sph}}/n)^{-1} \simeq \begin{cases} 10^6 T^{-1}\,\text{GeV}^{-1} & (T > T_C) \\ 10^5 T^{-1} e^{E_{\text{sph}}/T}\,\text{GeV}^{-1} & (T < T_C) \end{cases}$$

[*cf.* $E_{\text{sph}} \simeq 10\text{TeV}$ for $v_0 = 246\text{GeV}$]



If $v(T_C) \ll 200\text{GeV}$ (eg. 2nd order EWPT), $^{\exists}T_{\text{dec}}$, $s.t.$

$$T_{\text{dec}} < T < T_C \implies \Gamma_{\text{sph}}^{(b)}(T) > H(T)$$

wash-out of $B+L$ even in the broken phase

$\star$ Quantum numbers in equilibrium

Q_i: conserved quantum number $\qquad [H, Q_i] = 0$

equilibrium partition function: $Z(T, \mu) \equiv \mathrm{Tr}\left[e^{-(H - \sum_i \mu_i Q_i)/T}\right]$

$$\Rightarrow \langle Q_i \rangle (T, \mu) = T \frac{\partial}{\partial \mu_i} \log Z(T, \mu)$$

$\longrightarrow$ relations among μ's $\Longleftrightarrow$ relations among Q's

In the SM, $Q_i = \frac{1}{N} B - L_i$ without lepton-flavor mixing.

1st-principle calculation of $Z(T, \mu)$

- path integral over *all* fields
- *nonperturbative* $B + L$ violation

$\Downarrow$

- perturbation
- free-field approximation

[Shaposhnikov, et al, PLB387 ('96); PRD61 ('00)]

chemical potentials of the particles

number density of free particles (per degree of freedom)

$$\langle N \rangle = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\frac{1}{e^{(\omega_k-\mu)/T}\mp 1} - \frac{1}{e^{(\omega_k+\mu)/T}\mp 1}\right]$$

$$\overset{m\ll T}{\simeq} \frac{T^3}{2\pi^2}\int_0^\infty dx \left[\frac{x^2}{e^{x-\mu/T}\mp 1} - \frac{x^2}{e^{x+\mu/T}\mp 1}\right] \overset{|\mu|\ll T}{\simeq} \begin{cases} \frac{T^3}{3}\cdot\frac{\mu}{T}, & \text{(bosons)} \\ \frac{T^3}{6}\cdot\frac{\mu}{T}, & \text{(fermions)} \end{cases}$$

Quantum number densities in terms of μ

[Harvey & Turner, PRD42 ('90)]

SM with N generations and N_H Higgs doublets $(\phi^0\,\phi^-)$

W^-	$u_{L(R)}$	$d_{L(R)}$	$e_{iL(R)}$	ν_{iL}	ϕ^0	ϕ^-
μ_W	$\mu_{u_{L(R)}}$	$\mu_{d_{L(R)}}$	$\mu_{iL(R)}$	μ_i	μ_0	μ_-

gauge int., Yukawa int, quark mixings are in equilibrium.

$$\mu_\gamma = \mu_Z = \mu_{\text{gluon}} = 0$$

$$\Downarrow$$

$(3N+7)$ μ's

$$\text{gauge} \quad \Leftrightarrow \quad \mu_W = \mu_{d_L} - \mu_{u_L} = \mu_{iL} - \mu_i = \mu_- + \mu_0$$

$$\text{Yukawa} \quad \Leftrightarrow \quad \mu_0 = \mu_{u_R} - \mu_{u_L} = \mu_{d_L} - \mu_{d_R} = \mu_{iL} - \mu_{iR}$$

$2(N+2)$ relations $\Rightarrow$ $N+3$ independent μ's: $(\mu_W, \mu_0, \mu_{u_L}, \mu_i)$

sphaleron process in equilibrium: $|0\rangle \leftrightarrow \prod_i (u_L d_L d_L \nu_L)_i \Leftrightarrow N(\mu_{u_L} + 2\mu_{d_L}) + \sum_i \mu_i = 0$

Quantum number densities [in unit of $T^2/6$]

$$\begin{aligned}
B &= N(\mu_{u_L} + \mu_{u_R} + \mu_{d_L} + \mu_{d_R}) = 4N\mu_{u_L} + 2N\mu_W, \\
L &= \sum_i (\mu_i + \mu_{iL} + \mu_{iR}) = 3\mu + 2N\mu_W - N\mu_0 \\
Q &= \frac{2}{3}N(\mu_{u_L} + \mu_{u_R}) \cdot 3 - \frac{1}{3}N(\mu_{d_L} + \mu_{d_R}) \cdot 3 - \sum_i (\mu_{iL} + \mu_{iR}) - 2 \cdot 2\mu_W - 2N_H\mu_- \\
&= 2N\mu_{u_L} - 2\mu - (4N + 4 + 2N_H)\mu_W + (4N + 2N_H)\mu_0 \\
I_3 &= -(2N + N_H + 4)\mu_W \qquad \mu \equiv \sum_i \mu_i
\end{aligned}$$

- $T \gtrsim T_C$ (symmetric phase) We require $Q = I_3 = 0$. ($\mu_W = 0$)

$$B = \frac{8N + 4N_H}{22N + 13N_H}(B - L), \qquad L = -\frac{14N + 9N_H}{22N + 13N_H}(B - L)$$

- $T \lesssim T_C$ (broken phase) $Q = 0$ and $\mu_0 = 0$ ($\because$ ϕ^0 condensates.)

$$B = \frac{8N + 4(N_H + 2)}{24N + 13(N_H + 2)}(B - L), \qquad L = -\frac{16N + 9(N_H + 2)}{24N + 13(N_H + 2)}(B - L)$$

$\therefore$ Once $B - L = 0$ in the era when the sphaleron is in equilibrium,

$\Downarrow$

$$\boldsymbol{B = L = 0}$$

To have nonzero BAU,

(i) we must have $B - L$ before the sphaleron process decouples, or

(ii) $B + L$ must be created at the first-order EWPT, *and* the sphaleron process must decouple immediately after that.

N.B.

$\Delta(B+L) \neq 0$ process is in equilibrium, for $T_C \simeq 100\text{GeV} < T < 10^{12}\text{GeV}$.

If $\Delta L \neq 0$ process is in equilibrium in this range of T, $B = L = 0!$

To leave $B \neq 0$, $\Gamma_{\Delta L \neq 0} < H(T)$ for $T \in [T_C, 10^{12}\text{GeV}]$.

$\Longrightarrow$ constraints on models with $\Delta L \neq 0$ processes.

$\longrightarrow$
- lower (upper) bound on m_N (m_ν) [Fukugita & Yanagida, PRL89]
- Hasegawa's talk

Electroweak Baryogenesis

review articles:

- KF, Prog.Theor.Phys. 96 (1996) 475
- Rubakov and Shaposhnikov, Phys.Usp. 39 (1996) 461-502 (hep-ph/9603208)
- Riotto and Trodden, Ann. Rev. Nucl. Part. Sci. 49 (1999) 35 (hep-ph/9901362)
- Bernreuther, Lect.Notes Phys. 591 (2002) 237 (hep-ph/0205279)

$$T \simeq 100\text{GeV} \Rightarrow \quad H^{-1}(T) \simeq 10^{14}\text{GeV}^{-1} \ll \bar{t}_{EW} \simeq \frac{1}{\alpha_W T^2} \sim 10\text{GeV}^{-1}$$

$\therefore$ All the particles of the SM are in *kinetic* equilibrium.

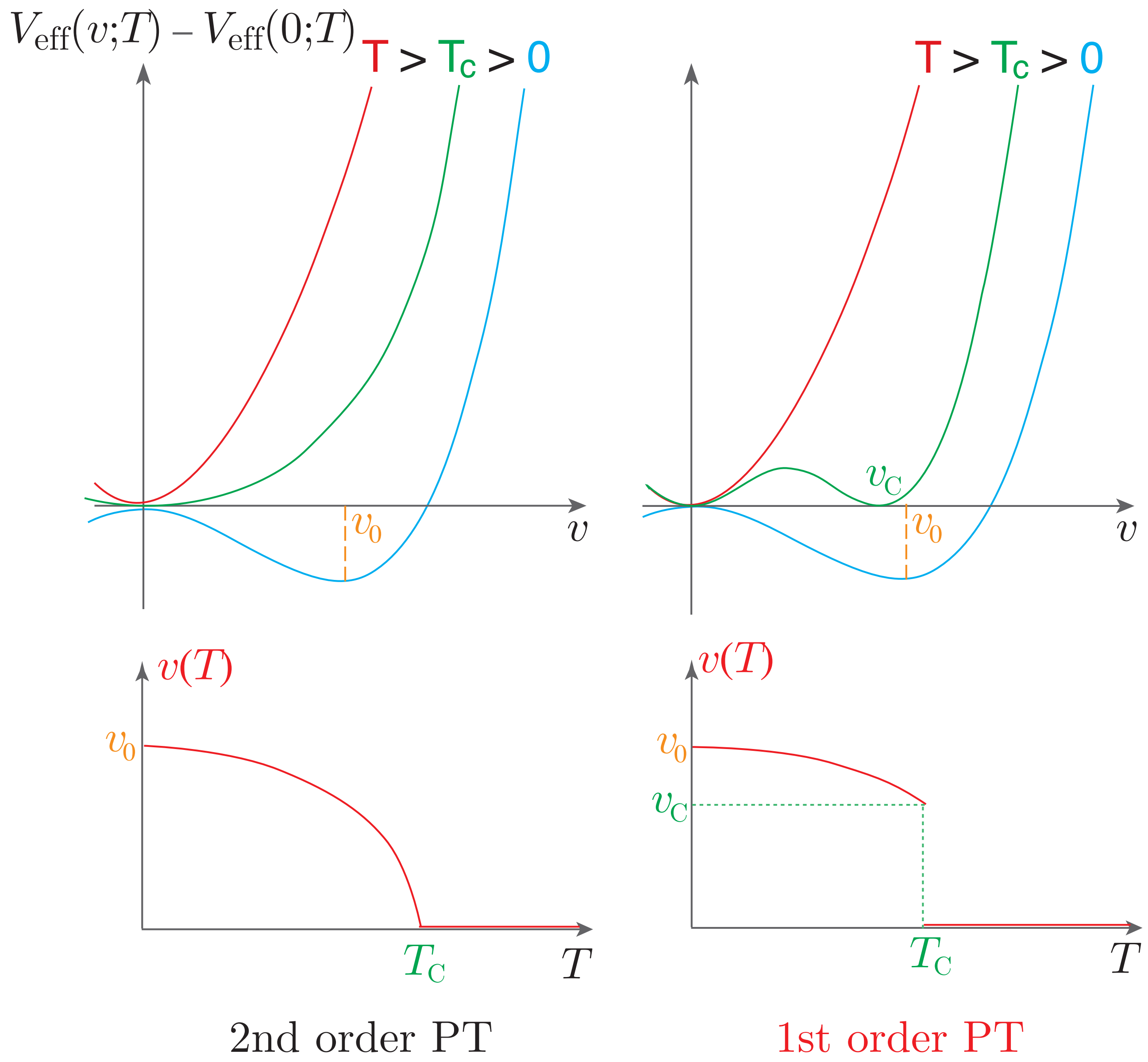
nonequilibrium state $\Longleftarrow$ **1st order EW phase transition**

study of the EWPT

$\star$ static properties $\Leftarrow$ effective potential $=$ free energy density

$$V_{\text{eff}}(v;T) = -\frac{1}{V} T \log Z = -\frac{1}{V} \log \text{Tr}\left[e^{-H/T}\right]_{\langle\phi\rangle = v}$$

$\star$ dynamics — formation and motion of the bubble wall when 1st order PT



Minimal SM

order parameter:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$$

$\therefore$ 1st order EWPT

$\Updownarrow$

$$v_C \equiv \lim_{T \uparrow T_C} \varphi(T) \neq 0$$

Minimal SM — perturbation at the 1-loop level

$$V_{\text{eff}}(\varphi;T) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2Bv_0^2\varphi^2 + B\varphi^4\left[\log\left(\frac{\varphi^2}{v_0^2}\right) - \frac{3}{2}\right] + \bar{V}(\varphi;T)$$

where $$B = \frac{3}{64\pi^2 v_0^4}(2m_W^4 + m_Z^4 - 4m_t^4),$$

$$\bar{V}(\varphi;T) = \frac{T^4}{2\pi^2}\left[6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)\right], \qquad (a_A = m_A(\varphi)/T)$$

$$I_{B,F}(a^2) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right).$$

high-temperature expansion $[m/T \ll 1]$

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right)\frac{a^4}{2} + O(a^6)$$

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6)$$

For $T > m_W, m_Z, m_t$,

$$V_{\text{eff}}(\varphi; T) \simeq D(T^2 - T_0^2)\varphi^2 - E\,T\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2), \qquad E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4}\left(2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right)$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2), \qquad \log\alpha_{F(B)} = 2\log(4)\pi - 2\gamma_E$$

At T_C, $^{\exists}$degenerate minima: $\varphi_C = \dfrac{2E\,T_C}{\lambda_{T_C}}$

$$\Gamma_{\text{sph}}^{(b)}/T_C^3 < H(T_C) \iff \frac{\varphi_C}{T_C} \gtrsim 1$$

$\Longrightarrow$ upper bound on λ $\quad [m_H = \sqrt{2}\lambda v_0]$

$$m_H \lesssim 46\text{GeV}$$

$\Longrightarrow$ MSM is excluded

⋆ Monte Carlo simulations

[MSM]

effective fermion mass : $m_f(T) \sim O(T) \leftarrow$ nonzero modes

$\therefore$ simulation only with the bosons

QFT on the lattice $\begin{cases} \text{scalar fields:} & \phi(x) \text{ on the sites} \\ \text{gauge fields:} & U_\mu(x) \text{ on the links} \end{cases}$

$$Z = \int [d\phi\, dU_\mu] \exp\{-S_E[\phi, U_\mu]\}$$

- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet   time-component of U_μ

  [Laine & Rummukainen, hep-lat/9809045]

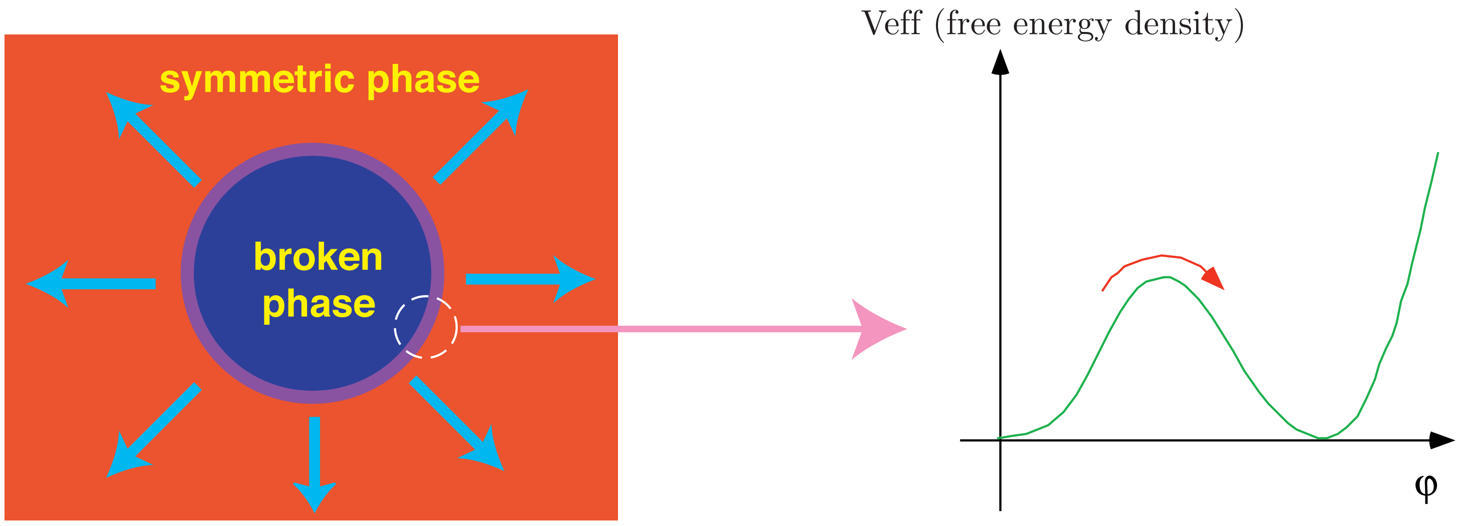
- 4-dim. $SU(2)$ system with a Higgs doublet   [Csikor, hep-lat/9910354]

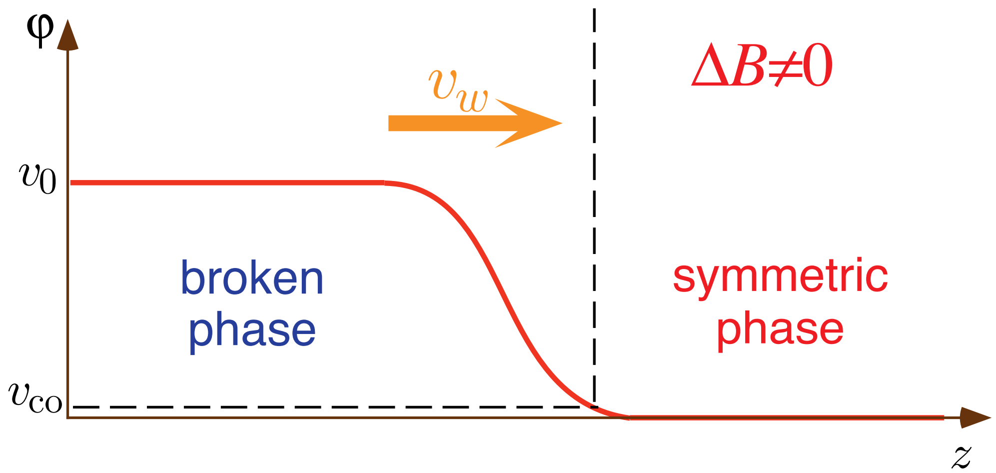
  EWPT is first order for $m_h < 66.5 \pm 1.4$GeV

Both the simulations found end-point of EWPT at

$$m_h = \begin{cases} 72.3 \pm 0.7 \text{ GeV} \\ 72.1 \pm 1.4 \text{ GeV} \end{cases} \Rightarrow \boxed{\text{no PT (cross-over) in the MSM !}}$$

strongly first-order phase transition





bubble wall $\Leftarrow$ classical config. of the gauge-Higgs system

Mechanism of the baryogenesis

$$\bar{t}_s \simeq 0.1\text{GeV}^{-1} \ll \bar{t}_{EW} \simeq 1\text{GeV}^{-1} \ll \bar{t}_{\text{sph}} \simeq 10^5\text{GeV}^{-1} \ll H^{-1} \simeq 10^{14}\text{GeV}^{-1}$$

EW bubble wall motion: $t_{\text{wall}} = \frac{l_w}{v_w} = \frac{(1-40)/T}{0.1-0.9} = (0.01-4)\text{GeV}^{-1}$

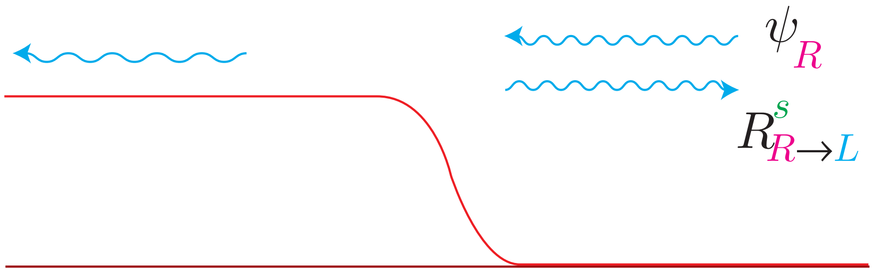
1. All the particles are in *kinetic equilibrium* at the same temperature, because of $H^{-1} \gg \bar{t}_{EW}$, far from the bubble wall.

2. Since $\lambda_Y > \lambda_{EW} \gg l_w$, the leptons and some of the quarks propagate almost freely before and after the scattering off the bubble wall.

3. Because of $t_{\text{wall}} \ll \bar{t}_{\text{sph}}$, the sphaleron process is out of *chemical equilibrium* near the bubble wall.

chiral charge accumulated in the sym. phase $\xrightarrow{\text{sphaleron}}$ $B \neq 0$

$$\dot{n}_B = -\frac{\mu_B}{T}\Gamma_{\text{sph}}, \qquad \mu_B \propto \text{chiral charge}$$



total flux injected into the *symmetric phase* region

$$F^i{}_Q = \frac{Q_L{}^i - Q_R{}^i}{4\pi^2\gamma} \int_{m_0}^{\infty} dp_L \int_0^{\infty} dp_T\, p_T \left[f_i{}^s(p_L, p_T) - f_i{}^b(-p_L, p_T) \right] \Delta R(m_0, p_L)$$

$$\begin{aligned} \Delta R &= R^s{}_{R\to L} - \bar{R}^s{}_{R\to L}, \\ f_i{}^s(p_L, p_T) &= \frac{p_L}{E} \frac{1}{\exp[\gamma(E - v_w p_L)/T] + 1} \\ f_i{}^b(-p_L, p_T) &= \frac{p_L}{E} \frac{1}{\exp[\gamma(E + v_w \sqrt{p_L^2 - m_0^2})/T] + 1} \end{aligned}$$

$Q_L - Q_R \neq 0$ and conserved in the sym. phase $\Longrightarrow$ $Y, \quad I_3$

N.B. For B, no F_B is generated, since it is vectorlike.

EW baryogenesis in the MSSM

- EW Phase Transition

  3 order parameters: $\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 + i v_3 \end{pmatrix}$

- CP Violation

  complex parameters: $\mu,\ M_{3,2,1},\ A,\ \mu B = m_3^2$

  $v_3 \neq 0$ — $v_3 = 0$ at the tree level

- sphaleron solution

2HDM	[Peccei, et al, PLB '91]
squarks vs sphaleron	[Moreno, et al, PLB '97]
NMSSM	[KF, et al, in progress]

⋆ Electroweak phase transition

light stop scenario [de Carlos & Espinosa, NPB '97]

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + \left(\frac{g_1^2}{24} - \frac{g_2^2}{8}\right)(v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 & \frac{y_t}{\sqrt{2}}(\mu v_d + A(v_2 - i v_3)) \\ * & m_{\tilde{t}_R}^2 - \frac{g_1^2}{6}(v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 \end{pmatrix}$$

$m_{\tilde{t}_L}^2 = 0$ or $m_{\tilde{t}_R}^2 = 0 \Longrightarrow$ smaller eigenvalue: $m_{\tilde{t}_1}^2 \sim O(v^2)$

$\therefore$ high-T expansion

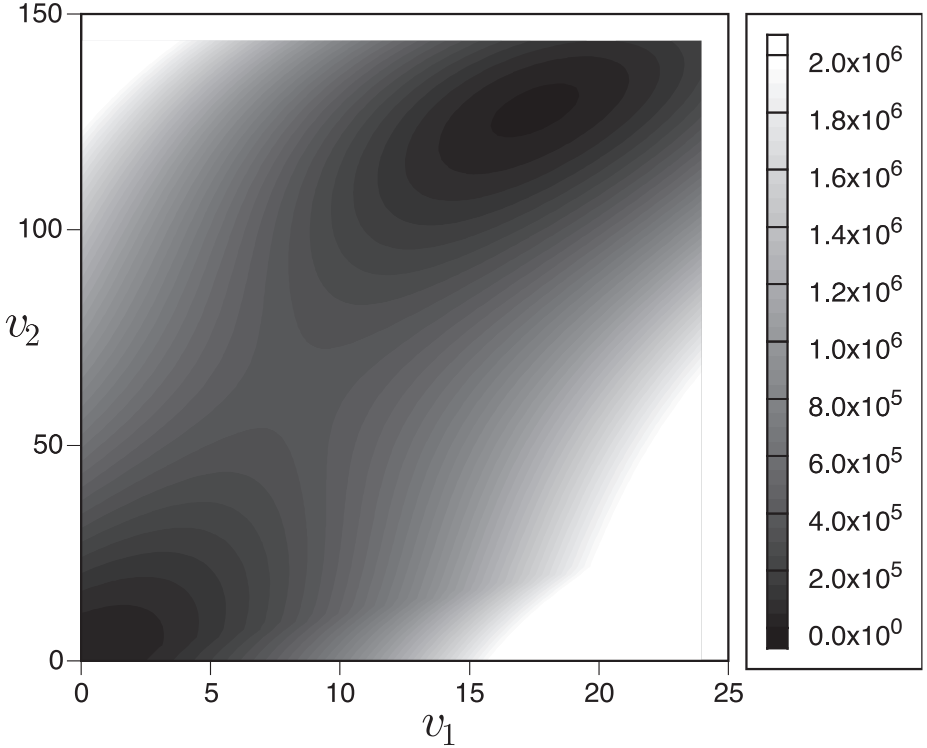
$$\bar{V}_{\tilde{t}}(\boldsymbol{v}; T) \Longrightarrow -\frac{T}{6\pi}(m_{\tilde{t}_1}^2)^{3/2} \sim T v^3$$

$\longrightarrow$ stronger 1st order PT

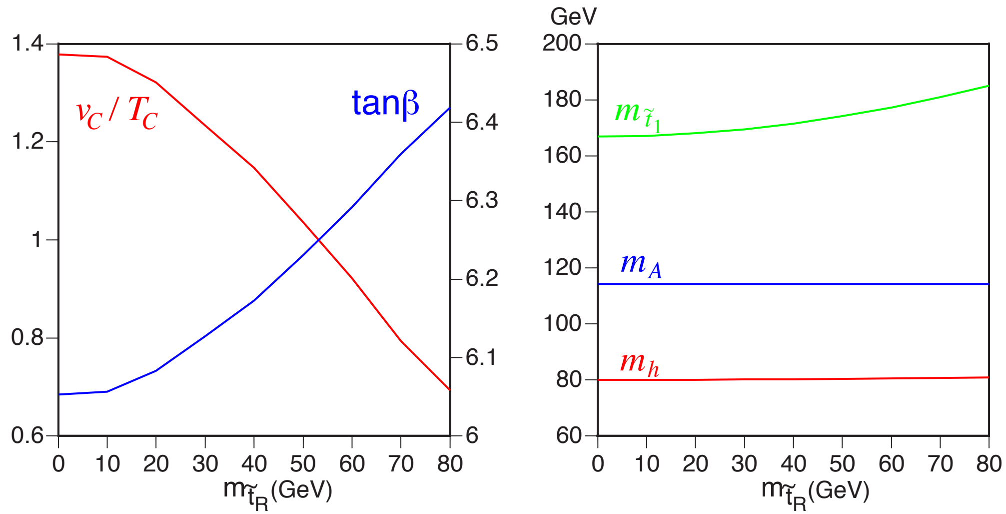
effective for larger y_t — smaller $\tan\beta$

An example: $\tan\beta = 6$, $m_h = 82.3$GeV, $m_A = 118$GeV, $m_{\tilde{t}_1} = 168$GeV

$T_C = 93.4$GeV, $v_C = 129$GeV [KF, PTP101]



$V_{\rm eff}(v_1, v_2, v_3 = 0; T_C)$



m_{t_R}-dependence $(\tan\beta = 6)$

⋆ Lattice MC studies

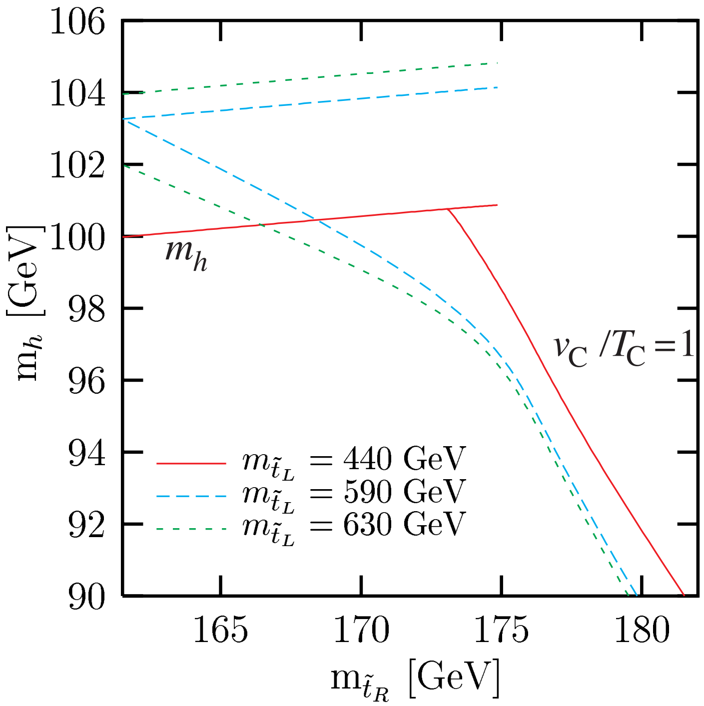
- 3d reduced model [Laine et al. hep-lat/9809045]

  strong 1st order for $m_{\tilde{t}_1} \lesssim m_t$ and $m_h \leq 110$GeV

- 4d model [Csikor, et al. hep-lat/0001087]

  with $SU(3)$, $SU(2)$ gauge bosons, 2 Higgs doublets, stops, sbottoms

  $A_{t,b} = 0$, $\tan\beta \simeq 6$ $\longrightarrow$ agreement with the perturbation theory within the errors

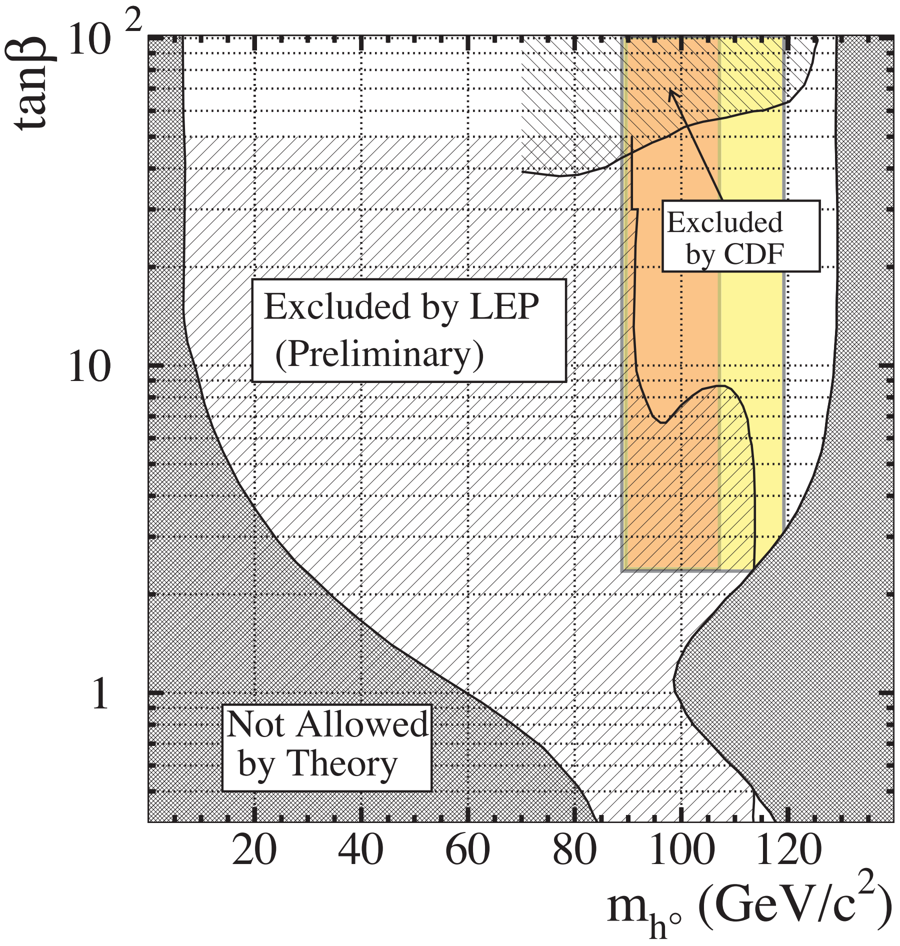


$m_A = 500$ GeV

$v_C/T_C > 1$
below the steeper lines

$\Downarrow$

max. $m_h = 103 \pm 4$ GeV
for $m_{\tilde{t}_L} \simeq 560$ GeV



[PDG, http://ccwww.kek.jp/pdg/]

light stop: $m_{t_R} = 0$

negative soft mass^2: $m_{t_R}^2 > -(65\mathrm{GeV})^2$

[Laine & Rummukainen, NPB597]

Effects of CP violation on the EWPT

[KF, Tao & Toyoda, PTP 109]

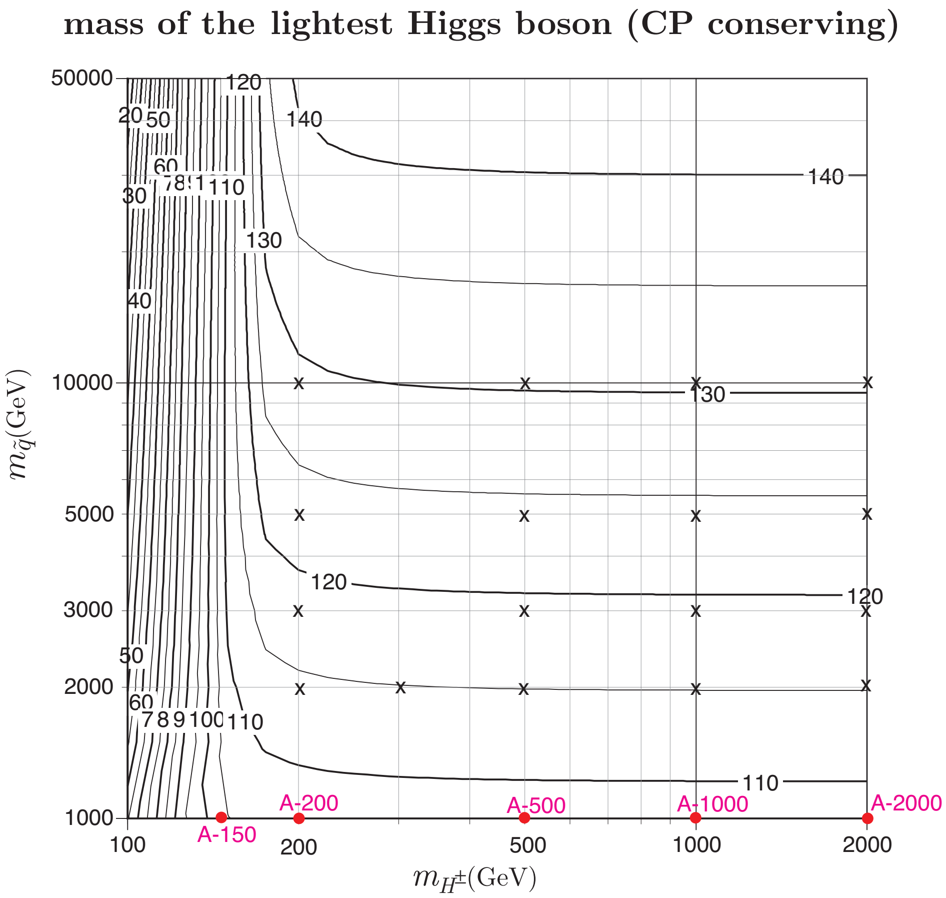
EWPT in the light-stop scenario $[m_{\tilde{t}_R} = 10\text{GeV}]$

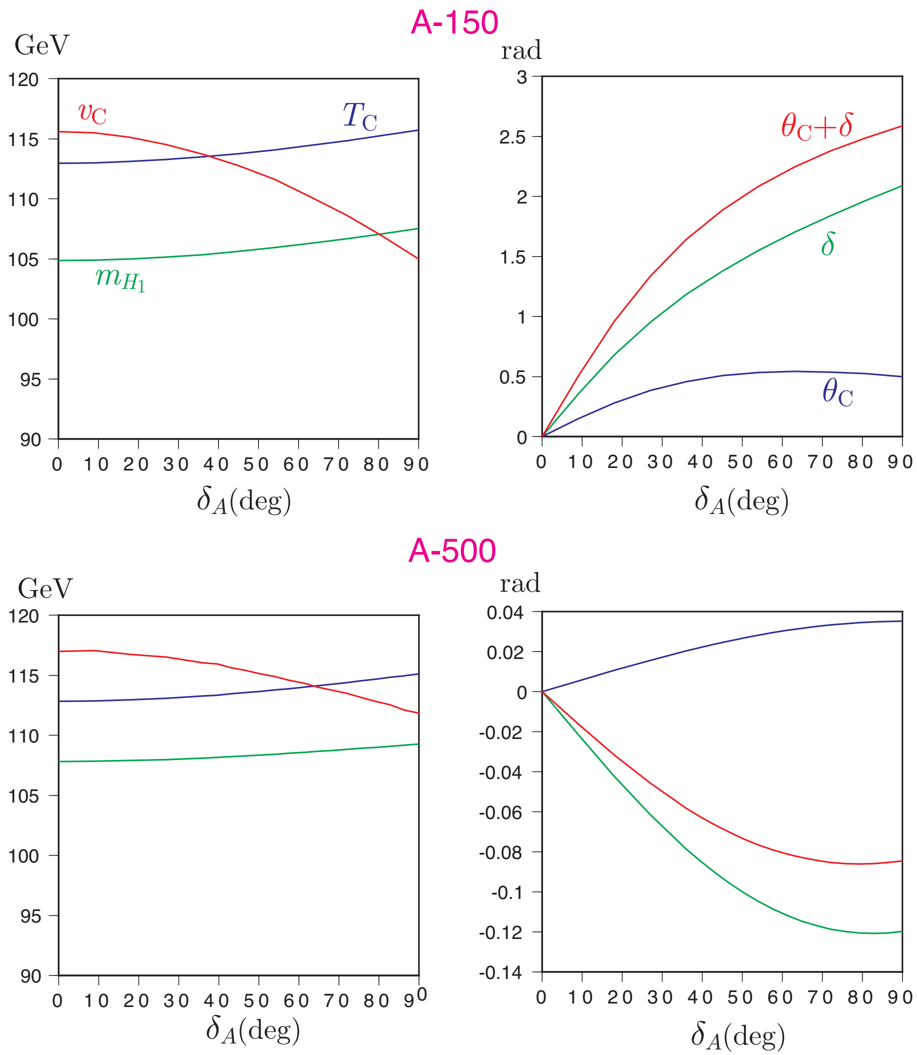
$\text{Im}(\mu A_t e^{i\theta}) \neq 0 \Rightarrow$
- ▷ scalar-pseudoscalar mixing [Carena, et al., NPB586]
- ▷ induces $\delta = \text{Arg}(m_3^2)$
- • weakens the EWPT

field-dependent mass^2 of the lighter stop:

$$\bar{m}_{\tilde{t}_1}^2 = \frac{1}{2}\Bigg[m_{\tilde{q}}^2 + m_{\tilde{t}_R}^2 + y_t^2 v_u^2 + \frac{g_2^2 + g_1^2}{4}(v_d^2 - v_u^2) - \sqrt{\left(m_{\tilde{q}}^2 - m_{\tilde{t}_R}^2 + \frac{x_t}{2}(v_d^2 - v_u^2)\right)^2 + y_t^2 \left|\mu v_d - A_t^* e^{-i\theta} v_u\right|^2}\Bigg]$$

$\tan\beta = 10,\ \mu = 1500\text{GeV},\ |A| = 150\text{GeV}$





Summary

Electroweak Baryogenesis

- **based on a testable model** $\longleftrightarrow$ stringent constraints
- **free from proton decay problem**

other attemps:

⋆ GUTs

⋆ Leptogenesis

⋆ Affleck-Dine

⋆ Gravitational Baryogeneisis

[Alexander, Peskin & Sheikh-Jabbari, hep-th/0405214]

⋆ Inflationary Baryogenesis [KF, Kakuto, Otsuki & Toyoda, PTP 105]

[Nanopoulos & Rangarajan, PRD 64]

viable models for EW baryogenesis

- Minimal SM — excluded !! ✗ $\left\{ \begin{array}{l} \text{strongly 1st-order EWPT (with acceptable } m_h\text{)} \\ \text{sufficient } CP \text{ violation} \end{array} \right.$

- MSSM

  ⋆ $\left. \begin{array}{l} m_h \leq 110\text{GeV} \ \text{ and } \ m_{\tilde{t}_1} \leq m_t \\ m_h \leq 120\text{GeV} \ \text{ if } \ m^2_{\tilde{t}_R} < 0\,? \end{array} \right\} \Longrightarrow$ 1st-order EWPT with $\dfrac{v_C}{T_C} > 1$

  ⋆ many sources of CP violation

  complex parameters: $\mu,\ M_2,\ M_1,\ A;\ \theta$ — $\mathrm{Im}(\mu A_t e^{i\theta})$ weakens the EWPT

- Other extensions of the MSM

  ▷ non-SUSY : 2HDM — many parameters not so constrained

  favored parameters for 1st-order PT and hhh-coupling → Senaha's talk

  ▷ Next-to-MSSM (NMSSM) = MSSM + Singlet chiral superfield

  strong 1st order PT without a light stop → Tao's talk