Sphaleron Process and L-to-B Conversion

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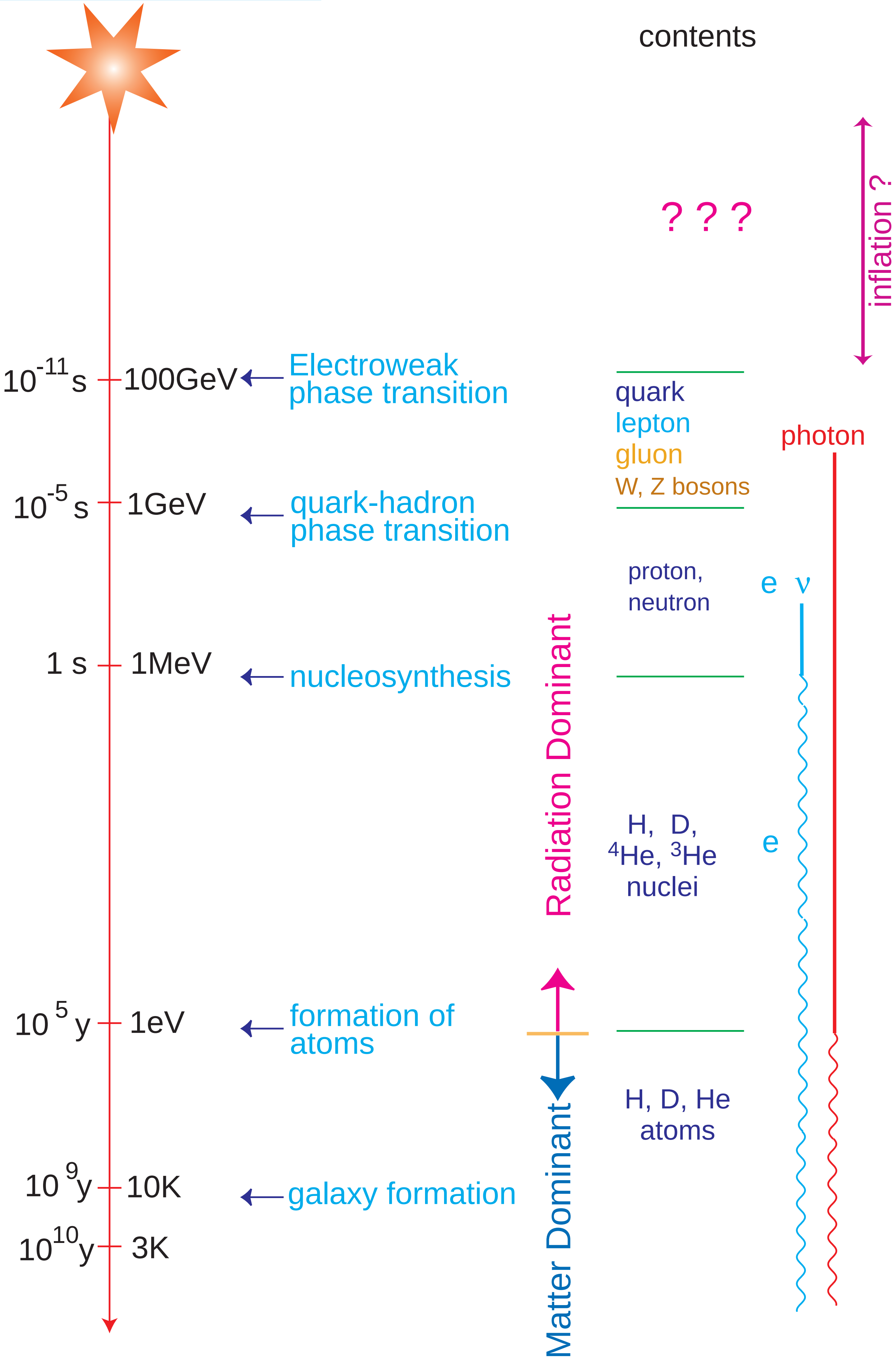
content

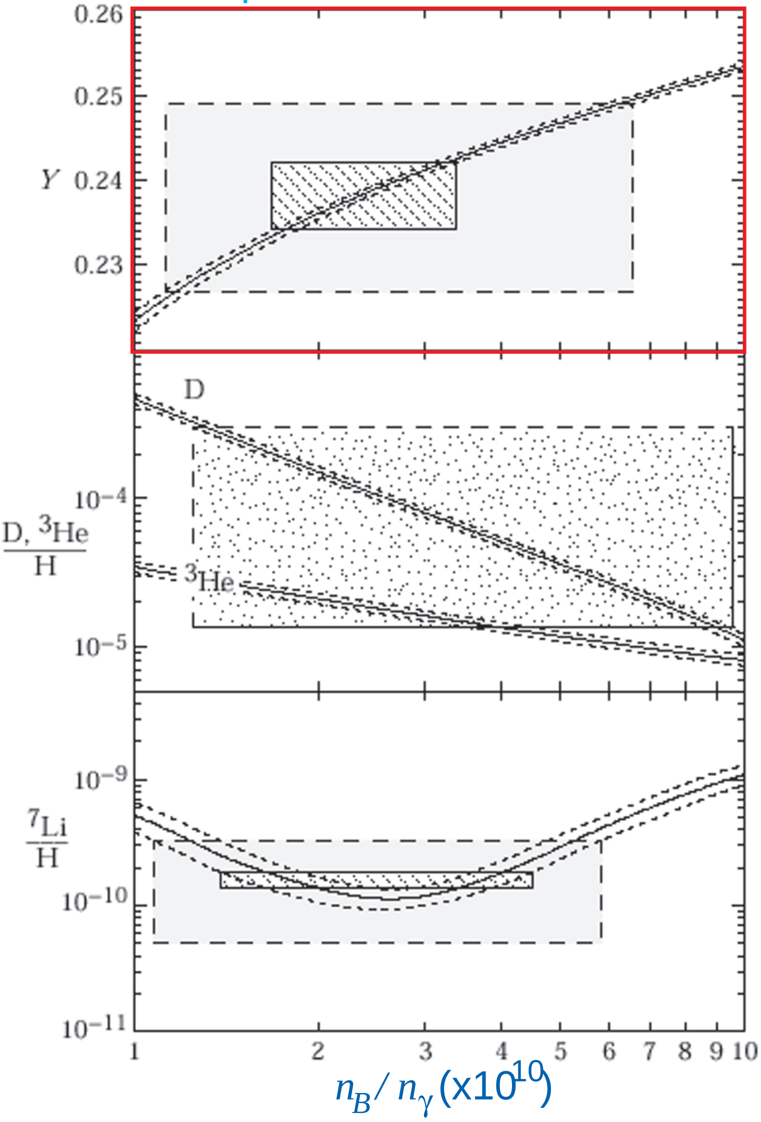
1. Introduction



Nucleosynthesis

[http://ccwww.kek.jp/pdg/2001/bigbangnucrpp.pdf]

SM prediction vs Observation



$Y = \dfrac{2\, n/p}{1 + n/p}$

$Y = 0.25 \longleftrightarrow n/p = 1/7$

- $T \gg 1\text{MeV}$: $n \leftrightarrow p + e + \bar{\nu}_e \Rightarrow n/p = 1$

- $T = T_F \simeq 1\text{MeV}$ $\quad \Gamma_{n\leftrightarrow p}(T_F) \simeq H$

$$\left(\frac{n}{p}\right)_{\text{freeze-out}} = e^{-(m_n - m_p)/T_F} \simeq \frac{1}{6}$$

- $T = 0.3 - 0.1\text{MeV}$

$$\frac{n}{p} \longrightarrow \frac{1}{6} - \frac{1}{7}$$ depending on $\dfrac{n_B}{n_\gamma}$ $\quad$ cf. $s \simeq 7 n_\gamma$

$$\frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} = (0.2 \ - \ 0.9) \times 10^{-10}$$

— constant after the decoupling of $\Delta B \neq 0$ process

evidence of the BAU [Steigman, Ann.Rev.Astron.Astrop.**14**('76)]

1. no anti-matter in cosmic rays from our galaxy
   some anti-matter consistent as secondary products

2. nearby clusters of galaxies are stable
   a cluster: $(1 \sim 100) M_{\text{galaxy}} \simeq 10^{12 \sim 14} M_\odot$

Starting from a B-symmetric universe ...

$$\frac{n_b}{s} \simeq \frac{n_{\bar{b}}}{s} \sim 8 \times 10^{-11} \quad \text{at } T = 38\text{MeV}$$

$$\sim 7 \times 10^{-20} \quad \text{at } T = 20\text{MeV}$$

$N\bar{N}$-annihilation decouple

At $T = 38$MeV,
mass within a causal region $= 10^2 M_\odot \ll 10^{12} M_\odot$.

$\Downarrow$

We must have the BAU $\dfrac{n_B}{s} = (0.2 - 0.9) \times 10^{-10}$
before the universe was cooled down to $T \simeq 38$MeV.

3 requirements for generation of BAU [Sakharov, '67]

(1) baryon number violation

(2) C and CP violation

(3) departure from equilibrium

- GUTs — out of equil. decay of heavy bosons

  [review: Kolb & Turner, The Early Universe]

- Electroweak baryogenesis

  anomalous $B+L$-violation — sphaleron process
  1st order EW phase transition
  CP violation in extended SM [review: KF, PTP '96]

- Leptogenesis [Fukugita & Yanagida, PL '86]

  $$\left.\begin{array}{c}\text{decoupling of heavy-}\nu\text{ decay}\\ CP\text{ violation in the lepton sector}\end{array}\right\} \Rightarrow \text{Leptogenesis}$$

  $$\overset{\text{sphaleron}}{\Longrightarrow} \text{BAU}$$

- Affleck-Dine mechanism in SUSY models [NPB '86]

  $\langle\text{squark}\rangle \neq 0$ or $\langle\text{slepton}\rangle \neq 0$ along (nearly) flat directions, at high temperature

  coherent motion of complex $\langle\tilde{q}\rangle$, $\langle\tilde{l}\rangle \neq 0$ B, C, CP viol.
  $\Longrightarrow$ B- and/or L-genesis

2. Sphaleron Process

⋆ Anomalous fermion number nonconservation

axial anomaly in the standard model

$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}]$$
$$\partial_\mu j^\mu_{B-L} = 0$$

$N_f =$ number of the generations, $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

integrating these equations,

$$\begin{aligned} B(t_f) - B(t_i) &= \int_{t_i}^{t_f} d^4x \, \frac{1}{2}\left[\partial_\mu j^\mu_{B+L} + \partial_\mu j^\mu_{B-L}\right] \\ &= \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x \left[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right] \\ &= N_f \left[N_{CS}(t_f) - N_{CS}(t_i)\right] \end{aligned}$$

where N_{CS} is the Chern-Simons number:

in the $A_0 = 0$ gauge,

$$N_{CS}(t) = \frac{1}{32\pi^2}\int d^3x \, \epsilon_{ijk}\left[g^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g'^2 B_{ij}B_k\right]_t$$

classical vacua of the gauge sector $\mathcal{E} = \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2) = 0$

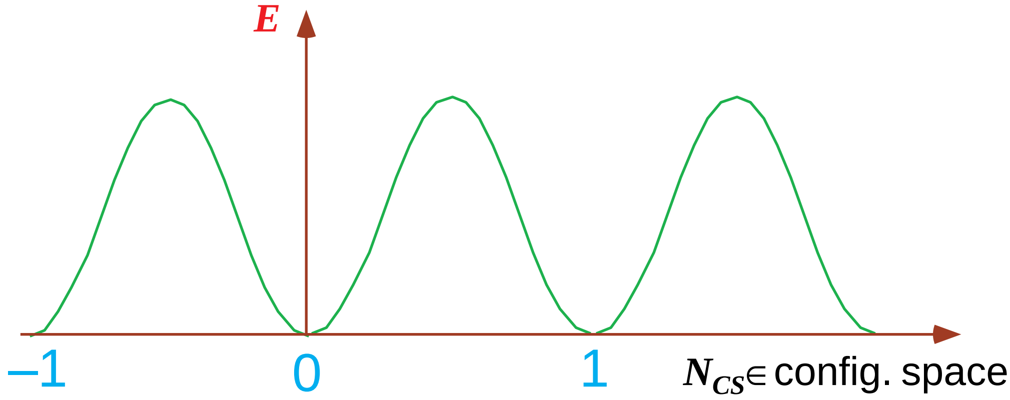
$\Longleftrightarrow F_{\mu\nu} = B_{\mu\nu} = 0$

$\Longleftrightarrow A = iU^{-1}dU$ and $B = dv$ with $U \in SU(2)$

$\therefore \ U(\boldsymbol{x}) : S^3 \ni \boldsymbol{x} \longrightarrow U \in SU(2) \simeq S^3$

$\pi_3(S^3) \simeq \mathbf{Z} \Rightarrow U(\boldsymbol{x})$ is classified by an integer N_{CS}.

energy functional vs configuration space



background U changes with $\Delta N_{CS} = 1$
$\Longrightarrow \Delta B = 1$ $(\Delta L = 1)$ in each (left-) generation

$\Longleftrightarrow$
- level crossing
- index theorem

Transition of the field config. with $\Delta B \neq 0$

▷ quantum tunneling	low temperature
▷ thermal activation	high temperature

transition rate with $\Delta N_{CS} = 1 \Longleftarrow$ WKB approx.

What is Sphaleron ?

sphaleros : $\sigma\varphi\alpha\lambda\epsilon\rho o\sigma$ = 'ready to fall'

a saddle-point solution of 4d $SU(2)$ gauge-Higgs system
[Klinkhammer & Manton, PRD30 ('84)]

$$E_{\text{sph}} = 8 - 14 \text{ TeV}$$

- ⋆ unstable
- ⋆ static (3d) solution with finite energy
- ⋆ Chern-Simons No. = "1/2" $\longrightarrow$ example below

$\Longrightarrow$ over-barrier transition at finite temperature

cf. instanton

- ⋆ stable
- ⋆ 4d solution with finite euclidean action
- ⋆ integer Pontrjagin index

$\Longrightarrow$ quantum tunneling

$$\text{tunneling amplitude} \simeq e^{-S_{\text{instanton}}}$$

§2.1 Fate of false vacuum at $T \neq 0$

decay rate of a false vacuum through quantum tunneling
by WKB approximation [Coleman, *Aspects of Symmetry*]

$$\begin{aligned} \Gamma &\simeq \frac{2}{\hbar} \mathrm{Im} E_0 \\ &\simeq \left(\frac{S_{\rm cl}}{2\pi\hbar}\right)^{1/2} e^{-S_{\rm cl}/\hbar} \left[1 + O(\hbar)\right] \end{aligned}$$

generalization to $T \neq 0$ case: Affleck, PRL 46 ('81)
Langer, Ann.Phys. 41 ('67) – classical

at finite-T,

$$\Gamma \propto \mathrm{Im}\ F$$

N.B.

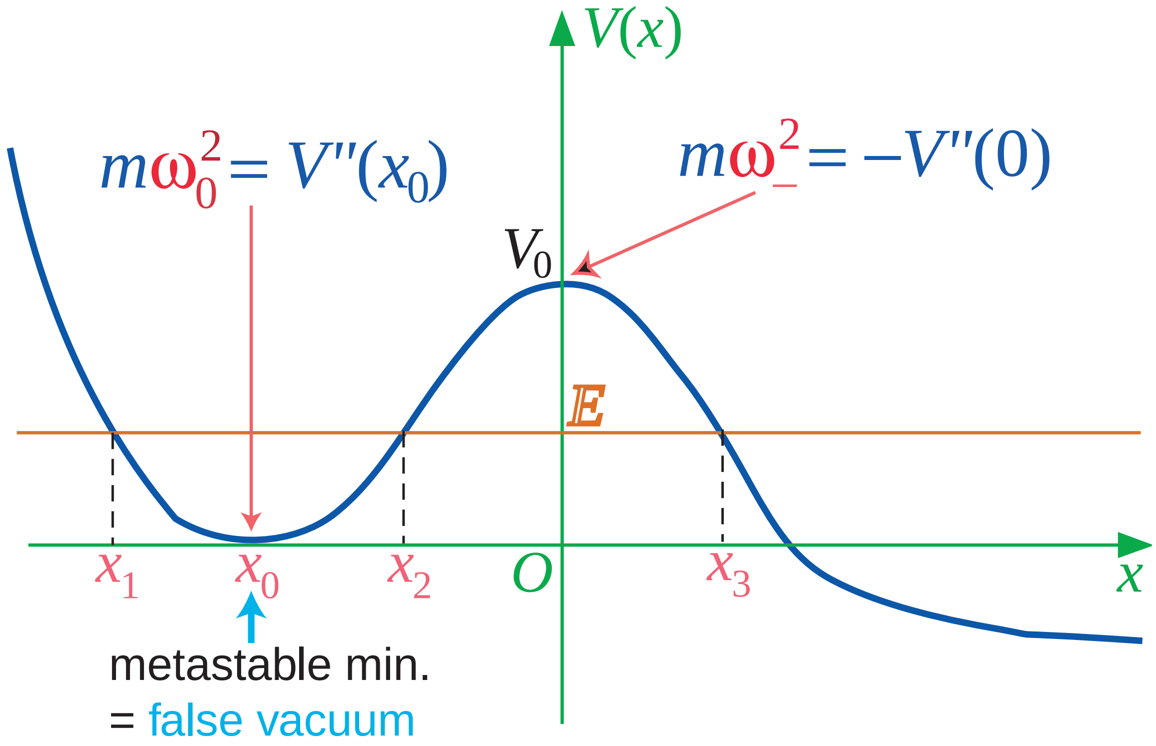
$$\left.\begin{array}{l} E_0 = \langle 0|H|0\rangle \\ F = -T \ln Z \end{array}\right\} \Longrightarrow \left\{\begin{array}{ll} \mathrm{Im}\, E_0 = 0 & ? \\ \mathrm{Im}\, F = 0 & ? \end{array}\right.$$

$\mathrm{Im}\, E_0$ or $\mathrm{Im}\, F$ are *defined* by the procedure by which we evaluate them.

Now we define Γ in a natural way and see how $\Gamma \propto \mathrm{Im}\, F$ holds.

1d Quantum Mechanics

$$H = \frac{p^2}{2m} + V(x)$$



metastable $\Longleftrightarrow$ $\frac{1}{2}\hbar\omega_0, \; T \ll V_0$

initial state = thermal equil. around x_0

Definition of Γ at T :

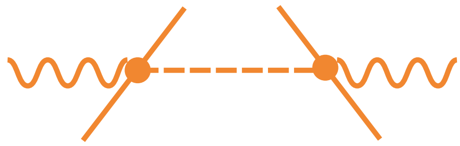
$$\Gamma \equiv \int_0^\infty dE \, \frac{e^{-\beta E}}{Z_0} \, \Gamma(E)$$

where

$$Z_0 \equiv \sum_{n=0}^{\infty} e^{-\beta\hbar\omega_0(n+1/2)} = \left[2 \sinh \frac{\beta\hbar\omega_0}{2} \right]^{-1}$$

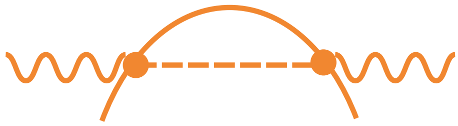
$$\Gamma(E) \equiv -\frac{i\hbar}{2m} \left(\psi^* \psi' - \psi^{*\prime} \psi \right) \qquad \text{prob. current}$$

$\psi(x) \Leftarrow$ WKB approximation [Landau-Lifshitz, Q.M.]



- $E < V_0$ linear turning pt.

$$\Gamma(E) \simeq \frac{1}{2\pi\hbar} \exp\left[-\frac{2}{\hbar}\int_{x_2(E)}^{x_3(E)} dx\sqrt{2m(V(x)-E)}\right]$$



- $E \gtrsim V_0$ parabolic barrier

$$\Gamma(E) \simeq \frac{1}{2\pi\hbar}\left\{1+\exp\left[-\frac{2\pi}{\hbar\omega_-}(E-V_0)\right]\right\}^{-1}$$

♠ Evaluation of Γ

(i) low temperature : $T = \beta^{-1} < \frac{\hbar\omega_-}{2\pi}$

E-integral in Γ is dominated by $E < V_0$

$\Gamma(E) \longleftarrow$ linear turning point approximation

$$\Gamma \simeq \frac{Z_0^{-1}}{2\pi\hbar}\int_0^\infty dE\, e^{-[\beta\hbar\cdot E+W(E)]/\hbar} = \frac{Z_0^{-1}}{2\pi\hbar}\int_0^\infty dE\, e^{-f(E)/\hbar}$$

with

$$W(E) \equiv 2\int_{x_2(E)}^{x_3(E)} dx\sqrt{2m(V(x)-E)}$$

semiclassical approximation at $\hbar \sim 0$

$\longrightarrow$ dominated by the saddle point : $f'(E_0) = 0$

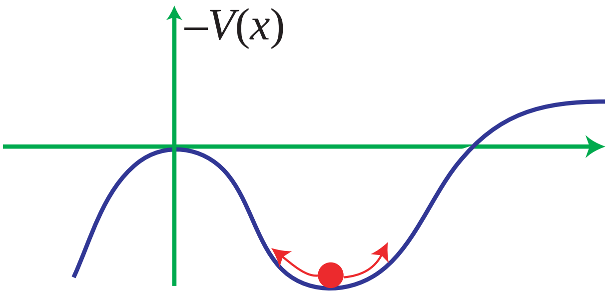
$$f'(E) = \beta\hbar - T(E) = 0$$

where

$$T(E) \equiv \int_{x_2(E)}^{x_3(E)} dx \sqrt{\frac{2m}{V(x) - E}}$$

$$= \text{period of classical orbit in } -V(x) \text{ with } -E$$

$$\min_{0 \leq E < \infty} \{T(E)\} = T(0) \simeq \frac{2\pi}{\omega_-}$$



$\beta\hbar \gtrsim \frac{2\pi}{\omega_-} \Longrightarrow {}^{\exists}E_0 > 0$ s.t. $\beta\hbar = T(E_0)$

Gaussian integral :

$$\Gamma \simeq \frac{Z_0^{-1}}{2\pi\hbar} e^{-[T(E_0)E_0 + W(E_0)]/\hbar} \left| \frac{2\pi\hbar}{T'(E_0)} \right|^{1/2}$$

where

Legendre trf.

$$T(E) \cdot E + W(E) = S(T(E))$$

$$= \text{action of the classical orbit with } -E$$

$$= \text{action of the bounce}$$

(ii) high temperature : $T = \beta^{-1} \gtrsim \frac{\hbar\omega_-}{2\pi}$

no solution to $f'(E) = 0$

$E \gtrsim V_0$ portion contributes to the E-intagral of Γ

$$\begin{aligned}\Gamma &\simeq \frac{Z_0^{-1}}{2\pi\hbar} \int_0^\infty dE\ e^{-\beta E} / \left[1 + e^{-2\pi(E-V_0)/(\hbar\omega_-)}\right] \\ &= \frac{Z_0^{-1}}{2\pi\hbar}\ e^{-\beta V_0} \int_{-V_0}^\infty dE\ e^{-\beta E} / \left[1 + e^{-2\pi E/(\hbar\omega_-)}\right]\end{aligned}$$

integrand $\to 0$ as $E \to -\infty$

$$\begin{aligned}&\simeq \frac{Z_0^{-1}}{2\pi\hbar}\ e^{-\beta V_0} \int_{-\infty}^\infty dE\ e^{-\beta E} / \left[1 + e^{-2\pi E/(\hbar\omega_-)}\right] \\ &= Z_0^{-1}\omega_- \cdot \frac{e^{-\beta V_0}}{4\pi \sin(\beta\hbar\omega_-/2)}\end{aligned}$$

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to summarize,

- $T = \beta^{-1} \lesssim \frac{\hbar\omega_-}{2\pi}$

$$\Gamma \simeq Z_0^{-1} \left|2\pi T'(E_0)\right|^{-1/2} e^{-S(E_0)/\hbar}$$

- $T = \beta^{-1} \gtrsim \frac{\hbar\omega_-}{2\pi}$

$$\Gamma \simeq Z_0^{-1}\omega_- \cdot \frac{e^{-\beta V_0}}{4\pi \sin(\beta\hbar\omega_-/2)}$$

♠ Evaluation of Im F

$$F = -\frac{1}{\beta} \ln Z$$

where

$$Z = \mathrm{Tr}\, e^{-\beta H} = \int_{\text{periodic bc}} [dx]\, e^{-S[x]/\hbar}$$

$$S[x] = \int_0^{\beta\hbar} dt \left[\frac{1}{2} m \dot{x}^2 + V(x)\right]$$

semiclassical approx. $\hbar \sim 0$: dominated by a classical path

$$\frac{\partial S}{\partial x} = -m\ddot{x}_{\rm cl} + V'(x_{\rm cl}) = 0$$

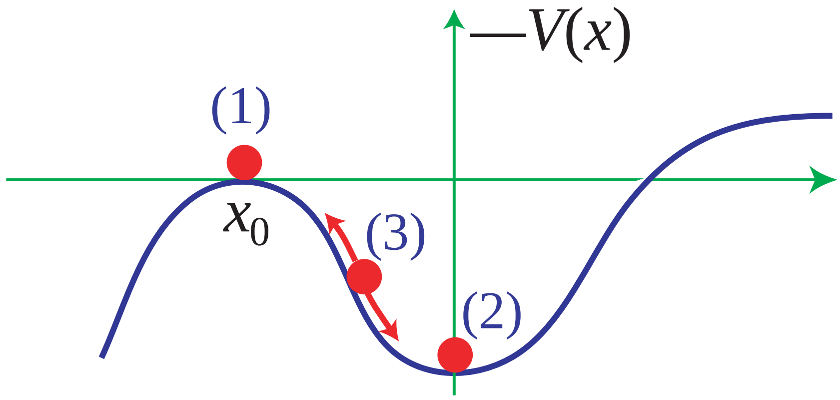
with bc $\quad x_{\rm cl}(0) = x_{\rm cl}(\beta\hbar)$

possible classical orbit

(1) $x_{\rm cl}(t) = x_0 \ ({}^\forall t)$

(2) $x_{\rm cl}(t) = 0 \ ({}^\forall t)$

(3) bounce
$x_{\rm cl}(t) = x_b(t)$
with $x_b(0) = x_b(\beta\hbar)$



(1) and (2) always exist
(3) is possible only when $\beta\hbar \gtrsim 2\pi/\omega_-$ [low temp. regime]

contributions to Z

(1) $x_{\rm cl}(t) = x_0$

$$Z^{(1)} \simeq e^{-S[x_{\rm cl}]/\hbar} \int [dy]\, e^{-\frac{1}{2\hbar}\int_0^{\beta\hbar} dt(\dot{y}^2+\omega_0^2 y^2)}$$

$$= \frac{1}{2\sinh(\beta\hbar\omega_0/2)} = Z_0$$

(2) $x_{\rm cl}(t) = 0$

$$Z^{(2)} \simeq e^{-S[x_{\rm cl}]/\hbar} \int [dy]\, e^{-\frac{1}{2\hbar}\int_0^{\beta\hbar} dt(\dot{y}^2-\omega_-^2 y^2)}$$

$$= e^{-\beta V_0} \cdot \frac{1}{2i} \cdot \frac{1}{2\sin(\beta\hbar\omega_-/2)}$$

$\Uparrow$ assumption of analytic continuation

(3) bounces

n-bounce : $x_b^{(n)} \longrightarrow$ dilute-gas approximation

- $S[x_b^{(n)}] \simeq n \cdot S[x_b]$
- $\det[-\partial_t^2 + V''(x_b^{(n)})] \simeq \left[\det(-\partial_t^2 + V''(x_b))\right]^n \equiv K^n$
- sum over the locations $\subset$ zero-mode integral

$$\int_0^{\beta\hbar} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{n-1}} dt_n = \frac{(\beta\hbar)^n}{n!}$$

$$Z^{(3)} \simeq \sum_{n=1}^{N(\beta)} \frac{(\beta\hbar)^n}{n!} K^n\, e^{-nS[x_b]} \qquad N(\beta) \simeq \frac{\beta\hbar}{2\pi/\omega_-}$$

$$K = \int_{1-\text{bounce}} [dy] \exp\left[-\frac{1}{2\hbar} \int_0^{\beta\hbar} dt (\dot{y}^2 + V''(x_b) y^2) \right]$$

$$= \left(\frac{S[x_b]}{2\pi\hbar} \right)^{1/2} \left[\det{}'(-\partial_t^2 + V''(x_b)) \right]^{-1/2}$$

⇑

Jacobian of the zero mode

Note that, as for the operator $-\partial_t^2 + V''(x_b)$,

- $\psi_0(t) = C\, \dot{x}_b(t)$ is zero mode.

  $\because\ (-\partial_t^2 + V''(x_b))\, \dot{x}_b(t) = \frac{d}{dt}[-\ddot{x}_b + V'(x_b)] \equiv 0$

- $\dot{x}_b(t)$ has a node. $\therefore\ {}^{\exists}$one negative mode

$$\left[\det{}'(-\partial_t^2 + V''(x_b)) \right]^{-1/2} = \frac{1}{2i} \left| \det{}'(-\partial_t^2 + V''(x_b)) \right|^{-1/2}$$

$$= \frac{1}{2i} \left| S[x_b] \cdot T'(E) \right|^{-1/2}$$

⇑

[Rajaraman, Phys.Rep. C21 ('75)]

$$\therefore\ Z^{(3)} \simeq \sum_{n=1}^{N(\beta)} \frac{1}{n!} \left[-\frac{i\beta\hbar}{2} \left(\frac{S[x_b]}{2\pi\hbar} \right)^{1/2} e^{-S[x_b]/\hbar} \times \left| S[x_b] \cdot T'(E) \right|^{-1/2} \right]^n$$

From

$$\mathrm{Im}\,F = -\frac{1}{\beta}\mathrm{Im}\left[\log Z_0 + \log\left(1 + \frac{Z^{(2)}}{Z_0} + \frac{Z^{(3)}}{Z_0}\right)\right]$$

we have

- low temperature : $\beta^{-1} < \hbar\omega_-/(2\pi)$

$$\mathrm{Im}\,F \simeq -\frac{1}{\beta Z_0}\mathrm{Im}\,Z^{(3)} \simeq Z_0^{-1}\,\frac{\hbar}{2}\,|2\pi\hbar T'|^{-1/2}\,e^{-S[x_b]/\hbar}$$

- high temperature : $\beta^{-1} > \hbar\omega_-/(2\pi)$

$$\mathrm{Im}\,F \simeq -\frac{1}{\beta Z_0}\mathrm{Im}\,Z^{(2)} \simeq Z_0^{-1}\,\frac{1}{4\beta\sin(\beta\hbar\omega_-/2)}\,e^{-\beta V_0}$$

Comparing these results to those obtained by the WKB approximation to the wave function,

⋆ $T < \dfrac{\hbar\omega_-}{2\pi}$:

$$\Gamma \simeq \frac{2}{\hbar}\,\mathrm{Im}\,F \simeq Z_0^{-1}\,|2\pi\hbar T'(E_0)|^{-1/2}\,e^{-S[x_b]/\hbar}$$

quantum tunnneling

⋆ $T > \dfrac{\hbar\omega_-}{2\pi}$:

$$\Gamma \simeq \frac{\omega_-\beta}{\pi}\,\mathrm{Im}\,F \simeq Z_0^{-1}\,\frac{\omega_-}{4\pi\sin(\beta\hbar\omega_-/2)}\,e^{-\beta V_0}$$

thermal activation

$\mathrm{Im}\,F$ is applicable to system with many degrees of freedom applied to 4-dim. $SU(2)$ gauge-Higgs system:

⋆ broken phase [Arnold & McLerran, P.R.D36 ('87)]

$$\Gamma_{\rm sph}^{(b)} \simeq k\,\mathcal{N}_{\rm tr}\,\mathcal{N}_{\rm rot}\,\frac{\omega_-}{2\pi}\left(\frac{\alpha_W(T)T}{4\pi}\right)^3 {\rm e}^{-E_{\rm sph}/T}$$

zero modes $\rightarrow \left\{ \begin{array}{l} \mathcal{N}_{\rm tr} = 26 \\ \mathcal{N}_{\rm rot} = 5.3 \times 10^3 \end{array} \right.$ for $\lambda = g^2$

$$\omega_-^2 \simeq (1.8 \sim 6.6) m_W^2 \quad \text{for } 10^{-2} \le \lambda/g^2 \le 10$$

$$k \simeq O(1)$$

⋆ symmetric phase — no mass scale

dimensional analysis :

$$\Gamma_{\rm sph}^{(s)} \simeq \kappa (\alpha_W T)^4$$

check by Monte Carlo simulation $\langle N_{CS}^2(t)\rangle = e^{-\Gamma V t}$ as $t \to \infty$

$\kappa = 1.09 \pm 0.04$ $SU(2)$ pure gauge system

[Ambjørn and Krasnitz, P.L.B362('95)]

'sphaleron transition' even in the symmetric phase

§2.2 Other Methods — nonequilibrium phenomenon

♣ classical stochastic approach

- Langer, Ann.Phys. 54 ('69)
- Ringwald, P.L. B201 ('88)

Fokker-Planck eq. for distribution function $\rho(q, p; t)$
$\longrightarrow$ stationary prob. flow $\equiv$ decay rate

♣ formal density operator approach

- Zubarev, "Nonequilibrium Statistical Thermodynamics"
- Khlebnikov, Shaposhnikov, N.P. B308 ('88)

formal solution to retarded Liouvill eq.
$\longrightarrow$ linear response approximation

♣ numerical approach — applicable in the symmetric phase

- Grigoriev, Rubakov, Shaposhnikov, P.L. B216 ('89)
- Ambjørn, Askgaard, Porter, Shaposhnikov, N.P. B353 ('91)
- Ambjørn, Krasnitz, P.L. B362 ('95)

classical hamiltonian lattice formalism in $A_0 = 0$ gauge
$\downarrow$
initial config. from classical statistical mechanics
generated by MC method with weight $e^{-\beta H(\phi,\pi)}$
$\downarrow$
classical time evolution from an initial config. (ϕ, π)
ergodicity
$\downarrow$
classical config. at ${}^\forall t \rightarrow N_{CS}(t)$

$$\begin{cases} \langle N_{CS} \rangle \\ \langle N_{CS}(t) N_{CS}(0) \rangle \sim \langle N_{CS} \rangle^2 + A\, e^{-\Gamma t} \end{cases}$$

§2.3 An Example — 2d $U(1)$ gauge-Higgs system

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - \lambda\left(|\phi|^2 - v^2\right)^2$$

instanton = vortex $\leftarrow \pi_1(U(1)) = \mathbf{Z}$

static solutions ($A_0 = 0$-gauge)

- vacuum with widing number $= N$

$$\phi(x) = v\, e^{i\alpha(x)}, \qquad A_1(x) = \frac{1}{g}\partial_x \alpha(x)$$

with $\alpha(\infty) - \alpha(-\infty) = 2\pi N$

$$\Delta Q_5 = \frac{g}{4\pi}\int_{t_i}^{t_f} dt\, dx\, \epsilon_{\mu\nu}F_{\mu\nu} = N_{CS}(t_f) - N_{CS}(t_i),$$

$$N_{CS}(t) = \frac{g}{2\pi}\int dx\, A_1(x) = N \quad \text{for the vacua}$$

- sphaleron solution

$$\phi_{\rm sph}(x) = e^{i\pi(1-y(x))/2} v\, y(x) = e^{i\theta(x)} v\, y(x),$$

$$A_1^{\rm sph}(x) = \frac{1}{g}\partial_x \theta(x)$$

$$y(x) \equiv \tanh(\sqrt{\lambda}vx) = \tanh(m_H x/2)$$

$$N_{CS} = \frac{g}{2\pi}\int dx\, A_1^{\rm sph}(x) = \frac{1}{2\pi}[\theta(\infty) - \theta(-\infty)] = \frac{1}{2}$$

N.B.
$\theta(x)$ is not the phase of $\phi_{\rm sph}(x)$.
$|\phi_{\rm sph}(x)|$ and $\mathrm{Arg}(\phi_{\rm sph}(x))$ are singular at $x = 0$.

 [Grigoriev, et al. P.L. B216 ('89)]

1d $U_1(x)$ and $\phi(x)$ on a ring

λ/g^2	#(lattice sites)	$T/E_{\rm sph}$	$\Gamma^{-1}_{\rm exp}$	$\Gamma^{-1}_{\rm th}$
0.5	400	0.103	62	138
0.5	200	0.100	140	190
0.395	200	0.094	180	190
0.32	200	0.101	90	125
0.264	200	0.100	103	114
0.264	400	0.100	34	58

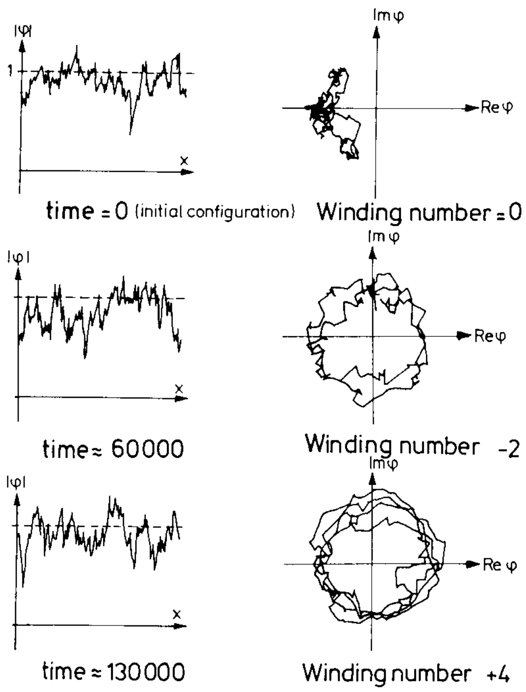


Fig. 1. States of the system ($T=0.07M_{\rm sph}$).

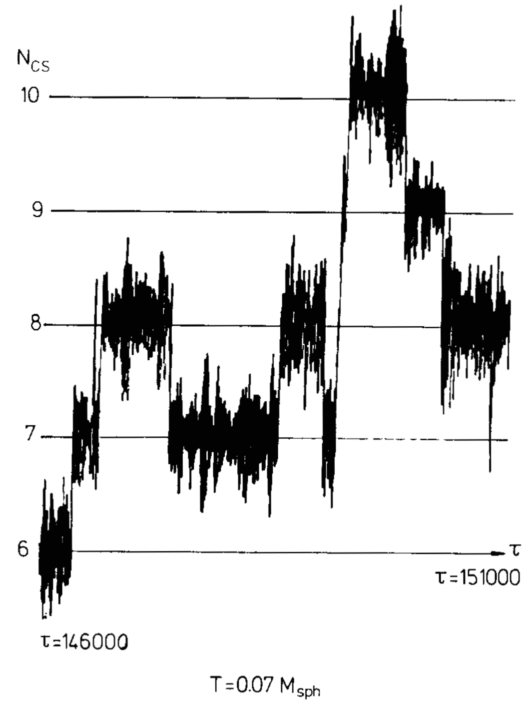
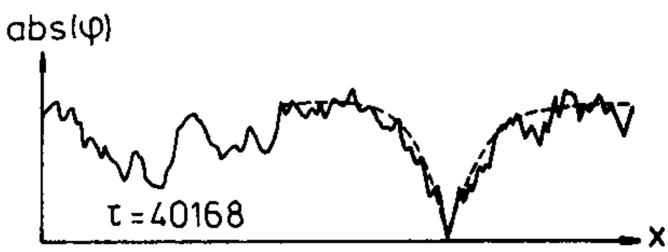
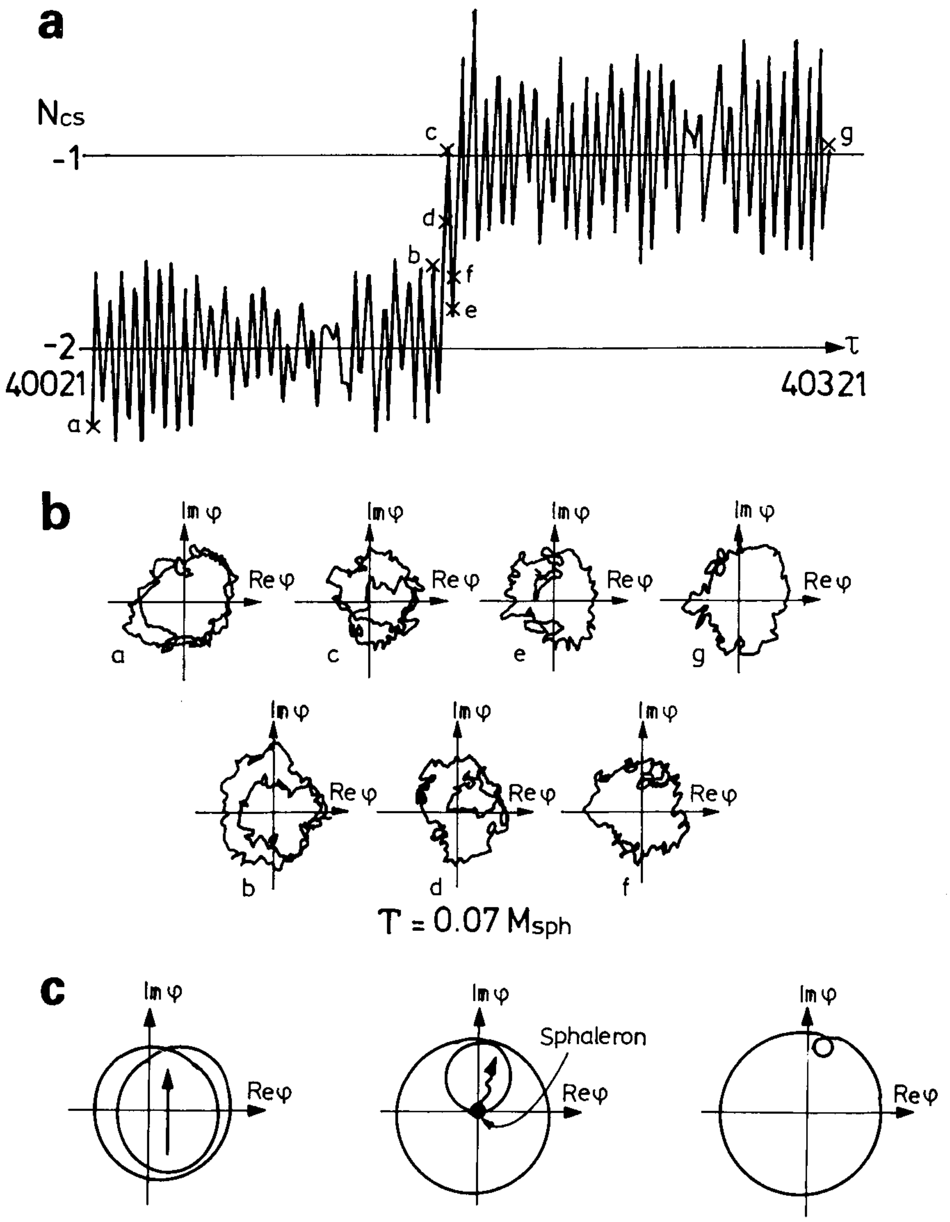


Fig. 2. Chern–Simons number as a function of time ($T=0.07\ M_{\rm sph}$).





ig. 3. Anatomy of the sphaleron transition: (a) Behaviour of the Chern–Simons number. (b) "Trajectories" of the scalar f
ifferent moments a–g; the parameter along the curve is the spatial coordinate x^1. (c) Schematic plot of the sequence of (b).

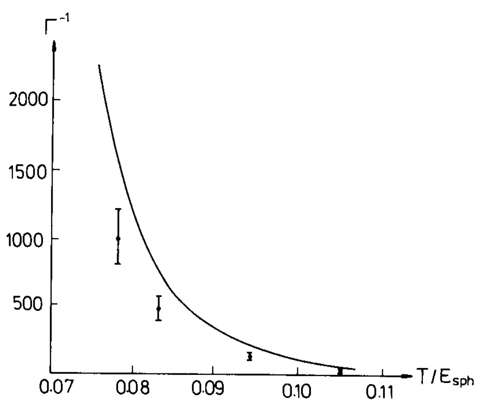


Fig. 5. Transition rate as a function of the temperature.

3. B and L in Hot Universe

sphaleron process at early universe

- ⋆ $\Gamma_{\rm sph} > H$? (H:Hubble paramter)
- ⋆ distribution of particles which take part in the process

Here, we focus on equilibrium physics.

nonequilibrium $\Longrightarrow$ B- and/or L-Genesis

§3.1 Time scales

Hubble parameter: $H \equiv \dfrac{\dot{a}(t)}{a(t)} \simeq \sqrt{\dfrac{8\pi G}{3}\rho(t)}$

$\rho(t)$: energy density $\rho = \dfrac{1}{V}\mathrm{Tr}\left[H\, e^{-H/T}\right]$ in equil.

We replace ρ by the sum of free particle contributions:

$$\rho = g\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{\omega_k}{e^{\omega_k/T}\mp 1} \overset{m\ll T}{\simeq} g\begin{cases}\dfrac{\pi^2}{30}T^4\\[1ex] \dfrac{7}{8}\dfrac{\pi^2}{30}T^4\end{cases}$$

$$\overset{m\gg T}{\simeq} g\, mn$$

where

g = degrees of freedom of each species

$\omega_k = \sqrt{\boldsymbol{k}^2+m^2}$

n = particle number density

For radiation-dominant universe,

$$\rho(T) = \frac{\pi^2}{30} g_* T^4 \quad \text{with} \quad g_* \equiv \sum_B g_B + \frac{7}{8} \sum_F g_F$$

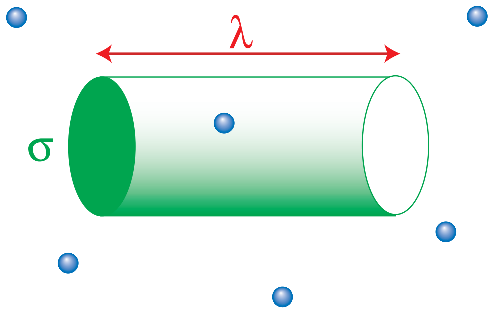
SM with N_f generations and N_H Higgs doublets,

$$g_* = 24 + 4N_H + \frac{7}{8} \times 30 N_f \stackrel{\text{MSM}}{=} 106.75$$

Then

$$\boxed{H \simeq \sqrt{\frac{8\pi G_N}{3}\rho} \simeq 1.66 \sqrt{g_*} \frac{T^2}{m_{Pl}}}$$

time scales of interactions



σ : cross section of some interaction

mean free path : $\lambda \cdot \sigma = \dfrac{1}{n}$

for $m \ll T$

$\lambda \simeq \bar{t} =$ mean free time

$$n = g \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{e^{\omega_k/T} \mp 1} \overset{m \ll T}{\simeq} g \begin{cases} \dfrac{\zeta(3)}{\pi^2} T^3 \\ \dfrac{3}{4} \dfrac{\zeta(3)}{\pi^2} T^3 \end{cases}$$

$$\overset{m \gg T}{\simeq} g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$$

$\zeta(3) = 1.2020569 \cdots$

For relativistic particles at T, $\sigma \simeq \dfrac{\alpha^2}{s} \simeq \dfrac{\alpha^2}{T^2}$, we have

$$\lambda \simeq \frac{10}{gT^3} \left(\frac{\alpha^2}{T^2} \right)^{-1} = \frac{10}{g\alpha^2 T}$$

For $T = 100\text{GeV}$, $H^{-1} \simeq 10^{14}\text{GeV}^{-1}$,

$\lambda_s \simeq \dfrac{1}{{\alpha_s}^2 T} \sim 1\,\text{GeV}^{-1}$ for strong interactions

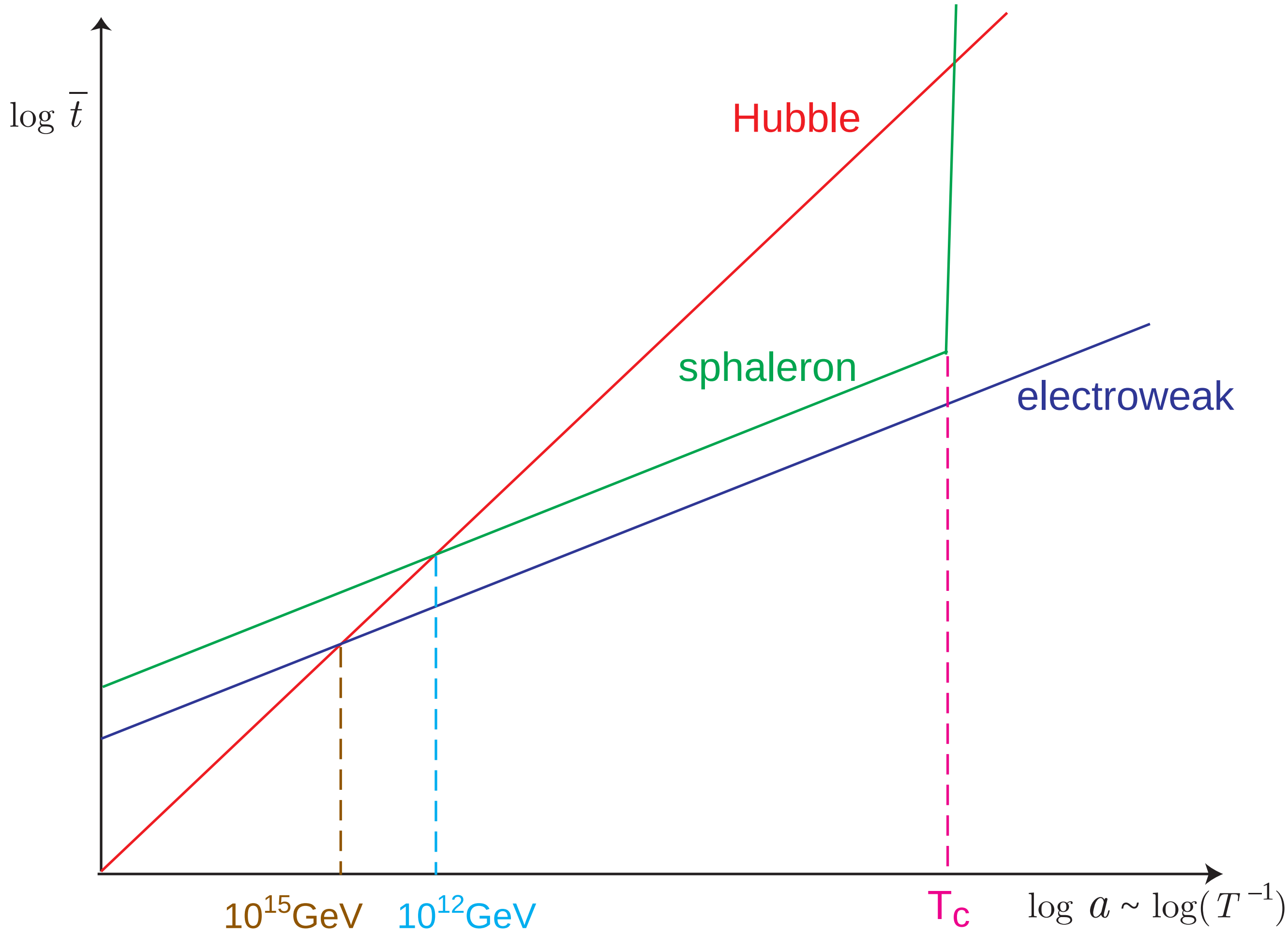
$\lambda_{EW} \simeq \dfrac{1}{{\alpha_W}^2 T} \sim 10\,\text{GeV}^{-1}$ for EW interactions

$\lambda_Y \simeq \left(\dfrac{m_W}{m_f} \right)^4 \lambda_{EW}$ for Yukawa interactions

time scale of sphaleron process

$$\bar{t}_{\rm sph} = (\Gamma_{\rm sph}/n)^{-1} \simeq \begin{cases} 10^6 T^{-1}\,\text{GeV}^{-1} & (T > T_C) \\ 10^5 T^{-1} e^{E_{\rm sph}/T}\,\text{GeV}^{-1} & (T < T_C) \end{cases}$$

[*cf.* $E_{\rm sph} \simeq 10$TeV for $v_0 = 246$GeV]



If $v(T_C) \ll 200$GeV (eg. 2nd order EWPT), $^{\exists}T_{\rm dec}$, *s.t.*

$$T_{\rm dec} < T < T_C \implies \Gamma^{(b)}_{\rm sph}(T) > H(T)$$

§3.2 Quantum numbers in equilibrium

Q_i: conserved quantum number $\qquad [H, Q_i] = 0$

equilibrium partition function:

$$Z(T,\mu) \equiv \mathrm{Tr}\left[e^{-(H-\sum_i \mu_i Q_i)/T}\right]$$

expectation value of Q_i:

$$\langle Q_i \rangle(T,\mu) = T\frac{\partial}{\partial \mu_i} \log Z(T,\mu)$$

relations among μ's $\Longrightarrow$ relations among Q's

In the SM, $Q_i = \frac{1}{N}B - L_i$ without lepton-flavor mixing.

1st-principle calculation of $Z(T,\mu)$

⋆ path integral over *all* fields

⋆ *nonperturbative* $B+L$ violation

$\Downarrow$

- perturbation [Khebnikov & Shaposhnikov, PLB387 ('96); Laine & Shaposhnikov, PRD61 ('00)]
- free-field approximation
  relation among chemical potentials of the particles

⋆ Massless free particle approximation

number density of free particles (per degree of freedom)

$$\langle N \rangle = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\frac{1}{e^{(\omega_k-\mu)/T}\mp 1}-\frac{1}{e^{(\omega_k+\mu)/T}\mp 1}\right]$$

$$\overset{m\ll T}{\simeq} \frac{T^3}{2\pi^2}\int_0^\infty dx\left[\frac{x^2}{e^{x-\mu/T}\mp 1}-\frac{x^2}{e^{x+\mu/T}\mp 1}\right]$$

$$\overset{|\mu|\ll T}{\simeq} \begin{cases} \frac{T^3}{3}\cdot\frac{\mu}{T}, & \text{(bosons)} \\ \frac{T^3}{6}\cdot\frac{\mu}{T}, & \text{(fermions)} \end{cases}$$

$s = \frac{2\pi^2}{45} g_* T^3$: entropy density

particle asymmetry $\frac{\langle N \rangle}{s} \sim \frac{|\mu|}{T} \simeq 10^{-10} \ll 1$

Quantum number densities in terms of μ

[Harvey & Turner, PRD42 ('90)]

SM with N generations and N_H Higgs doublets $(\phi^0\,\phi^-)$

W^-	$u_{L(R)}$	$d_{L(R)}$	$e_{iL(R)}$	ν_{iL}	ϕ^0	ϕ^-
μ_W	$\mu_{u_{L(R)}}$	$\mu_{d_{L(R)}}$	$\mu_{iL(R)}$	μ_i	μ_0	μ_-

gauge int., Yukawa int, quark mixings are in equilibrium.

$$\mu_\gamma = \mu_Z = \mu_{\text{gluon}} = 0$$

$$\Downarrow$$

$(3N+7)$ μ's

gauge $\Leftrightarrow$ $\mu_W = \mu_{d_L} - \mu_{u_L} = \mu_{iL} - \mu_i = \mu_- + \mu_0$

Yukawa $\Leftrightarrow$ $\mu_0 = \mu_{u_R} - \mu_{u_L} = \mu_{d_L} - \mu_{d_R} = \mu_{iL} - \mu_{iR}$

$2(N+2)$ relations

$\Rightarrow$ $N+3$ independent μ's: $(\mu_W, \mu_0, \mu_{u_L}, \mu_i)$

sphaleron process in equilibrium

$$|0\rangle \leftrightarrow \prod_i (u_L d_L d_L \nu_L)_i \Leftrightarrow N(\mu_{u_L} + 2\mu_{d_L}) + \sum_i \mu_i = 0$$

Quantum number densities [in unit of $T^2/6$]

$$\begin{aligned}
B &= N(\mu_{u_L} + \mu_{u_R} + \mu_{d_L} + \mu_{d_R}) = 4N\mu_{u_L} + 2N\mu_W, \\
L &= \sum_i (\mu_i + \mu_{iL} + \mu_{iR}) = 3\mu + 2N\mu_W - N\mu_0 \\
Q &= \frac{2}{3}N(\mu_{u_L} + \mu_{u_R}) \cdot 3 - \frac{1}{3}N(\mu_{d_L} + \mu_{d_R}) \cdot 3 \\
&\quad - \sum_i (\mu_{iL} + \mu_{iR}) - 2 \cdot 2\mu_W - 2N_H \mu_- \\
&= 2N\mu_{u_L} - 2\mu - (4N + 4 + 2N_H)\mu_W + (4N + 2N_H)\mu_0 \\
I_3 &= \frac{1}{2}N(\mu_{u_L} - \mu_{d_L}) \cdot 3 + \frac{1}{2}\sum_i (\mu_i - \mu_{iL}) \\
&\quad - 2 \cdot 2\mu_W - 2 \cdot \frac{1}{2}N_H(\mu_0 + \mu_-) \\
&= -(2N + N_H + 4)\mu_W
\end{aligned}$$

$$\mu \equiv \sum_i \mu_i$$

- $T \gtrsim T_C$ (symmetric phase)

  We require $Q = I_3 = 0$. ($\mu_W = 0$)

$$B = \frac{8N + 4N_H}{22N + 13N_H}(B-L)$$

$$L = -\frac{14N + 9N_H}{22N + 13N_H}(B-L)$$

- $T \lesssim T_C$ (broken phase)

  $Q = 0$ and $\mu_0 = 0$ ($\because$ ϕ^0 condensates.)

$$B = \frac{8N + 4(N_H+2)}{24N + 13(N_H+2)}(B-L)$$

$$L = -\frac{16N + 9(N_H+2)}{24N + 13(N_H+2)}(B-L)$$

In any case, $B = L = 0$, if $(B-L)_{\text{primordial}} = 0$.

To have nonzero BAU,

(i) we must have $B - L$ before the sphaleron process decouples, or

(ii) $B + L$ must be created at the first-order EWPT, and the sphaleron process must decouple immediately after that.

⋆ Corrections due to mass

[Dreiner & Ross, NPB410 ('93)]

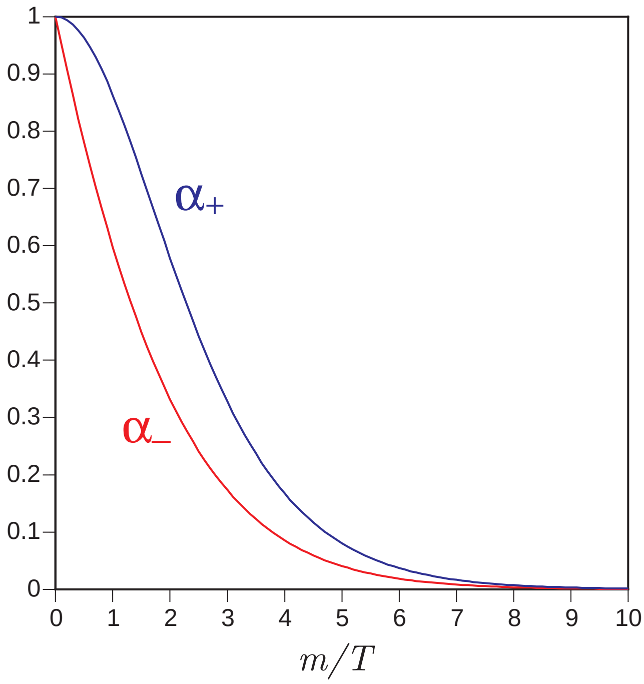
$$\langle N \rangle = \int_0^\infty \frac{dx}{2\pi^2} \left[\frac{x^2}{e^{\sqrt{x^2+m^2/T^2}-\mu/T} \mp 1} - \frac{x^2}{e^{\sqrt{x^2+m^2/T^2}+\mu/T} \mp 1} \right]$$

$$\stackrel{|\mu| \ll T}{\simeq} \langle N \rangle_{m=0} \cdot \alpha_{\mp}(m/T)$$

where

$$\alpha_-(a) \equiv \frac{3}{\pi^2} \int_0^\infty dx \frac{x^2 e^{\sqrt{x^2+a^2}}}{(e^{\sqrt{x^2+a^2}} - 1)^2}$$

$$\alpha_+(a) \equiv \frac{6}{\pi^2} \int_0^\infty dx \frac{x^2 e^{\sqrt{x^2+a^2}}}{(e^{\sqrt{x^2+a^2}} + 1)^2}$$



quantum number densities (in unit of $T^2/6$)

$$
\begin{aligned}
Q &= \sum_{i=1}^{N}\left[3\cdot\frac{2}{3}\alpha_{u_i}(\mu_{u_L}+\mu_{u_R}) - 3\cdot\frac{1}{3}\alpha_{d_i}(\mu_{d_L}+\mu_{d_R}) \right. \\
&\qquad\qquad\qquad\qquad \left. -\alpha_i(\mu_{iL}+\mu_{iR})\right] \\
&\qquad -1\cdot 2\cdot 2\alpha_W\mu_W - N_H\cdot 2\alpha_-\mu_- \\
I_3 &= \sum_{i=1}^{N}\left[\frac{3}{2}(\alpha_{u_i}\mu_{u_L} - \alpha_{d_i}\mu_{d_L}) + \frac{1}{2}(\mu_i - \alpha_i\mu_{iL})\right] \\
&\qquad -1\cdot 2\cdot 2\alpha_W\mu_W - \frac{1}{2}N_H\cdot 2(\alpha_0\mu_0 + \alpha_-\mu_-) \\
B &= 3\cdot\frac{1}{3}\sum_{i=1}^{N}\left[\alpha_{u_i}(\mu_{u_L}+\mu_{u_R}) + \alpha_{d_i}(\mu_{d_L}+\mu_{d_R})\right] \\
L &= \sum_{i=1}^{N}\left[\mu_i + \alpha_i(\mu_{iL}+\mu_{iR})\right]
\end{aligned}
$$

By use of the equilibrium relations among μ's, and introducing

$$
\Delta_l = N - \sum_i \alpha_i, \qquad \mu = \sum_i \mu_i, \qquad \Delta\mu = \mu - \sum_i \alpha_i\mu_i,
$$

$$
\Delta_u = N - \sum_i \alpha_{u_i}, \qquad \Delta_d = N - \sum_i \alpha_{d_i},
$$

5 unknowns $(\mu_{u_L}, \mu_W, \mu_0, \mu, \Delta\mu)$ before the use of sphaleron equilibrium: $N(\mu_{u_L} + 2\mu_W) + \mu = 0$

$$
\begin{aligned}
Q &= 2(N - 2\Delta_u + \Delta_d)\mu_{u_L} \\
&\quad -2(2N - \Delta_d - \Delta_l + 2\alpha_W + N_H\alpha_-)\mu_W \\
&\quad +(4N - 2\Delta_u - \Delta_d - \Delta_l + 2N_H\alpha_-)\mu_0 - 2(\mu - \Delta\mu), \\
I_3 &= \frac{3}{2}(\Delta_d - \Delta_u)\mu_{u_L} + \frac{1}{2}\Delta\mu - N_H(\alpha_0 - \alpha_-)\mu_0 \\
&\quad +(-2N + \frac{3}{2}\Delta_d + \frac{1}{2}\Delta_l - 4\alpha_W - N_H\alpha_-)\mu_W, \\
B &= 2(2N - \Delta_u - \Delta_d)\mu_{u_L} + 2(N - \Delta_d)\mu_W + (\Delta_d - \Delta_u)\mu_0, \\
L &= 3\mu - 2\Delta\mu + 2(N - \Delta_l)\mu_W - (N - \Delta_l)\mu_0
\end{aligned}
$$

- $T \gtrsim T_C$ (symmetric phase)

  quarks, leptons, W: massless

  $$\Rightarrow \Delta_u = \Delta_d = \Delta_l = \Delta\mu = 0,\ \alpha_W = 1$$

  $m_{\phi^0} = m_{\phi^-} \Rightarrow \alpha_0 = \alpha_-$

$$
\begin{aligned}
B &= \frac{8N + 4N_H\alpha_0}{22N + 13N_H\alpha_0}(B - L) \\
L &= -\frac{14N + 9N_H\alpha_0}{22N + 13N_H\alpha_0}(B - L)
\end{aligned}
$$

the same as those in the massless approx. if $\alpha_0 = 1$.

- $T \lesssim T_C$ (broken phase)

T	Δ_u	Δ_d	Δ_l	α_W
80GeV	0.47	4.2×10^{-4}	7.5×10^{-5}	0.60
100GeV	0.35	2.7×10^{-4}	4.8×10^{-5}	0.66

$$\therefore \quad \Delta_d, \Delta_l \ll \Delta_u < 1$$

Then

$$B = \left(2 + \frac{N}{2\alpha_W + N_H \alpha_-}\right)(2N - \Delta_u)\mu_{u_L} + \frac{N}{2\alpha_W + N_H \alpha_-}\Delta\mu,$$

$$L = -\left[9 + \frac{8(2N - \Delta_u)}{2\alpha_W + N_H \alpha_-}\right] N \mu_{u_L} - 2\left(1 + \frac{2N}{2\alpha_W + N_H \alpha_-}\right)\Delta\mu$$

$\Rightarrow$ $B + L \not\propto B - L$

$\therefore$ $B - L = 0$ does not necessarily imply $B + L = 0$ and $B = 0$

Suppose that $B - L = 0$. (at $^\forall t$) $\Longrightarrow$ $\mu_{u_L} = (\cdots)\Delta\mu$

$$B = \left[-\frac{\left(4N - 2\Delta_u + \frac{4N(2N-\Delta_u)}{2\alpha_W + N_H\alpha_-}\right)(2\alpha_W + N_H\alpha_- + 3N)}{(13N - 2\Delta_u)(\alpha_W + N_H\alpha_-/2) + 6N(2N - \Delta_u)} + \frac{2N}{2\alpha_W + N_H\alpha_-} \right] \Delta\mu$$

flavor asymmetry in L_i's ($\mu_i \neq \mu_j$)

$\Downarrow$

$B \neq 0$, even when $B - L = 0$

Simplified toy model

$\begin{pmatrix} p_i \\ n_i \end{pmatrix}, \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}, W^- \quad (i = 1 - N)$ 'nucleons' well mixed

chemical potential: $\mu_p, \mu_n, \mu_i, \mu_{ie}, \mu_W$

chemical equil.: $\bar{p}_i + n_i \rightleftharpoons W^- \rightleftharpoons \bar{\nu}_i + e_i$

$\rightarrow \mu_W = \mu_n - \mu_p = \mu_{ie} - \mu_i$ $\therefore$ indep. (μ_p, μ_i, μ_W)

sphaleron process: $\prod_i (n_i \nu_i) \rightleftharpoons |0\rangle \rightarrow N(\mu_p + \mu_W) + \mu = 0$

$$\begin{aligned}
Q &= (N - \Delta_p)\mu_p - (N - \Delta_e + 4\alpha_W)\mu_W - (\mu - \Delta\mu), \\
B &= (2N - \Delta_p - \Delta_n)\mu_p + (N - \Delta_e)\mu_W, \\
L &= 2\mu - \Delta\mu + (N - \Delta_e)\mu_W
\end{aligned}$$

In the 'broken phase', $Q = 0$ and sphaleron equil. lead to

$$\begin{aligned}
\mu_W &= \frac{1}{N + 4\alpha_W}\left[(N - \Delta_p)\mu_p - (\mu - \Delta\mu)\right], \\
\mu &= \frac{1}{N+1}\left[\frac{N(2N + 4\alpha_W - \Delta_p)}{N + 4\alpha_W}\mu_p + \Delta\mu\right]
\end{aligned}$$

Then B and L are linear combinations of μ_p and $\Delta\mu$.

$$B - L = 0 \Longrightarrow B = L = \text{const.} \times \Delta\mu$$

Sphaleron process is suppressed by the least n_{ν_i}(?)

If we assumed $n_i \nu_i \rightleftharpoons |0\rangle$ for each flavor, $\mu_n + \mu_i = 0$, and $B = L = 0$ when we assume $B - L = 0$.

4. Discussions

▷ With sphaleron process in equilibrium, BAU can be generated from nonzero $B-L$.

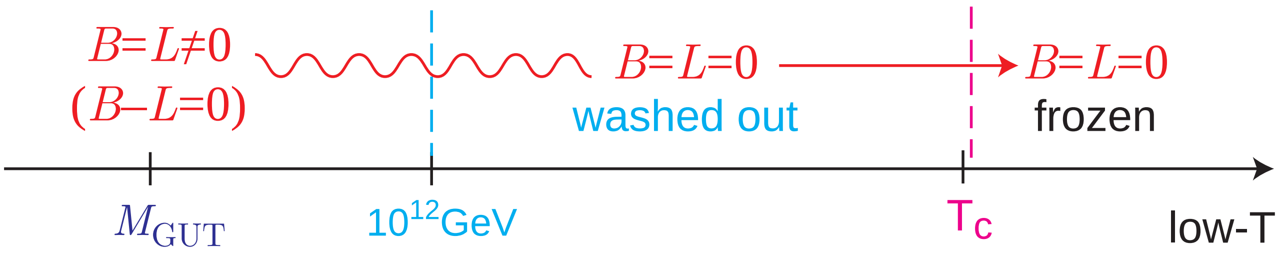
- Leptogenesis [Fukugita & Yanagida, PLB174 ('86)]

  mass scale and CP violation in the heavy ν-sector

  ⇒ Morozumi's and Endoh's talks

- $(B-L)$-violating GUTs

  SU(5) X, SSB of $U(1)_{B-L} \in G_{\mathrm{GUT}}$



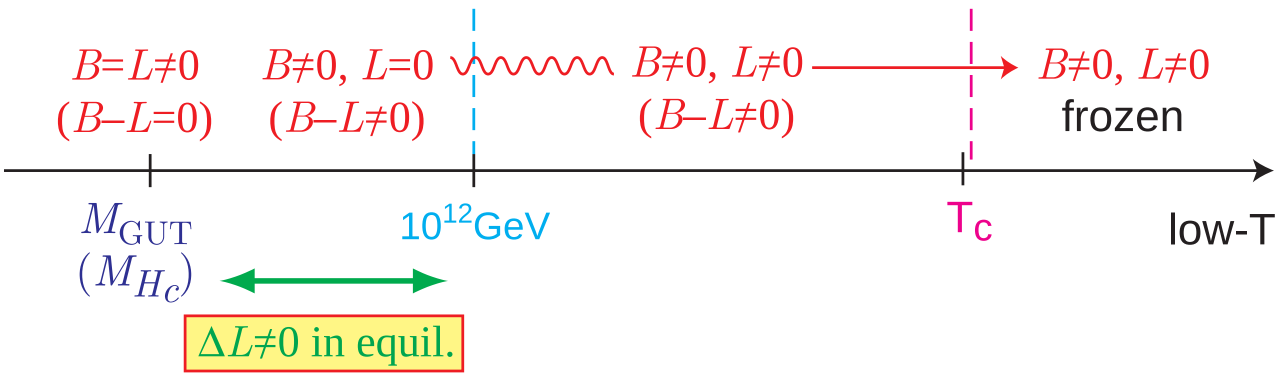
- Affleck-Dine mechanism

  initial condition for $\langle \tilde{q} \rangle$ [Dine, et al. NPB458 ('96)]

  Q-ball formation [Kasuya & Kawasaki, hep-ph/0106119]

▷ "Resurrection of $(B-L)$-conserving GUT B-genesis"

[Fukugita & Yanagida, hep-ph/0203194]



$\Delta L \neq 0$-processes are in equil. at $T \gg 10^{12}$GeV.

$\Longleftarrow$ (experimentally indicated ν-mass)

We must require that

the processes decouple before T lowers to 10^{12}GeV.

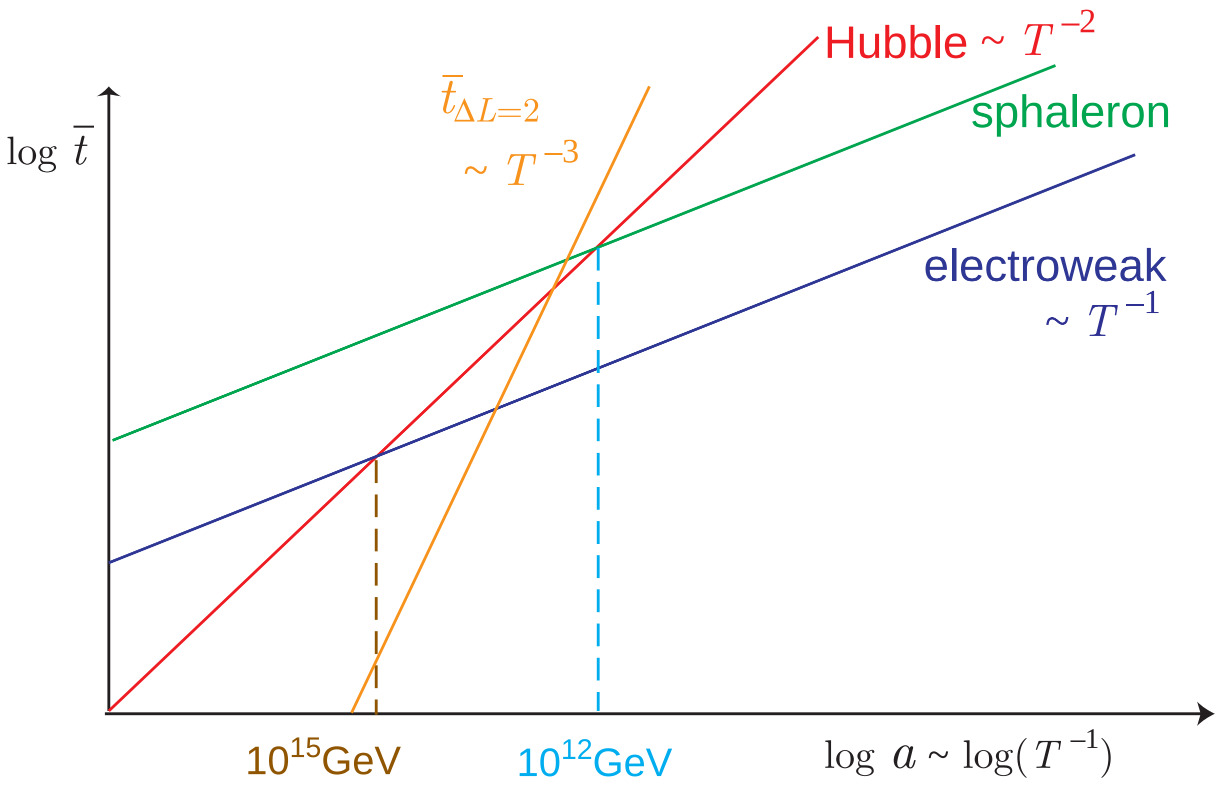
otherwise, $B = L = 0$.

e.g.,

$$\mathcal{L}_{\rm eff} = \frac{g_i^2}{m_{N_i}} l_i \phi\, l_i \phi \quad \Rightarrow \quad \Gamma_{\Delta L=2} \simeq \frac{0.12 g_i^4 T^3}{4\pi m_{N_i}^2}$$

$\Gamma_{\Delta L=2} < H(T)$ at $T < 10^{12}$GeV

$\Rightarrow$ lower bound on $m_{N_i} \Longleftrightarrow m_{\nu_i} < 0.8$eV



$$\log \bar{t}_{\Delta L=2} = 3 \log(T^{-1}) + 2 \log(m_{N_i}) + \cdots$$

▷ Effects of nonzero mass

B-reproduction at $T \in [T_{\rm dec}, T_C)$, if $^\exists$flavor-asym. L_i

(1) production of $L_i \neq L_j$

(2) decoupling of LF-mixing before T_C

Nonvanishing mass at high temperatures

right-handed Majorana mass

soft-SUSY-breaking mass

$\cdots\cdots$

$\Downarrow$

modification of particle number densities

$\Downarrow$

estimation of B ?