Electroweak Baryogenesis

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§1. Introduction

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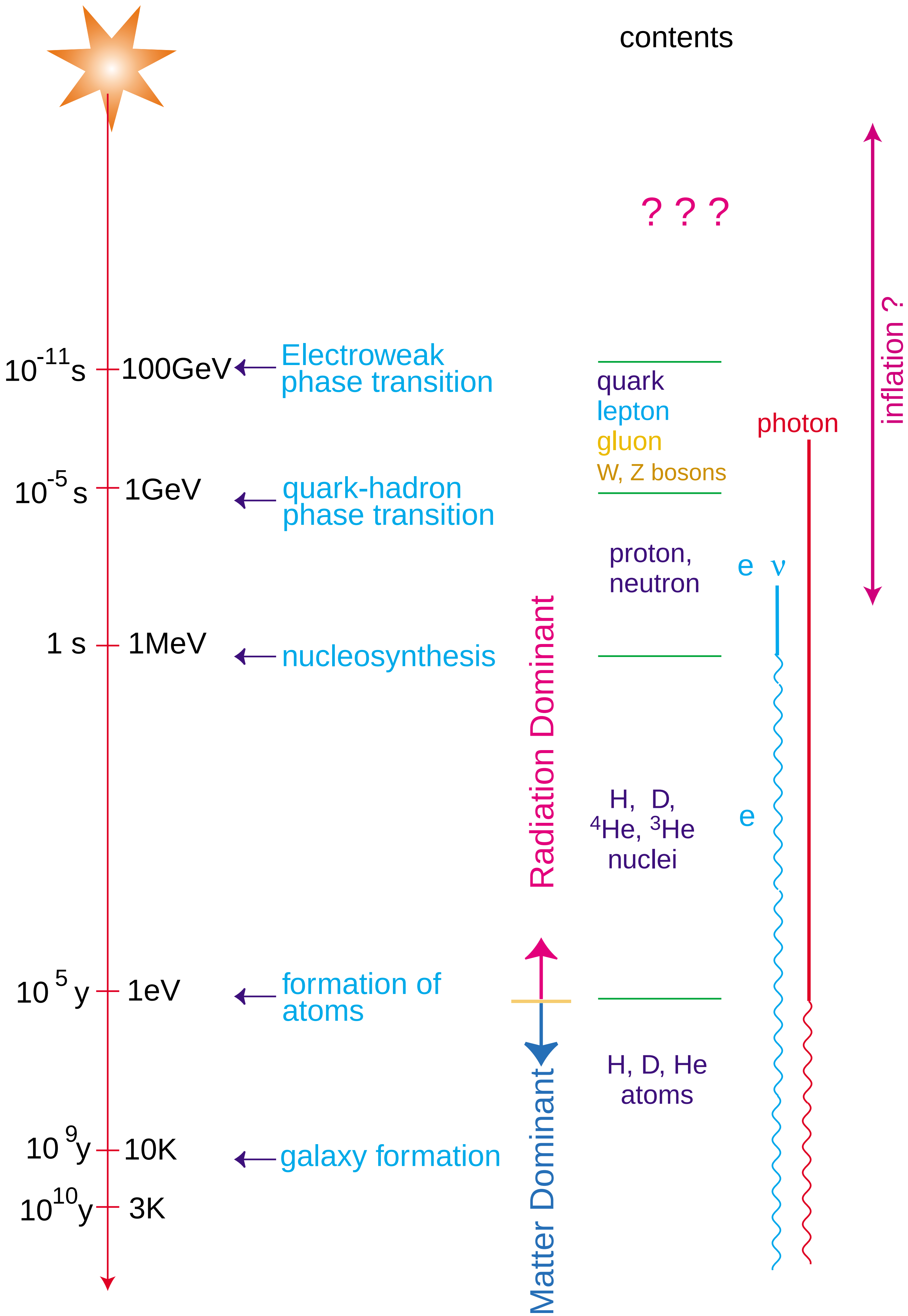
§5. Summary

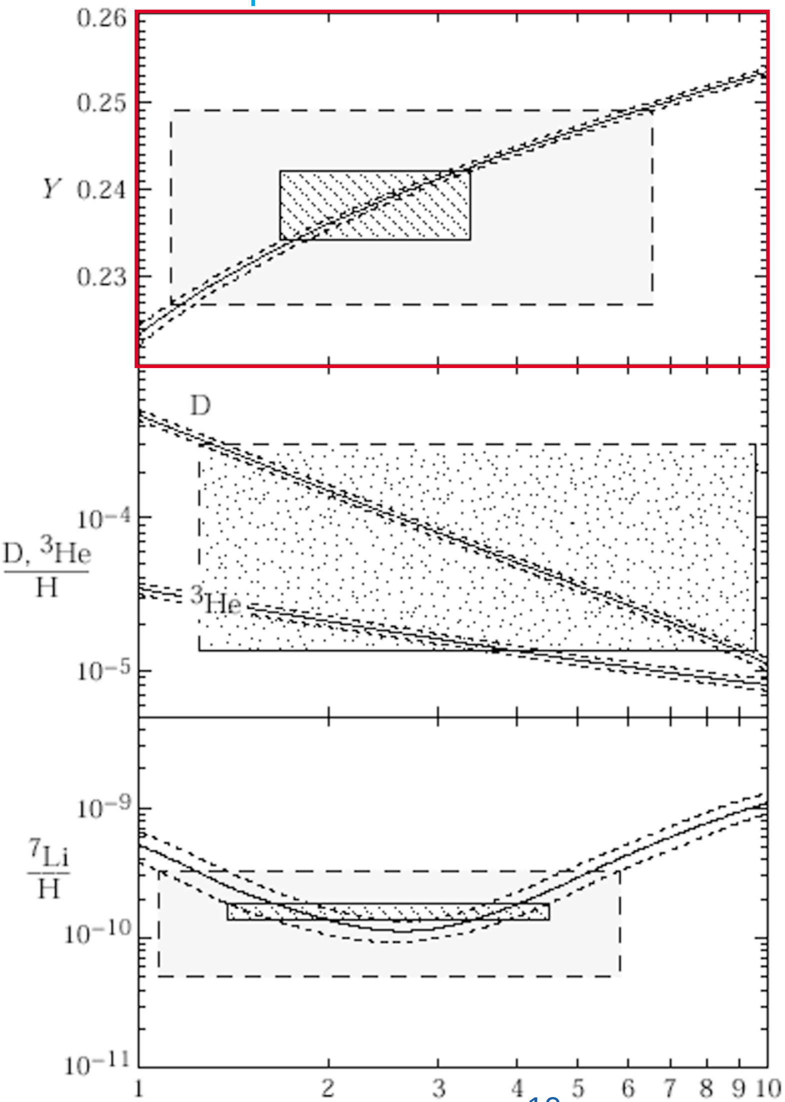


contents
? ? ?
inflation ?
10^{-11}s
100GeV
Electroweak phase transition
quark
lepton
gluon
W, Z bosons
photon
10^{-5} s
1GeV
quark-hadron phase transition
proton, neutron
e ν
1 s
1MeV
nucleosynthesis
Radiation Dominant
H, D, ^{4}He, ^{3}He nuclei
e
10^5 y
1eV
formation of atoms
H, D, He atoms
10^9 y
10K
galaxy formation
10^{10} y
3K
Matter Dominant

Nucleosynthesis

[http://ccwww.kek.jp/pdg/2000/bigbangnucrpp.pdf]

SM prediction vs Observation



n_B / n_γ (x10^{10})

$$Y = \frac{2\, n/p}{1 + n/p}$$

$Y = 0.25 \longleftrightarrow n/p = 1/7$

- $T \gg 1\text{MeV}$: $n \leftrightarrow p + e + \bar{\nu}_e \Rightarrow n/p = 1$

- $T = T_F \simeq 1\text{MeV}$ $\quad \Gamma_{n\leftrightarrow p}(T_F) \simeq H$

$$\left(\frac{n}{p}\right)_{\text{freeze-out}} = e^{-(m_n - m_p)/T_F} \simeq \frac{1}{6}$$

- $T = 0.3 - 0.1\text{MeV}$

$$\frac{n}{p} \longrightarrow \frac{1}{6} - \frac{1}{7} \quad \text{depending on } \frac{n_B}{n_\gamma} \qquad \text{cf. } s \simeq 7 n_\gamma$$

$$\frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} = (0.2 \; - \; 0.9) \times 10^{-10}$$

— constant after the decoupling of $\Delta B \neq 0$ process

evidence of the BAU [Steigman, Ann.Rev.Astron.Astrop.**14**('76)]

1. no anti-matter in cosmic rays from our galaxy
   some anti-matter consistent as secondary products
2. nearby clusters of galaxies are stable
   a cluster: $(1 \sim 100) M_{\rm galaxy} \simeq 10^{12\sim 14} M_\odot$

Starting from a B-symmetric universe ...

$$\frac{n_b}{s} \simeq \frac{n_{\bar{b}}}{s} \quad \sim \quad 8 \times 10^{-11} \quad \text{at } T = 38\text{MeV}$$

$$\sim \quad 7 \times 10^{-20} \quad \text{at } T = 20\text{MeV}$$

$N\bar{N}$-annihilation decouple

At $T = 38$MeV,
mass within a causal region $= 10^2 M_\odot \ll 10^{12} M_\odot$.

$\Downarrow$

We must have the BAU $\frac{n_B}{s} = (0.2 - 0.9) \times 10^{-10}$
before the universe was cooled down to $T \simeq 38$MeV.

3 requirements for generation of BAU [Sakharov, '67]

(1)	baryon number violation
(2)	C and CP violation
(3)	departure from equilibrium

$\because$ (2) If C *or* CP is conserved, no B is generated:
This is because B is odd under C and CP.

indeed . . .

ρ_0 : baryon-symmetric initial state of the universe $s.t.$

$$\langle n_B \rangle_0 = \mathrm{Tr}[\rho_0 n_B] = 0$$

time evolution of $\rho \Leftrightarrow$ Liouville eq.: $i\hbar \dfrac{\partial \rho}{\partial t} + [\rho, H] = 0$

If H is C- *or* CP-invariant, $[\rho, \mathcal{C}] = 0$ *or* $[\rho, \mathcal{CP}] = 0$

[spontaneous CP viol. $\Longrightarrow [\rho, \mathcal{CP}] \neq 0$]

Since $\mathcal{C}B\mathcal{C}^{-1} = -B$ and $\mathcal{CP}B(\mathcal{CP})^{-1} = -B$

$$\langle n_B \rangle = \mathrm{Tr}[\rho n_B] = \mathrm{Tr}[\rho\, \mathcal{C} n_B \mathcal{C}^{-1}] = -\mathrm{Tr}[\rho n_B]$$

or

$$\langle n_B \rangle = \mathrm{Tr}[\rho\, \mathcal{CP} n_B (\mathcal{CP})^{-1}] = -\mathrm{Tr}[\rho n_B]$$

$\therefore$ Both C and CP must be violated to have $\langle n_B \rangle \neq 0$.

Candidates for generating BAU

- $^{\exists}$Majorana neutrino $\Rightarrow$ L-violating interaction

  [Fukugita & Yanagida, PL '86]

  $$\left.\begin{array}{c}\text{decoupling of heavy-}\nu\text{ decay}\\ CP\text{ violation in the lepton sector}\end{array}\right\} \Rightarrow \text{Leptogenesis}$$

  $$\overset{\text{sphaleron}}{\Longrightarrow}\ \text{BAU}$$

  [recent review: Buchmüller & Plümacher, hep-ph/0007176]

- Affleck-Dine mechanism in a supersymmetric model

  [Affleck & Dine, NPB '86]

  $\langle$squark$\rangle \neq 0$ or $\langle$slepton$\rangle \neq 0$ along (nearly) flat directions, at high temperature

  coherent motion of complex $\langle\tilde{q}\rangle$, $\langle\tilde{l}\rangle \neq 0$ B, C, CP viol.

  $\Longrightarrow$ B- and/or L-genesis

- **Electroweak Baryogenesis**

  (1) $\Delta(B+L) \neq 0$ $\left\{\begin{array}{l}\text{enhanced by sphaleron at } T > T_C\\ \text{suppressed by instanton at } T = 0\end{array}\right.$

  (2) C-violation (chiral gauge)

  CP-violation: KM phase or extension of the MSM

  (3) first-order EWPT with expanding bubble walls

- topological defects

  EW string, domain wall $\sim$ EW baryogenesis

  effective volume is too small, mass density of the universe

2. Sphaleron process

⋆ Anomalous fermion number nonconservation

axial anomaly in the standard model

$$\begin{aligned}\partial_\mu j^\mu_{B+L} &= \frac{N_f}{16\pi^2}[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}] \\ \partial_\mu j^\mu_{B-L} &= 0\end{aligned}$$

$N_f =$ number of the generations, $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

integrating these equations,

$$\begin{aligned}&B(t_f) - B(t_i) \\ &= \int_{t_i}^{t_f} d^4x\, \frac{1}{2}\left[\partial_\mu j^\mu_{B+L} + \partial_\mu j^\mu_{B-L}\right] \\ &= \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x \left[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right] \\ &= N_f\left[N_{CS}(t_f) - N_{CS}(t_i)\right]\end{aligned}$$

where N_{CS} is the Chern-Simons number:
in the $A_0 = 0$ gauge,

$$N_{CS}(t) = \frac{1}{32\pi^2}\int d^3x\, \epsilon_{ijk}\Big[g^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g'^2 B_{ij}B_k\Big]_t$$

— gauge-dependent

classical vacua of the gauge sector $\quad \mathcal{E} = \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2) = 0$

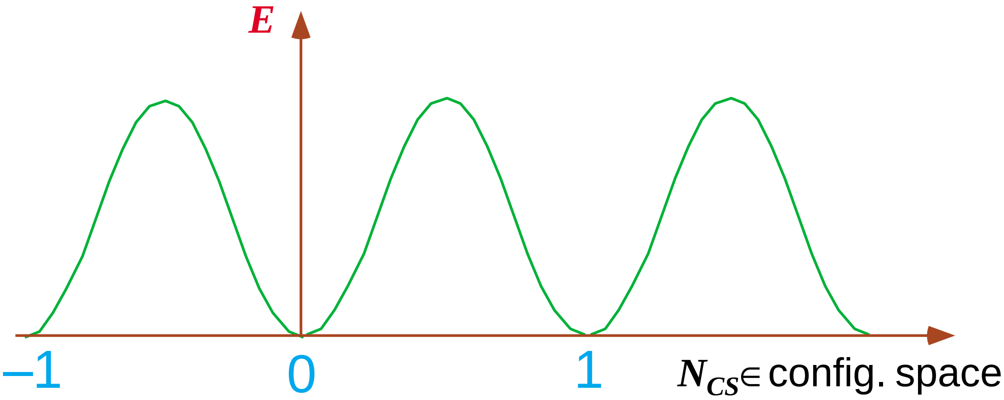
$\Longleftrightarrow \; F_{\mu\nu} = B_{\mu\nu} = 0$

$\Longleftrightarrow \; A = iU^{-1}dU$ and $B = dv$ with $U \in SU(2)$

$\therefore \; U(\boldsymbol{x}) : S^3 \ni \boldsymbol{x} \longrightarrow U \in SU(2) \simeq S^3$

$\pi_3(S^3) \simeq \mathbf{Z} \Rightarrow U(\boldsymbol{x})$ is classified by an integer N_{CS}.

energy functional vs configuration space



background U changes with $\Delta N_{CS} = 1$
$\Longrightarrow \Delta B = 1 \; (\Delta L = 1)$ in each (left-) generation

$\Longleftrightarrow$
- level crossing
- index theorem

Transition of the field config. with $\Delta B \neq 0$?

▷ quantum tunneling	low temperature
▷ thermal activation	high temperature

transition rate with $\Delta N_{CS} = 1 \Longleftarrow$ WKB approx.

$T = 0$

(valley or constrained) instanton = *finite euclidean action*

tunneling probability $\sim e^{-2S_{\rm inst}} = e^{-8\pi^2/g^2}$

for EW theory, $e^{-2S_{\rm inst}} \simeq e^{-378} = 10^{-164}$

[cf. QCD — θ-vacuum]

$T \neq 0$ [Affleck, P.R.L.46('81)]

$^{\exists}$classical static saddle-point solution with *finite energy*

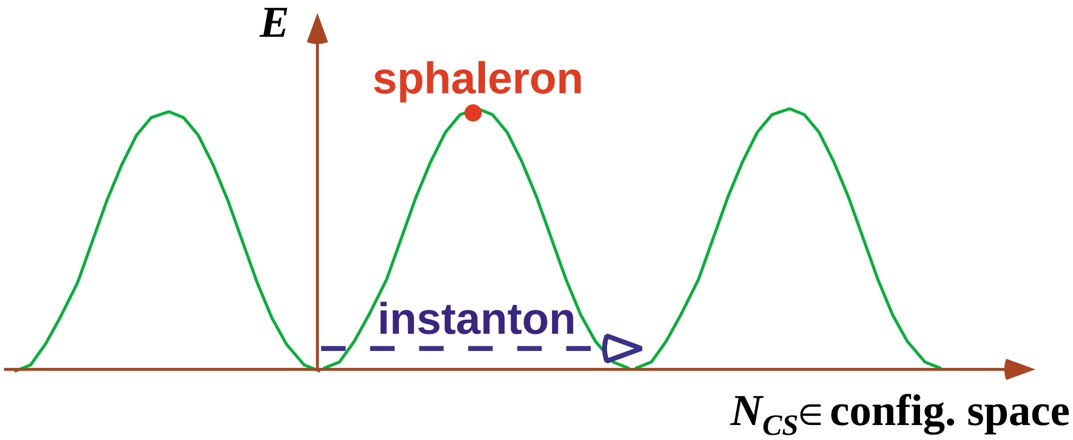
$\Updownarrow$

top of the energy barrier dividing two classical vacua

$\|$

sphaleron solution [Manton, P.R.D28('83)]

$\sigma\varphi\alpha\lambda\epsilon\rho o\sigma$ = 'ready to fall'



$$E_{\rm sph}(T=0) = \frac{2M_W}{\alpha_W} B\left(\frac{\lambda}{g^2}\right) \simeq 10\text{TeV}$$

λ:the Higgs self coupling, $\alpha_W = g^2/(4\pi)$

$1.5 \leq B \leq 2.7$ for $\lambda/g^2 \in [0, \infty)$

⋆ Transition rate

[Arnold and McLerran, P.R.D36('87)]

♣ $\omega_-/(2\pi) \lesssim T \lesssim T_C$

ω_-:negative-mode freq. around the sphaleron

$$\Gamma^{(b)}_{\rm sph} \simeq k\,\mathcal{N}_{\rm tr}\,\mathcal{N}_{\rm rot}\,\frac{\omega_-}{2\pi}\left(\frac{\alpha_W(T)T}{4\pi}\right)^3 {\rm e}^{-E_{\rm sph}/T}$$

zero modes $\rightarrow$ $\begin{cases} \mathcal{N}_{\rm tr} = 26 \\ \mathcal{N}_{\rm rot} = 5.3 \times 10^3 \end{cases}$ for $\lambda = g^2$

$\omega_-^2 \simeq (1.8 \sim 6.6) m_W^2$ for $10^{-2} \leq \lambda/g^2 \leq 10$

$k \simeq O(1)$

♣ $T \gtrsim T_C$ symmetric phase — no mass scale

dimensional analysis :

$$\Gamma^{(s)}_{\rm sph} \simeq \kappa(\alpha_W T)^4$$

check by Monte Carlo simulation $\langle N_{CS}^2(t)\rangle = e^{-2\Gamma V t}$ as $t \rightarrow \infty$

$\kappa > 0.4$ $SU(2)$ gauge-Higgs system

[Ambjørn, et al. N.P.B353('91)]

$\kappa = 1.09 \pm 0.04$ $SU(2)$ pure gauge system

[Ambjørn and Krasnitz, P.L.B362('95)]

'sphaleron transition' even in the symmetric phase.

$\star$ Washout of $B+L$ [Kuzmin, Rubakov, Shaposhnikov, PLB, '85]

sphaleron process is in equilibrium $\Longleftrightarrow$ $\Gamma_{\rm sph} > H$

At $T = T_C \simeq 100$GeV,

$$H(T_C) = \frac{\dot{R}(t)}{R(t)} \simeq 1.7\sqrt{g_*}\frac{T_C^2}{m_{Pl}} \simeq 10^{-13}\text{GeV}$$

$g_* \sim 100$: effective massless degrees of freedom

At $T > T_C$,

$$\Gamma_{\rm sph} \simeq \Gamma_{\rm sph}^{(s)}/T^3 \simeq \kappa\alpha_W^4 T \sim 10^{-4}\text{GeV} \gg H(T_C)$$

$\Longrightarrow$ $B+L$-changing process in equilibrium

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relic baryon number after the washout

[Harvey & Turner, PRD, '90]

particle number density $[m/T \ll 1$ and $\mu/T \ll 1]$

$$\begin{aligned} n_+ - n_- &= \int \frac{d^3\boldsymbol{k}}{(2\pi)^2}\left[\frac{1}{\mathrm{e}^{\beta(\omega_k-\mu)} \mp 1} - \frac{1}{\mathrm{e}^{\beta(\omega_k+\mu)} \mp 1}\right] \\ &\simeq \begin{cases} \dfrac{T^3}{3}\dfrac{\mu}{T} & \text{for bosons} \\ \dfrac{T^3}{6}\dfrac{\mu}{T} & \text{for fermions,} \end{cases} \end{aligned}$$

chemical equilibrium — all the gauge int., Yukawa, sphaleron

W^-	$u_{L(R)}$	$d_{L(R)}$	$e_{iL(R)}$	ν_{iL}	ϕ^0	ϕ^-
μ_W	$\mu_{u_{L(R)}}$	$\mu_{d_{L(R)}}$	$\mu_{iL(R)}$	μ_i	μ_0	μ_-

gauge int. $\Leftrightarrow$ $\mu_W = \mu_{d_L} - \mu_{u_L} = \mu_{iL} - \mu_i = \mu_- + \mu_0$

$|0\rangle \leftrightarrow u_L d_L d_L \nu_L$ $\Leftrightarrow$ $N_f(\mu_{u_L} + 2\mu_{d_L}) + \sum_i \mu_i = 0$

Quantum number densities [in unit of $T^2/6$]

$$
\begin{aligned}
B &= N_f(\mu_{u_L} + \mu_{u_R} + \mu_{d_L} + \mu_{d_R}) = 4N_f\mu_{u_L} + 2N_f\mu_W, \\
L &= \sum_i (\mu_i + \mu_{iL} + \mu_{iR}) = 3\mu + 2N_f\mu_W - N_f\mu_0 \\
Q &= \frac{2}{3}N_f(\mu_{u_L} + \mu_{u_R}) \cdot 3 - \frac{1}{3}N_f(\mu_{d_L} + \mu_{d_R}) \cdot 3 \\
&\quad -\sum_i (\mu_{iL} + \mu_{iR}) - 2 \cdot 2\mu_W - 2m\mu_- \\
&= 2N_f\mu_{u_L} - 2\mu - (4N_f + 4 + 2m)\mu_W + (4N_f + 2m)\mu_0 \\
I_3 &= \frac{1}{2}N_f(\mu_{u_L} - \mu_{d_L}) \cdot 3 + \frac{1}{2}\sum_i (\mu_i - \mu_{iL}) \\
&\quad -2 \cdot 2\mu_W - 2 \cdot \frac{1}{2}m(\mu_0 - \mu_-) \\
&= -(2N_f + m + 4)\mu_W
\end{aligned}
$$

$\mu \equiv \sum_i \mu_i$, m : number of Higgs doublets

- symmetric phase $\Longrightarrow Q = I_3 = 0$

$$B = \frac{8N_f + 4m}{22N_f + 13m}(B-L), \quad L = -\frac{14N_f + 9m}{22N_f + 13m}(B-L)$$

- broken phase $\Longrightarrow Q = 0$ and $\mu_0 = 0$

$$\begin{aligned} B &= \frac{8N_f + 4m + 8}{24N_f + 13m + 26}(B-L) \\ L &= -\frac{16N_f + 9m + 18}{24N_f + 13m + 26}(B-L) \end{aligned}$$

$\therefore$ If $(B-L)_{\text{primordial}} = 0$, $B = L = 0$ at present !

To have nonzero BAU,

(i) we must have $B-L$ before the sphaleron process decouples, or

(ii) $B+L$ must be created at the first-order EWPT, and the sphaleron process must decouple immediately after that.

(i) $\Leftarrow$ GUTs, Majorana ν, Affleck-Dine

(ii) $=$ Electroweak Baryogenesis

3. Electroweak phase transition (EWPT)

rate of any interaction at T : $\Gamma(T) > H(T)$

$\Longrightarrow$ equilibrium thermodynamics can be applied to study static properties

- transition temperature T_C
- order of the phase transition
- latent heat and surface tension for 1st order PT

$\Uparrow$

free energy density = effective potential: $V_{\text{eff}}(\varphi; T)$

Minimal SM (MSM)

order parameter = Higgs VEV: $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \varphi \end{pmatrix}$

$\therefore$ 1st order EWPT $\Longleftrightarrow$ $\varphi_C \equiv \lim_{T \uparrow T_C} \varphi(T) \neq 0$

⋆ Perturbative calculation

$$V_{\text{eff}}(\varphi; T) = V_0(\varphi) + \bar{V}(\varphi; T)$$

where

$$\bar{V}(\varphi; T) = \frac{T^4}{2\pi^2}\left[6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)\right]$$

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{4\pi}\cdots$$

For $T > m_W, m_Z, m_t$,

$$V_{\text{eff}}(\varphi; T) \simeq D(T^2 - T_0^2)\varphi^2 - E\,T\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2)$$

$$E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2)$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4}\left(2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right)$$

with $\log \alpha_B = 2\log 4\pi - 2\gamma_E$ and $\log \alpha_F = 2\log \pi - 2\gamma_E$

At T_C, $\quad \varphi_C = \dfrac{2E\,T_C}{\lambda_{T_C}}$

$$\Gamma_{\text{sph}}^{(b)}/T_C^3 < H(T_C) \quad \Longleftrightarrow \quad \frac{\varphi_C}{T_C} \gtrsim 1$$

$\Longrightarrow$ upper bound on λ $\qquad [m_H = \sqrt{2}\lambda v_0]$

$$m_H \lesssim 46\text{GeV}$$

$\longleftrightarrow$ inconsistent with the lower bound $m_H > 95.3\text{GeV}$ 107.7GeV(LEP)

⋆ Monte Carlo simulations

[MSM]

effective fermion mass : $m_f(T) \sim O(T) \leftarrow$ nonzero modes

$\therefore$ simulation only with the bosons

QFT on the lattice $\begin{cases} \text{scalar fields:} & \phi(x) \text{ on the sites} \\ \text{gauge fields:} & U_\mu(x) \text{ on the links} \end{cases}$

$$Z = \int [d\phi\, dU_\mu] \exp\{-S_E[\phi, U_\mu]\}$$

- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet

  $\Uparrow$

  time-component of the gauge field

  only zero-freq. modes of the bosons survive as $T \longrightarrow$ large

  matching finite-T Green's functions with 4-dim. theory

  $\Rightarrow$ T-dependent parameters

  [Laine & Rummukainen, hep-lat/9809045]

- 4-dim. $SU(2)$ system with a Higgs doublet

  [Csikor, hep-lat/9910354]

  EWPT is first order for $m_h < 66.5 \pm 1.4$GeV

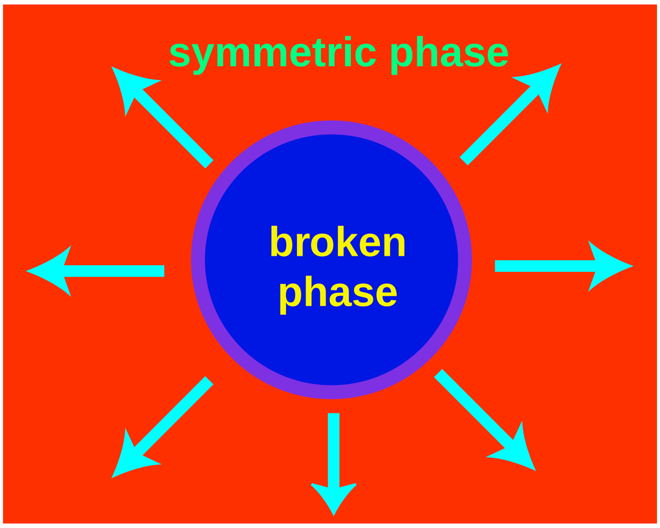
Both the simulations found end-point of EWPT at

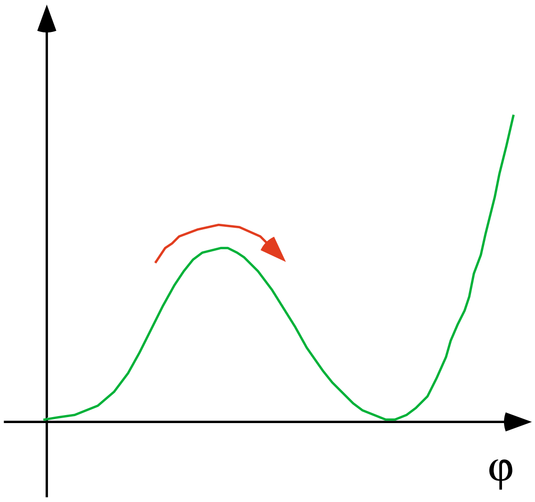
$$m_h = \begin{cases} 72.3 \pm 0.7 \text{ GeV} \\ 72.1 \pm 1.4 \text{ GeV} \end{cases} \Rightarrow \boxed{\text{no PT in the MSM !}}$$

no out-of-equilibrium state at the EWPT

⋆ Dynamics of the phase transition

first-order EWPT accompanying bubble nucleation/growth





Suppose that $V_{\text{eff}}(\varphi; T_C)$ is known.
nucleation rate per unit time and unit volume:

$$I(T) = I_0 \mathrm{e}^{-\Delta F(T)/T}$$

where

$$\Delta F(T) = \frac{4\pi}{3} r^3 [p_s(T) - p_b(T)] + 4\pi r^2 \sigma$$

with

$$p_s(T) = -V_{\text{eff}}(0; T), \qquad p_b(T) = -V_{\text{eff}}(\varphi(T); T)$$

supercooling $\longrightarrow$ $p_s(T) < p_b(T)$

$\sigma \simeq \int dz (d\varphi/dz)^2$: surface energy density

radius of the critical bubble : $r_*(T) = \dfrac{2\sigma}{p_b(T) - p_s(T)}$

How the EWPT proceeds ? [Carrington and Kapsta, P.R.D47('93)]

$f(t)$: fraction of space converted to the broken phase

$$f(t) = \int_{t_C}^{t} dt' \, I(T(t'))[1 - f(t')]V(t', t)$$

where

$V(t', t)$: volume of a bubble at t which was nucleated at t'

$$V(t', t) = \frac{4\pi}{3}\left[r_*(T(t')) + \int_{t'}^{t} dt'' \, v(T(t''))\right]^3$$

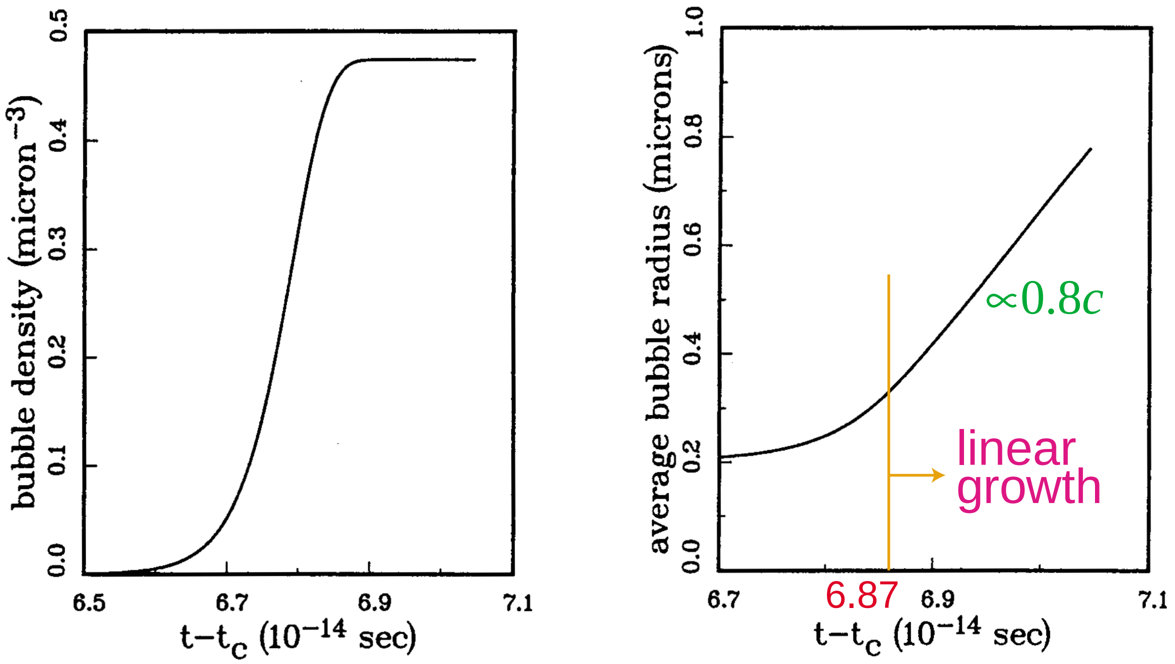
$T = T(t) \Leftarrow \rho = (\pi^2/30)g_* T^4 \propto R^{-4}$ for RD universe

$v(T)$: wall velocity

- one-loop $V_{\rm eff}$ of MSM with $m_H = 60$GeV and $m_t = 120$GeV

At $t = 6.5 \times 10^{-14}$ sec, bubbles began to nucleate.

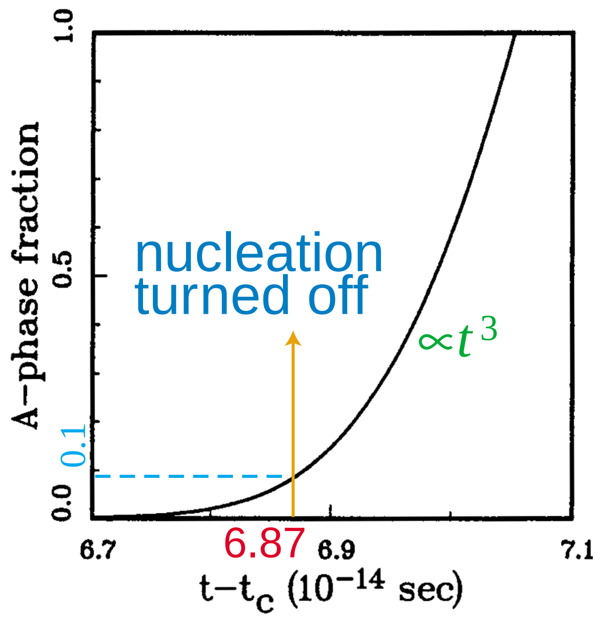
[A characteristic time scale of the EW processes is $O(10^{-26})$sec.]

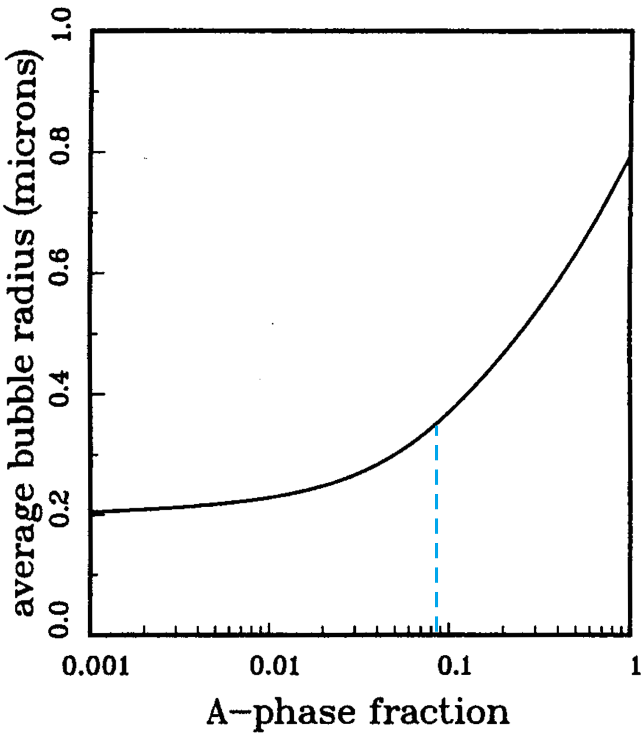


horizon size : $H^{-1} \simeq 7.1 \times 10^{12}$ GeV^{-1} $= 0.14$ cm

$r = 0.3\mu$m $\Rightarrow$ #(bubbles within a horizon) $\simeq 3 \times 10^{11}$

very small supercooling : $\dfrac{T_C - T_N}{T_C} \simeq 2.5 \times 10^{-4}$





90% of the universe is converted by bubble growth

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weakly first order $\Longleftrightarrow$ small φ_C and/or lower barrier height

$\Longrightarrow$
- nucleation dominance over growth
- thick bubble wall
- large fluctuation between the two phases

4. Electroweak baryogenesis

⋆ various time scales at $T \simeq T_C$

At $T = 100\text{GeV}$, MFP: $\lambda \simeq 1/n\sigma \simeq$ MFT

$$\lambda_s \simeq \frac{1}{10^3 \alpha_s^2} \sim 0.1\text{GeV}^{-1} \quad \text{for strong interactions}$$

$$\lambda_{EW} \simeq \frac{1}{10^3 \alpha_W^2} \sim 1\text{GeV}^{-1} \quad \text{for electroweak interactions}$$

$$\lambda_Y \simeq \left(\frac{m_W}{m_f}\right)^4 \lambda_{EW} \quad \text{for Yukawa interactions}$$

- expansion of the universe: $H^{-1}(T) \simeq 10^{14}\text{GeV}^{-1}$
- sphaleron process: $\bar{t}_{\text{sph}} \simeq (\Gamma_{\text{sph}}/n)^{-1} \sim 10^5\text{GeV}^{-1}$
- EW bubble wall thickness and velocity:

$$l_w \simeq \frac{1 \sim 40}{T} \simeq 0.01 \sim 0.4\text{GeV}^{-1}$$

$$v_w \simeq 0.1 \sim 0.9$$

[Liu, McLerran and Turok, PRD,'92]

time scale of the EW bubble wall motion

$$t_{\text{wall}} = \frac{l_w}{v_w} \simeq 0.01 \sim 4\text{GeV}^{-1}.$$

$\therefore$ $\lambda_s < \lambda_W < \lambda_Y \lesssim t_{\rm wall} \ll \bar{t}_{\rm sph} \ll H^{-1}(T)$

1. All the particles are in *kinetic equilibrium* at the same temperature, because of $H^{-1} \gg \bar{t}_{EW}$, far from the bubble wall.

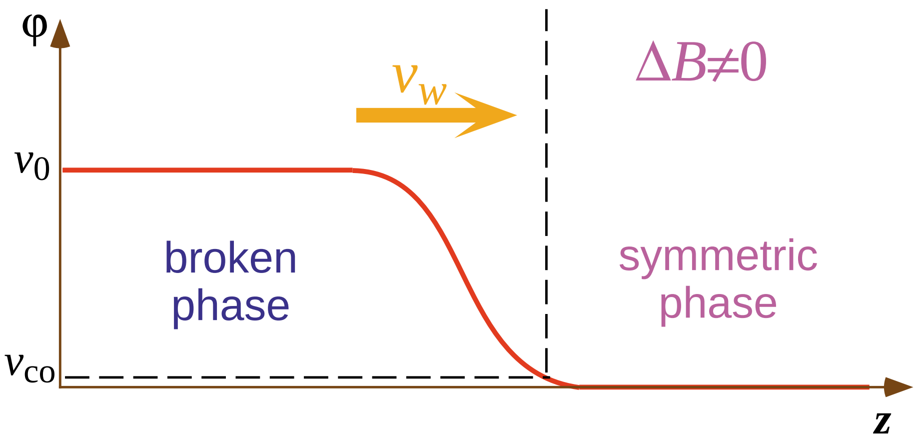
2. The Yukawa interactions of the light fermions ($m_f <$ 0.1GeV) are out of *chemical equilibrium*.

3. Some of the flavor-changing interactions are out of *chemical equilibrium* because of small KM matrix elements.

4. Since for the leptons $\lambda_Y > \lambda_{EW} \gg l_w$, the leptons propagate almost freely before and after the scattering off the bubble wall.

5. Because of $t_{\rm wall} \ll \bar{t}_{\rm sph}$, the sphaleron process is out of *chemical equilibrium* near the bubble wall.

⋆ Mechanism



$v_{co} \simeq 0.01 v_0 \Longleftarrow E_{\rm sph}/T_C \simeq 1$

bubble wall $\Leftarrow$ classical config. of the gauge-Higgs system

- interactions between the particles and the bubble wall
- accumulation of chiral charge in the symmetric phase

$\Downarrow$

generation of baryon number through sphaleron process

$\Downarrow$

decoupling of sphaleron process in the broken phase

2 scenarios:

- spontaneous baryogenesis + diffusion
  classical, adiabatic
- charge transport scenario
  quantum mechanical, nonlocal

Both need CP violation other than KM matrix

$\Longleftrightarrow$ extension of the MSM

two-Higgs-doublet model, MSSM, . . .

⋆ Charge transport mechanism

[Nelson, el al. NPB,'92]

CP violation in the Higgs sector [spacetime-dependent]

⇓

difference in reflections of chiral fermions and antifermions

⇓

net chiral charge flux into the symmetric phase

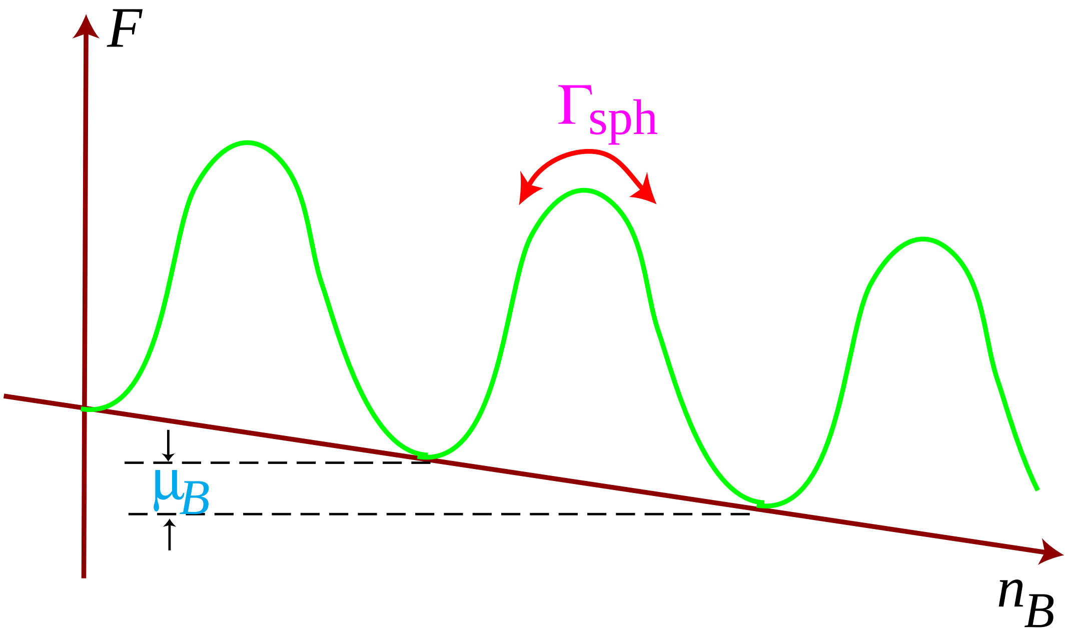
⇓

change of distribution functions by the chiral charge
with the sphaleron process out of equilibrium

⟸ Boltzmann equations

bubble wall velocity ≃ const. ⇒ constant chiral charge flux

⟹ bias on free energy along B [stationary nonequilibrium]

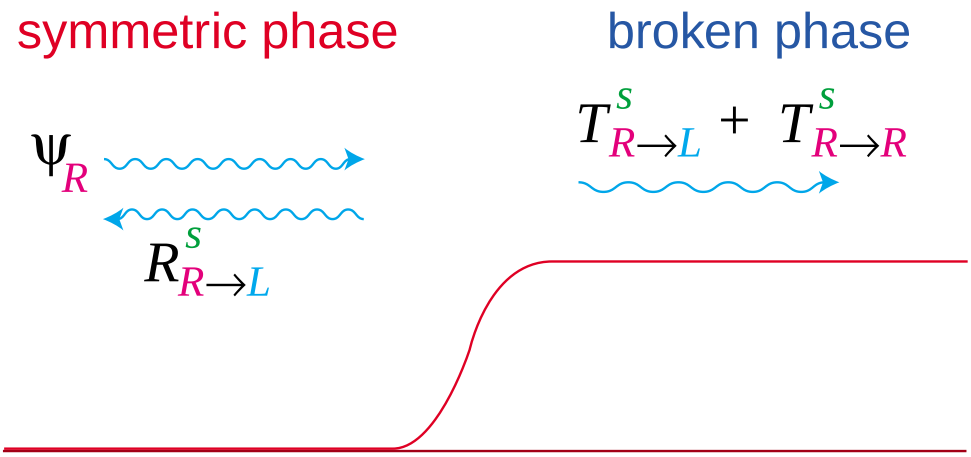


$$\dot{n}_B \simeq -\frac{\mu_B \Gamma_{\rm sph}}{T}$$

fermion scattering-off CP-violating bubble wall

$$i\partial\!\!\!/\psi(x) - m(x) P_R\, \psi(x) - m^*(x) P_L\, \psi(x) = 0$$

where $\quad -f\langle\phi(x)\rangle = m(x) \ \in \mathbf{C}$ $\quad$ through the Yukawa int.



$Q^i_{L(R)}$: charge of a L(R)-handed fermion of species i

$R^s{}_{R\to L}$: reflection coeff. for the R-handed fermion incident from the symmetric phase region

$\bar{R}^s{}_{R\to L}$: the same as above for the R-handed antifermion

$\langle$injected charge into symmetric phase$\rangle$ brought by the fermions and antifermions in the symmetric phase :

$$\begin{aligned}
&\Delta Q_i{}^s \\
&= [(Q_R{}^i - Q_L{}^i) R^s{}_{L\to R} + (-Q_L{}^i + Q_R{}^i)\bar{R}^s{}_{R\to L} \\
&+(-Q_L{}^i)(T^s{}_{L\to L} + T^s{}_{L\to R}) - (-Q_R{}^i)(\bar{T}^s{}_{R\to L} + \bar{T}^s{}_{R\to R})] f^s{}_{Li} \\
&+[(Q_L{}^i - Q_R{}^i) R^s{}_{R\to L} + (-Q_R{}^i + Q_L{}^i)\bar{R}^s{}_{L\to R} \\
&+(-Q_R{}^i)(T^s{}_{R\to L} + T^s{}_{R\to R}) - (-Q_L{}^i)(\bar{T}^s{}_{L\to L} + \bar{T}^s{}_{L\to R})] f^s{}_{Ri}
\end{aligned}$$

the same brought by transmission from the broken phase :

$$\begin{aligned}\Delta Q_i^{\,b} \quad = \quad & Q_L^{\,i}({T^b}_{L\to L}{f^b}_{Li} + {T^b}_{R\to L}{f^b}_{Ri}) \\ & + Q_R^{\,i}({T^b}_{L\to R}{f^b}_{Li} + {T^b}_{R\to R}{f^b}_{Ri}) \\ & + (-Q_L^{\,i})({\bar{T}^b}_{R\to L}{f^b}_{Li} + {\bar{T}^b}_{L\to L}{f^b}_{Ri}) \\ & + (-Q_R^{\,i})({\bar{T}^b}_{R\to R}{f^b}_{Li} + {\bar{T}^b}_{L\to R}{f^b}_{Ri})\end{aligned}$$

by use of

unitarity: $\quad {R^s}_{L\to R} + {T^s}_{L\to L} + {T^s}_{L\to R} = 1,$ etc.

reciprocity: $\quad {T^s}_{R\to L} + {T^s}_{R\to R} = {T^b}_{L\to L} + {T^b}_{R\to L},$ etc.

$$f_{iL}^{s(b)} = f_{iR}^{s(b)} \equiv f_i^{s(b)}$$

we obatin

$$\boxed{\Delta Q^s{}_i + \Delta Q^b{}_i = (Q_L^{\,i} - Q_R^{\,i})(f^s{}_i - f^b{}_i)\Delta R}$$

where

$$\boxed{\Delta R \equiv R^s{}_{R\to L} - \bar{R}^s{}_{R\to L}}$$

which depends on

- profile of the bubble wall
  - wall thickness, height
  - CP phase
- momentum of the incident particle

total flux injected into the *symmetric phase* region

$$F^i{}_Q = \frac{Q_L{}^i - Q_R{}^i}{4\pi^2\gamma} \int_{m_0}^{\infty} dp_L \int_0^{\infty} dp_T\, p_T$$
$$\times \left[f_i{}^s(p_L, p_T) - f_i{}^b(-p_L, p_T) \right] \Delta R(\frac{m_0}{a}, \frac{p_L}{a})$$

where

$$f_i{}^s(p_L, p_T) = \frac{p_L}{E} \frac{1}{\exp[\gamma(E - v_w p_L)/T] + 1}$$
$$f_i{}^b(-p_L, p_T) = \frac{p_L}{E} \frac{1}{\exp[\gamma(E + v_w \sqrt{p_L^2 - m_0^2})/T] + 1}$$

the fermion flux densities in the symmetric and broken phases.

m_0 : fermion mass in the broken pase
v_w : wall velocity
p_T : transverse momentum
$1/a$: wall width

$$\gamma = 1/\sqrt{1 - v_w^2}$$
$$E = \sqrt{p_L^2 + p_T^2}$$

available charge :

$Q_L - Q_R \neq 0$
conserved in the symmetric phase
$\Big\} \Longrightarrow$ $Y, \quad I_3$

change of the state by the injection of the flux

Assume

- bubble is macroscopic and expand with const. velocity
- deep in the sym. phase, elementary processes are fast enough to realize a new stationary state
- the sphaleron process is out of equilibrium near the bubble wall

$\Longrightarrow$ chemical potential argument

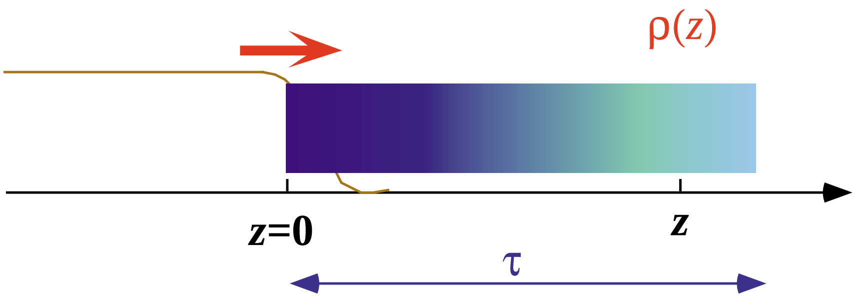
$$\mu_B = \frac{Y}{2(m+5/3)T^2}$$

Integrating the equation for $\dot{n}_B$,

$$n_B = -\frac{\Gamma_{\rm sph}}{T}\int dt\,\mu_B = -\frac{\Gamma_{\rm sph}}{2(m+5/3)T^3}\int dt\,Y$$

where

$$\int dt\,Y = \int_{-\infty}^{z/v_w} dt\,\rho_Y(z-v_w t) = \frac{1}{v_w}\int_0^\infty dz\,\rho_Y(z).$$



$$\frac{1}{v_w}\int_0^\infty dz\,\rho_Y(z) \simeq \frac{F_Y\tau}{v_w}$$

generated BAU :

$$\frac{n_B}{s} \simeq \mathcal{N} \frac{100}{\pi^2 g_*} \cdot \kappa \alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T$$

$$\mathcal{N} \sim O(1)$$

$$\tau T \simeq \begin{cases} 1 & \text{for quarks} \\ 10^2 \sim 10^3 & \text{for leptons} \end{cases}$$

MC simulation $\Longrightarrow$ forward scattering enhanced :

for top quark

$\tau T \simeq 10 \sim 10^3$ max. at $v_w \simeq 1/\sqrt{3}$

for this optimal case [top quark]

$$\frac{n_B}{s} \simeq 10^{-3} \cdot \frac{F_Y}{v_w T^3}$$

$\Longrightarrow$ $F_Y/(v_w T^3) \sim O(10^{-7})$ would be sufficient to explain the BAU.

⋆ **Example** [Nelson et al., NPB, '92]

$$m(z) = m_0 \frac{1+\tanh(az)}{2} \exp\left(-i\pi \frac{1-\tanh(az)}{2}\right)$$

— no CP violation in the broken phase $[z \sim \infty]$

- Calculation of $\Delta R \longrightarrow$ chiral charge flux
  - (i) perturbative method [FKOTT, PRD,'94]
  - (ii) numerical method [CKN, NPB '92, FKOT, PTP,'96]

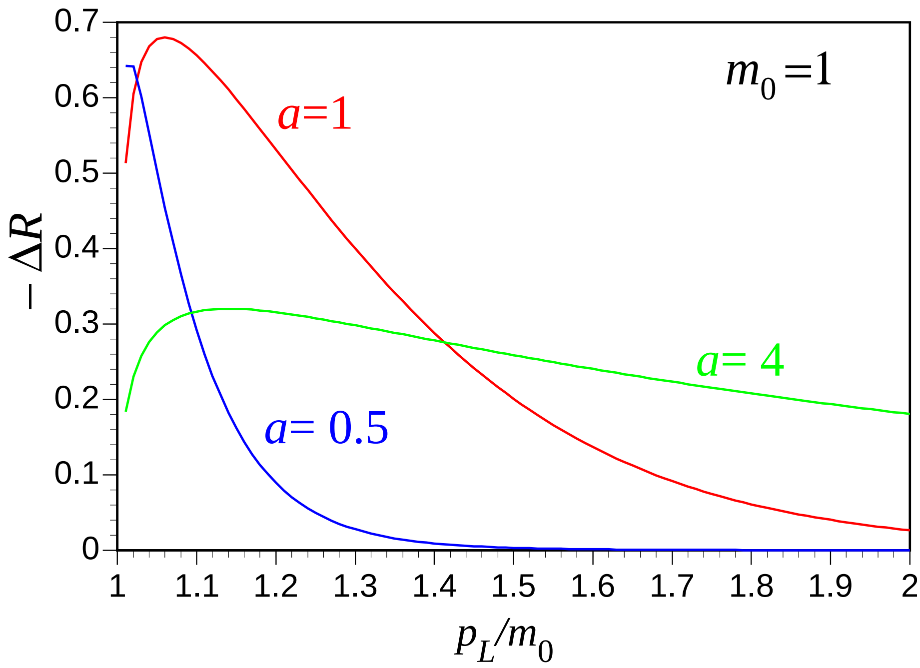
- $\Delta R \equiv {R^s}_{R\to L} - {\bar{R}^s}_{R\to L}$

  wall width $\simeq$ wave length of the carrier $\Rightarrow \Delta R \sim O(1)$

  $\Downarrow$

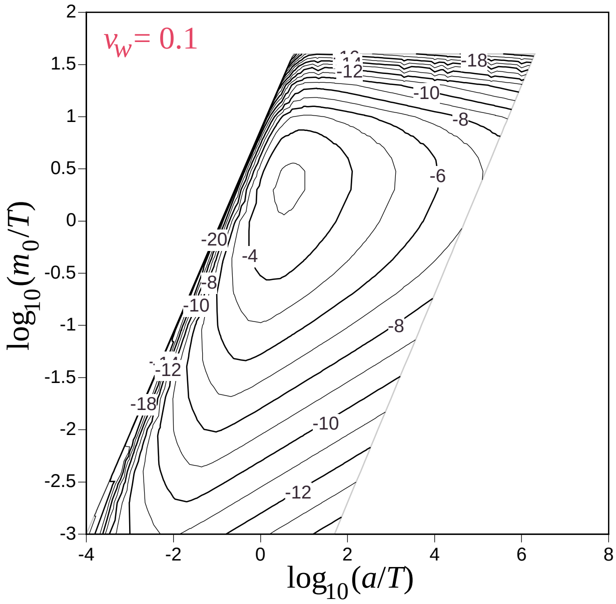
  stronger Yukawa coupling does *not* always implies larger flux

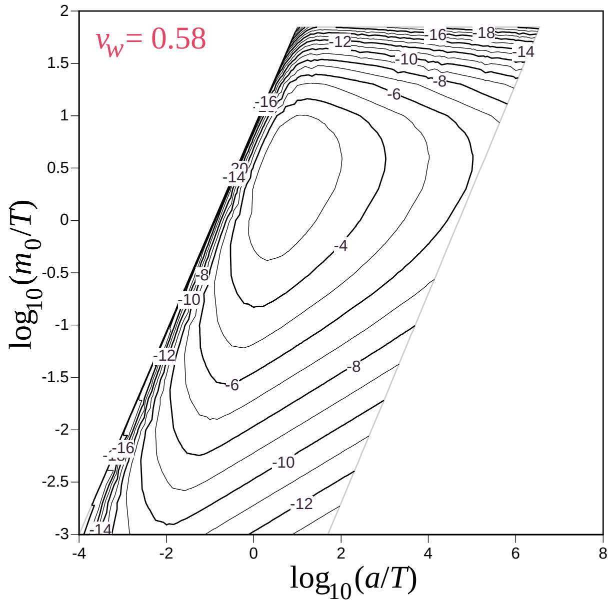
  for larger energy, ΔR decays exponentially

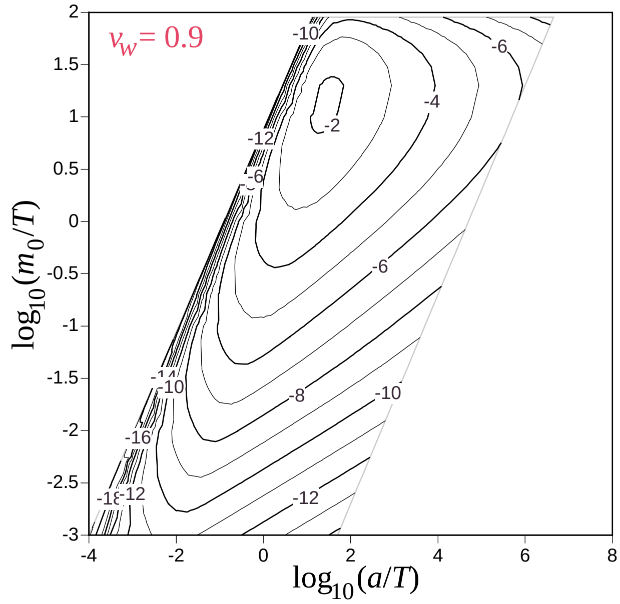


- chiral charge flux $\quad T = 100$ GeV

normalized as $\dfrac{F_Q}{T^3(Q_L - Q_R)}$ [dimensionless]







$$\frac{n_B}{s} \simeq \mathcal{N}\frac{100}{\pi^2 g_*} \cdot \kappa\alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T$$

$$\simeq 10^{-3} \cdot \frac{F_Y}{v_w T^3}$$ for an optimal case (top quark)

⋆ CP Violation

complex parameters in

- Yukawa coupling $y\bar{\psi}_R \chi_L \phi$ (∼ charged-curr. int.)
- scalar self coupling — extension of the SM

  2 Higgs doublets

  scalar partners of quarks and leptons in SUSY models

  & relative phase(s) between Higgs VEVs

CP violation at $T = 0$ (broken phase)
constrained by experiments — *e.g.* nEDM

CP violation relevant to EW baryogenesis
complex parameters

Higgs phases in the transient region from broken to symmetric phases

⇑

minimizing $E = \int d^3\boldsymbol{x} \left[\frac{1}{2}(\nabla\phi_i)^2 + V_{\text{eff}}(\phi_i; T_C) \right]$

with b.c. connecting the two phases

enhancement by **Transitional CP Violaion**

K.F., Kakuto, Otsuki, Toyoda, PTP98 ('97); 102 ('99)

6. Summary

Electroweak Baryogenesis

⋆ based on physics which are known or will be check in near future
but
Minimal SM

× { strongly 1st-order EWPT (with acceptable m_h)
  sufficient CP violation }

$\Longrightarrow$ extension of the SM MSSM, 2HDM, $\cdots$

⋆ issues in Thermo Field Theory

▷ equilibrium — order and T_C of the EW phase transition

3 (or more)-dim. space of order parameters

▷ nonequilibrium

sphaleron transition
dynamics of the EWPT
generation of B