Preheating and Charge Generation

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§ 1. Introduction

Big Bang宇宙論で未解決の問題

Horizon problem
Flatness problem
Density perturbation
} ⟸ inflation

Baryon asymmetry
Dark matter
Initial value problem
...

Baryon Asymmetry of the Universe
- バリオン数の破れ
- CとCPの破れ
- 非平衡状態

⋆ GUTs

⋆ Affleck-Dine

⋆ Leptogenesis

⋆ Electroweak Baryogenesis
sphaleron process, 1st order EW phase transition

Inflation後の**非平衡状態**を利用するもの

▷ Affleck-Dine

▷ reheating

▷ preheating

Inflation

[Kolb-Turner, The Early Universe]

— コヒーレントなスカラー場(inflaton 場)のダイナミクス

1 de Sitter期 — inflaton の potential energy

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \cdots\right)$$

$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + V'(\phi) = 0$$

slow-roll in a nearly flat potential $\Longrightarrow V(\phi) \simeq$ const. $\gg \dot{\phi}^2$

$\longrightarrow a(t) \propto e^{Ht}$ with constant H

2 reheating期 — inflaton のエネルギーから熱へ

$V(\phi)$ の最小値のまわりでの inflaton 場の振動

⇓

inflaton 場が軽い粒子へ崩壊することによる振動の減衰

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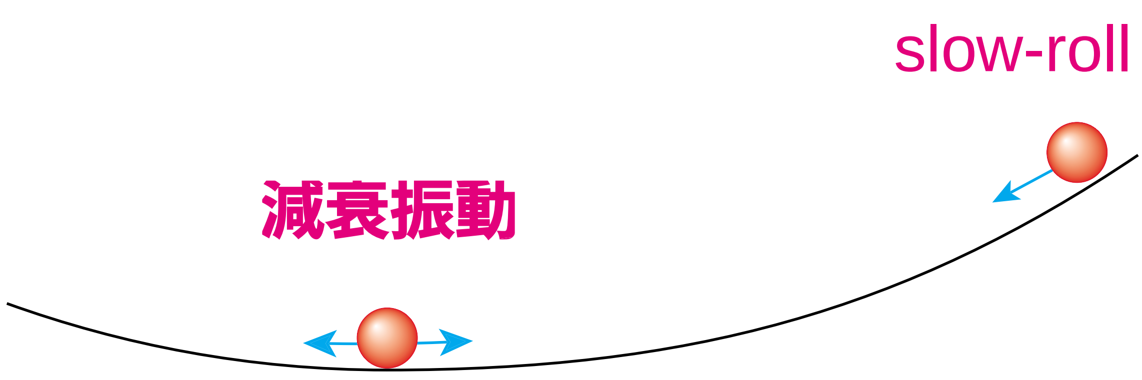
軽い粒子の熱平衡分布: $T_{\rm rh}$ (再加熱温度)



requirements for successful inflation

(1) number of e-folds

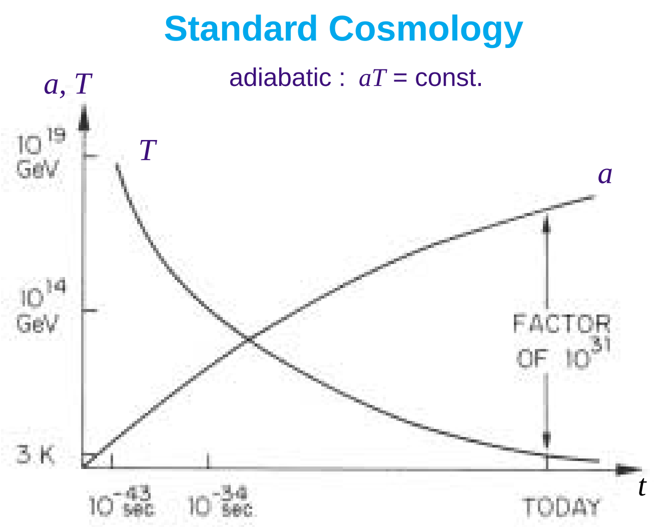
horizon problem $\Rightarrow H_i^{-1} \times e^{N_{\rm tot}} \times \dfrac{a_0}{a_{\rm osc}} > H_0^{-1}$

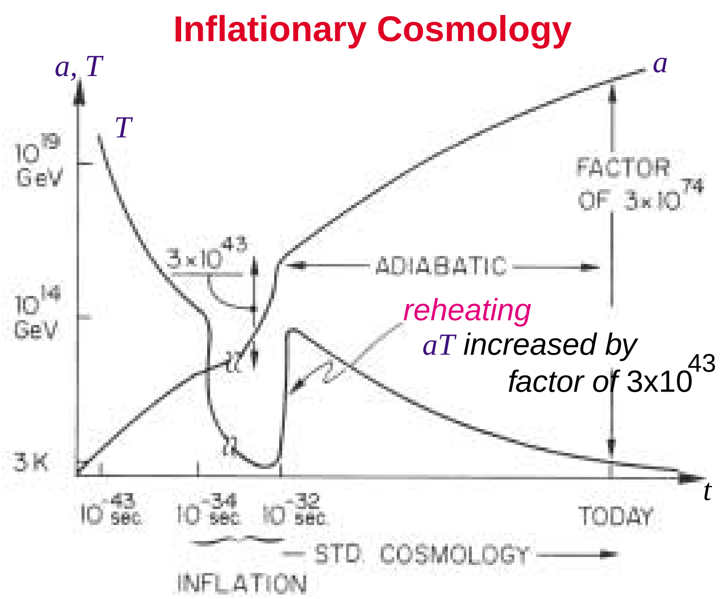
$$\therefore \quad N_{\rm min} \simeq 53 + \frac{2}{3}\log\left(\frac{M}{10^{14}{\rm GeV}}\right) + \frac{1}{3}\log\left(\frac{T_{\rm rh}}{10^{10}{\rm GeV}}\right)$$

where $V(\phi_{\rm ini}) \simeq M^4$

(2) density perturbation $\longrightarrow \delta T/T \simeq 2\times 10^{-5}$

$$\left(\frac{\delta\rho}{\rho}\right)_{\rm reenter} \simeq \frac{1}{2\pi}\left(\frac{V(\phi)\,H}{\dot{\phi}^2}\right)_{\rm bye} \simeq \frac{1}{m_P}\left(\frac{V^{3/2}(\phi)}{V'(\phi)}\right)_{\rm bye}$$





現在の宇宙の粒子、エントロピーは reheating で生成された。

現在の宇宙の粒子がinflation後に生成されたとすれば、
バリオン数もその時期に出来たと考えるのは自然。

particle creation after inflation

- reheating — perturbaive decay of the inflaton
- preheating — depending on paremters in the model
  - ▷ exponentially increasing particle number
  - ▷ large quantum fluctuation in low-energy modes

$\Downarrow$

nonthermal phase transition

Khlebnikov, et al. PRL81('98) — 1st order PT
Tkachev, et al. PL440('98) — string formaiton

new SUSY-breaking effects

Anderson, et al. PRL77('96) — Affleck-Dine

. . . . . .

§ 2. Review of Preheating

reheating = 「古典的スカラー場の軽い粒子への崩壊」

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{2}(\partial\chi)^2 + \bar{\psi} i \partial\!\!\!/ \psi$$
$$-\frac{1}{2}g^2\phi^2\chi^2 - f\bar{\psi}\psi\phi$$

崩壊率 (0-momentum ϕ 粒子の崩壊)

$$\Gamma_\phi = \Gamma(\phi \to \chi\chi) + \Gamma(\phi \to \psi\psi) = \frac{g^4\langle\phi\rangle^2}{8\pi m_\phi} + \frac{f^2 m_\phi}{8\pi}$$

EOM for ϕ:

$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + \Gamma_\phi\dot{\phi}(t) + V'(\phi) = 0$$

疑問点

(1) 「古典的場の崩壊」を**場の量子論**でどう取り扱うか？

(2) g^2 や f が小さくてもinflaton振幅が大きいときに摂動論が使えるか？ (e.g., $|g\phi| > m_\chi$ のとき)

⇓

古典的スカラー場 $\phi(t)$ を背景とする場の理論と考える

⇕

preheating

[Kofman, Linde, Starobinsky, PRD56('97)]

EOM for the inflaton with $V(\phi) \simeq \frac{1}{2}m^2\phi^2$:

$$\Rightarrow \phi(t) = \Phi(t)\sin(mt) \propto \frac{1}{t}\sin(mt)$$

mode equation for $\chi_k(t)$:

$$\ddot{\chi}_k(t) + 3H(t)\dot{\chi}_k(t) + \left(\frac{k^2}{a^2} + g^2\Phi^2(t)\sin^2(mt)\right)\chi_k(t) = 0$$

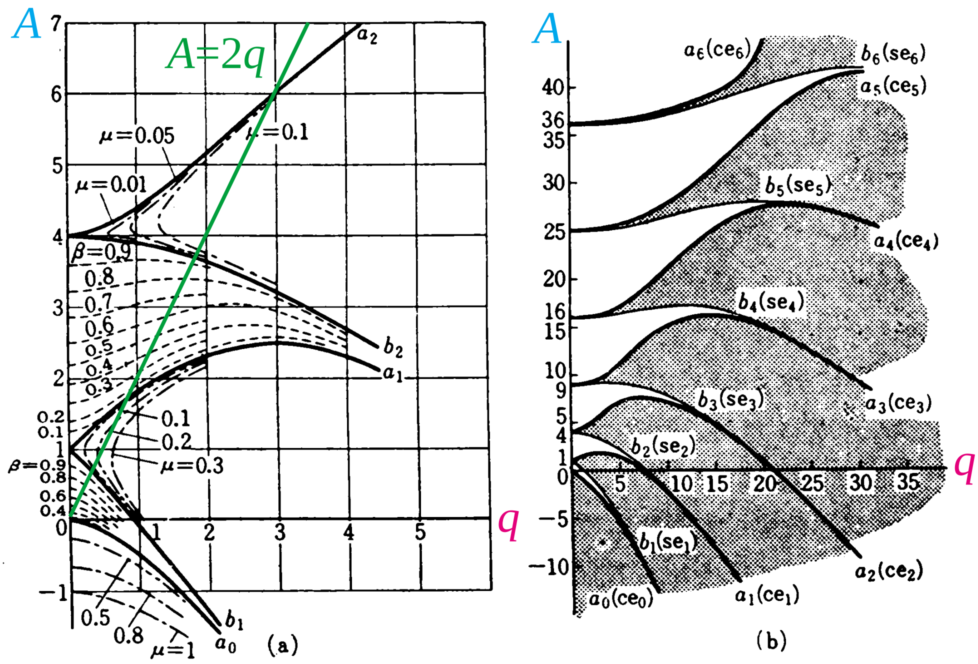
In the Minkowski spacetime $(a(t) \equiv 1,\ \Phi(t) = \text{const.})$

$$\chi_k''(z) + (A_k - 2q\cos 2z)\chi_k(z) = 0$$

where $z = mt$,

$$A_k \equiv \frac{k^2}{m^2} + \frac{g^2\Phi^2}{2m^2} = \frac{k^2}{m^2} + 2q, \qquad q \equiv \frac{g^2\Phi^2}{4m^2}$$

Mathieu equation



Mathieu の微分方程式の解の安定域

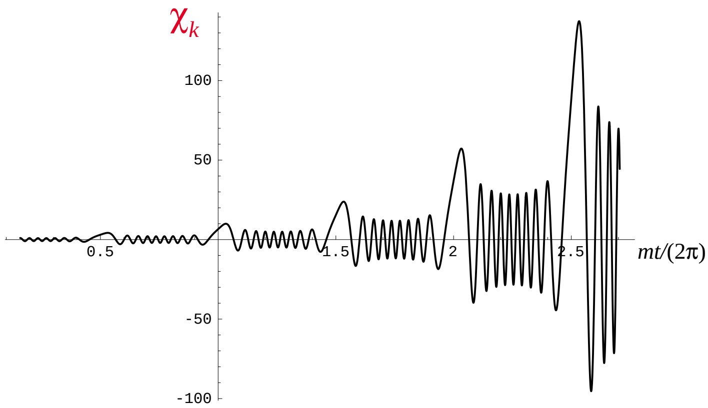
Parametric Resonance

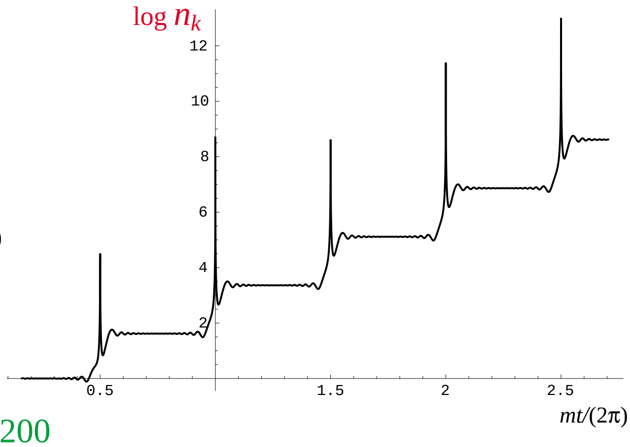
wave function in a periodic potential

$$= \begin{cases} \text{Bloch wave} \\ \text{exponentially growing or damping waves} \end{cases}$$

For $q \gg 1$, the waves are in broad resonance

a solution in a resonance band





q=200

$$n_k \equiv \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2}$$

n_k changes only at t where $\Phi(t) = 0$

$$\Leftrightarrow \dot{\omega}(t) \gtrsim \omega^2(t) \text{ with } \omega(t) = \sqrt{k^2 + g^2\Phi^2(t)\sin^2(mt)}$$

$$\left.\begin{matrix} |\chi_k(t)| \\ n_k(t) \end{matrix}\right\} \text{exponentially increase with } t \text{ stepwise.}$$

$\Longrightarrow$ seccessive scatterings by a periodic potential

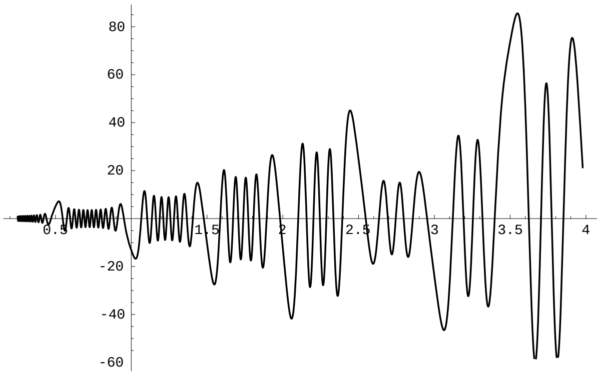
$\Longrightarrow$ descent equation for n_k

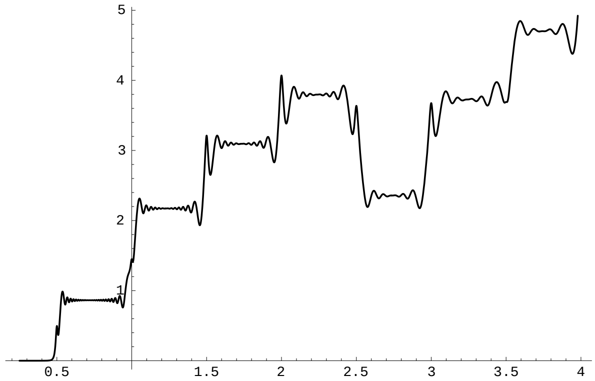
We must take into account ...

▷ **膨張宇宙の効果** $a(t)$, $\Phi(t)$

narrow resonance $q \lesssim O(1)$ $\longrightarrow$ resonanceが即終了

broad resonace $q \gg 1$ $\longrightarrow$ stochastic resonance





それでも、successive scatteringの描像は使える

$$n_k^{j+1} \simeq \left(1 + 2e^{-\pi\kappa_j^2} - 2\sin\hat{\theta}\, e^{-\pi\kappa_j^2/2}\sqrt{1+e^{-\pi\kappa_j^2}}\right) n_k^j$$

ここで$\hat{\theta}$はrandom phase、

$$\kappa_j \equiv \frac{k}{a_j k_{*j}}、\quad k_{*j} \equiv \sqrt{gm\Phi_j} = \sqrt{2}\, m q_j^{1/4}$$

($j \leftrightarrow j$-th zero of $\phi(t)$)

▷ **生成されたχ粒子の** back reaction

$$\begin{cases} \rho \simeq \rho_\phi \rightarrow \rho_\chi & \text{:damping the oscillation} \\ m_\phi^2 \simeq m^2 + g^2\langle\chi^2\rangle & \text{:increase } \phi\text{-frequency} \end{cases}$$

▷ χ**粒子と**ϕ**粒子の** rescattering

$\Delta m_\chi^2(k) = g^2\langle\delta\phi^2\rangle_k >$ resonance width

$\Longrightarrow$ terminates the resonance

state after preheating

- large occupation number of χ with small k

  resonance band $\Leftrightarrow \pi\kappa^2 < 1 \Leftrightarrow \kappa < \frac{1}{\sqrt{\pi}} \simeq 0.56$

- large quantum fluctuation of χ

  e.g.

$$m = 10^{-6} m_P, \quad \Phi_0 = \frac{m_P}{5}, \quad g = 10^{-3\sim-1}$$

$\Rightarrow$ resonance terminates after about 10 ϕ-oscillations

$\sqrt{\langle\chi^2\rangle} \simeq 3 \times 10^{16}$GeV for $g = 3 \times 10^{-4}$

$\longleftrightarrow$ thermal fluctuation at $T = 10^{17}$GeV

$\Downarrow$

nonthermal symmetry restoration nonthermal heavy particle production

Evolution of this state;

⋆ decay to light particles — conventional reheating process

⋆ relaxation to thermal distribution

numerical simulation [Felder & Kofman, hep-ph/0011160]

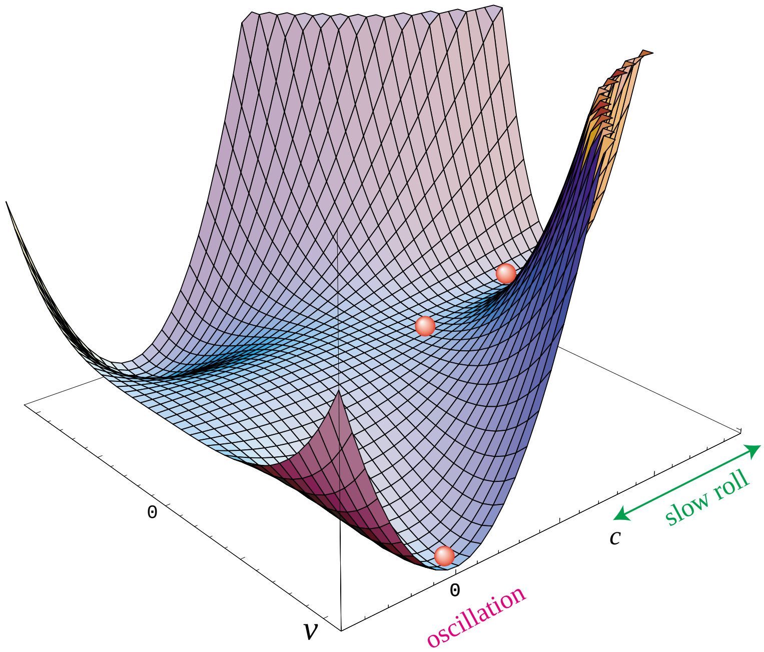
relaxation time $\ll \dfrac{1}{n\sigma_{\rm int}}$ ($\because$ large occupation no.)

Application to EW baryogenesis

inflation mode with
$T_{\rm rh}$ of EW scale

= hybrid inflation

$$\begin{cases} \sigma & : \text{inflaton} \\ \phi & : \text{Higgs scalar} \end{cases}$$



Garcia-Bellido et al. PRD60 ('99)

large fluctuation of long-wavelength Higgs and gauge fields

$$T_{\rm eff} \simeq 350\text{GeV}$$

⇒ enhanced sphaleron transition (conjecture)

assuming CP-viol. operator $\dfrac{\delta_{CP}}{M^2}\phi^\dagger\phi\,\dfrac{3\alpha_W}{8\pi}F_{\mu\nu}\tilde{F}^{\mu\nu}$

$$\frac{n_B}{s} \simeq 3\times 10^{-8}\delta_{CP}\frac{v^2}{M^2}\left(\frac{T_{\rm eff}}{T_{\rm rh}}\right)^3$$

We need a check by MC simulation of the sphaleron transition.

similar to the finite-T $\Gamma_{\rm sph}$

§ 3. Charge Generation

[K.F., Otsuki, Kakuto, Toyoda, hep-ph/0010266]

Extension to the case of n-component complex scalar fields

$$\begin{aligned}\mathcal{L} &= \partial_\mu \chi_a^* \partial^\mu \chi_a - g_a^2 \phi^2(t) \chi_a^* \chi_a \\ &\qquad - \chi_a^* V_{ab}(t) \chi_b - \frac{1}{2} \left(\chi_a W_{ab}(t) \chi_b + \text{c.c.} \right),\end{aligned}$$

$\phi(t)$: oscillating background

"effective potential": $V_{ab}(t) = V_{ba}^*(t)$, $W_{ab}(t)$

induced by couplings to ϕ and/or by radiative and finite-T corrections

$$\begin{array}{rcl} W_{ab}(t) = 0 & \Rightarrow & \text{global } U(1) \\ \mathrm{Im}V_{ab}(t) \neq 0 \text{ or } \mathrm{Im}W_{ab}(t) \neq 0 & \Rightarrow & \text{C and CP violation} \end{array}$$

We assume that

- ▷ charge is generated when $\phi(t) = 0$, as particles are created.
- ▷ $V_{ab}(t)$ and $W_{ab}(t)$ can be treated perturbatively.

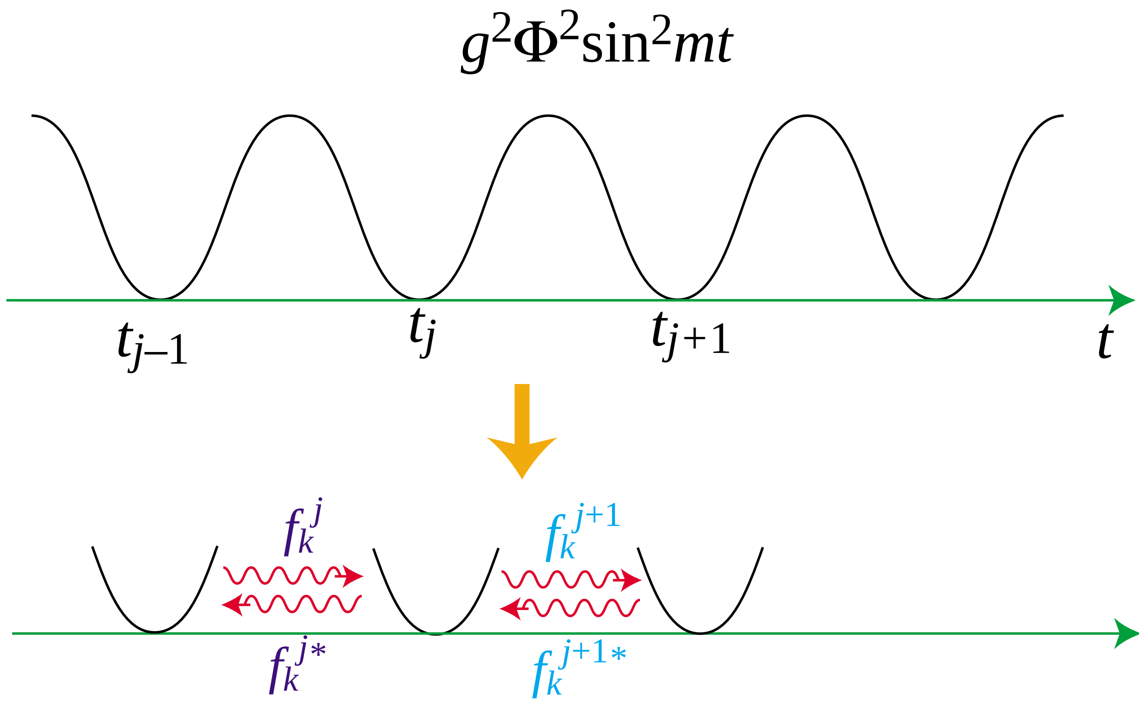
successive scattering approximation (for broad resonance)

for $t_{j-1} \ll t \ll t_j$, $(t_j = \pi j/m)$

$$\chi_a(x) = \int d^3\boldsymbol{k} \left(a^j_{a\boldsymbol{k}} f^j_{ak}(t) e^{i\boldsymbol{kx}} + b^{j\dagger}_{a\boldsymbol{k}} f^{j*}_{ak}(t) e^{-i\boldsymbol{kx}} \right)$$

ここで mode 関数 $f^j_k(t)$ は次の方程式の解:

$$\ddot{f}^j_k(t) + \left(k^2 + g_a^2 \Phi^2 \sin^2 mt\right) f^j_k(t) = 0$$



- t_j の近傍以外では**断熱近似**

$$f^j_k(t) \simeq \frac{1}{\sqrt{2\omega_a(t)}} e^{-i\int_0^t dt' \omega_a(t)}$$

を用いる。$\left(\omega_a(t) = \sqrt{k^2 + g_a^2\Phi^2 \sin^2 mt}\right)$

- t_j の近傍では、$\sin^2 mt$ を他の関数で近似して散乱問題を解く。 $\left(\sin^2 mt \simeq 2\tanh^2\left(\frac{m(t-t_j)}{\sqrt{2}}\right)\right)$

各$\phi(t)$のゼロ点毎の散乱により正振動モードと負振動モードが混合する

$$\begin{pmatrix} 0 \\ \vdots \\ f^0_{ak}(t) \\ \vdots \\ 0 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{pmatrix} \alpha^j_{a1} f^j_{1k}(t) + \beta^j_{a1} f^{j*}_{1k}(t) \\ \vdots \\ \alpha^j_{ab} f^j_{bk}(t) + \beta^j_{ab} f^{j*}_{bk}(t) \\ \vdots \\ \alpha^j_{an} f^j_{nk}(t) + \beta^j_{an} f^{j*}_{nk}(t) \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ \vdots \\ f^{0*}_{ak}(t) \\ \vdots \\ 0 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{pmatrix} \tilde{\beta}^j_{a1} f^j_{1k}(t) + \tilde{\alpha}^j_{a1} f^{j*}_{1k}(t) \\ \vdots \\ \tilde{\beta}^j_{ab} f^j_{bk}(t) + \tilde{\alpha}^j_{ab} f^{j*}_{bk}(t) \\ \vdots \\ \tilde{\beta}^j_{an} f^j_{nk}(t) + \tilde{\alpha}^j_{an} f^{j*}_{nk}(t) \end{pmatrix}$$

CP violation $\Longrightarrow \alpha^j_{ab} \neq \tilde{\alpha}^j_{ab},\ \beta^j_{ab} \neq \tilde{\beta}^j_{ab}$

Bogoliubov 変換

$$a^j_{a\boldsymbol{k}} = a^0_{b\boldsymbol{k}} \alpha^j_{ba} + b^{0\dagger}_{b\boldsymbol{k}} \tilde{\beta}^j_{ba}$$
$$b^{j\dagger}_{a\boldsymbol{k}} = a^0_{b\boldsymbol{k}} \beta^j_{ba} + b^{0\dagger}_{b\boldsymbol{k}} \tilde{\alpha}^j_{ba}$$

Bogoliubov 係数が満たすべき条件 commutation rel.

($n \times n$ 行列表記で)

$$\alpha^{j\dagger}\alpha^j - \tilde{\beta}^{j\dagger}\tilde{\beta}^j = \tilde{\alpha}^{j\dagger}\tilde{\alpha}^j - \beta^{j\dagger}\beta^j = 1, \quad \beta^{j\dagger}\alpha^j - \tilde{\alpha}^{j\dagger}\tilde{\beta}_j = 0$$

$|0^0\rangle$ ($a^0_{a\boldsymbol{k}}|0^0\rangle = b^0_{a\boldsymbol{k}}|0^0\rangle$) に対して第 j 区間で生成される**粒子数密度と charge 密度**

$$
\begin{aligned}
n_k^j &\equiv \frac{1}{V}\langle 0^0|\sum_{a=1}^{n}\left(a^{j\dagger}_{a\boldsymbol{k}}a^{j}_{a\boldsymbol{k}} + b^{j\dagger}_{a\boldsymbol{k}}b^{j}_{\boldsymbol{k}}\right)|0^0\rangle \\
&= \mathrm{Tr}\left(\tilde{\beta}^{j\dagger}\tilde{\beta}^{j} + \beta^{j\dagger}\beta^{j}\right) \\
j_k^j &\equiv \frac{1}{V}\langle 0^0|\sum_{a=1}^{n}Q_a\left(a^{j\dagger}_{a\boldsymbol{k}}a^{j}_{a\boldsymbol{k}} - b^{j\dagger}_{a\boldsymbol{k}}b^{j}_{a\boldsymbol{k}}\right)|0^0\rangle \\
&= \mathrm{Tr}\left[Q\left(\tilde{\beta}^{j\dagger}\tilde{\beta}^{j} - \beta^{j\dagger}\beta^{j}\right)\right]
\end{aligned}
$$

$$Q = \mathrm{diag}\left(Q_1, Q_2, \cdots, Q_n\right)$$

$n = 1$ の場合、Bogoliubov 係数の条件は

$$\left|\alpha^j\right|^2 = \left|\tilde{\beta}^j\right|^2 + 1, \quad \left|\tilde{\alpha}^j\right|^2 = \left|\beta^j\right|^2 + 1$$

$$\left|\alpha^j\right|^2\left|\beta^j\right|^2 = \left|\tilde{\alpha}^j\right|^2\left|\tilde{\beta}^j\right|^2$$

これから

$$\left|\beta^j\right|^2 = \left|\tilde{\beta}^j\right|^2 \quad \Rightarrow \quad \boxed{j_k^j = 0}$$

$\Longleftrightarrow$ heavy particle の崩壊では CP violation は 2 つ以上の channel の干渉として現れる

$$\left|\mathcal{A}_1 + e^{i\theta}\mathcal{A}_2\right|^2$$

Example

$n = 2$: $m_1 = m_2 \equiv m$, $V_{11} = V_{22}$, $W_{ab} = 0$

$U(1)$-sym.: $\chi_a \mapsto e^{i\alpha}\chi_a$ and discrete sym.: $\chi_1 \leftrightarrow \chi_2$

$$\begin{cases} n_k^j = \sum_{a,b=1}^{2} \left(\left|\beta_{ab}^j\right|^2 + \left|\tilde{\beta}_{ab}^j\right|^2 \right) \\ j_{1k}^j = \left|\tilde{\beta}_{11}^j\right|^2 + \left|\tilde{\beta}_{21}^j\right|^2 - \left|\beta_{11}^j\right|^2 - \left|\beta_{21}^j\right|^2 \quad \text{charge of } \chi_1 \\ j_{2k}^j = \left|\tilde{\beta}_{12}^j\right|^2 + \left|\tilde{\beta}_{22}^j\right|^2 - \left|\beta_{12}^j\right|^2 - \left|\beta_{22}^j\right|^2 \quad \text{charge of } \chi_2 \end{cases}$$

$$\kappa \equiv \frac{k}{k_*} = \frac{k}{\sqrt{g\Phi m}} \lesssim \frac{1}{\sqrt{\pi}} \quad \Longrightarrow \text{ in the resonance band}$$

For definiteness, take

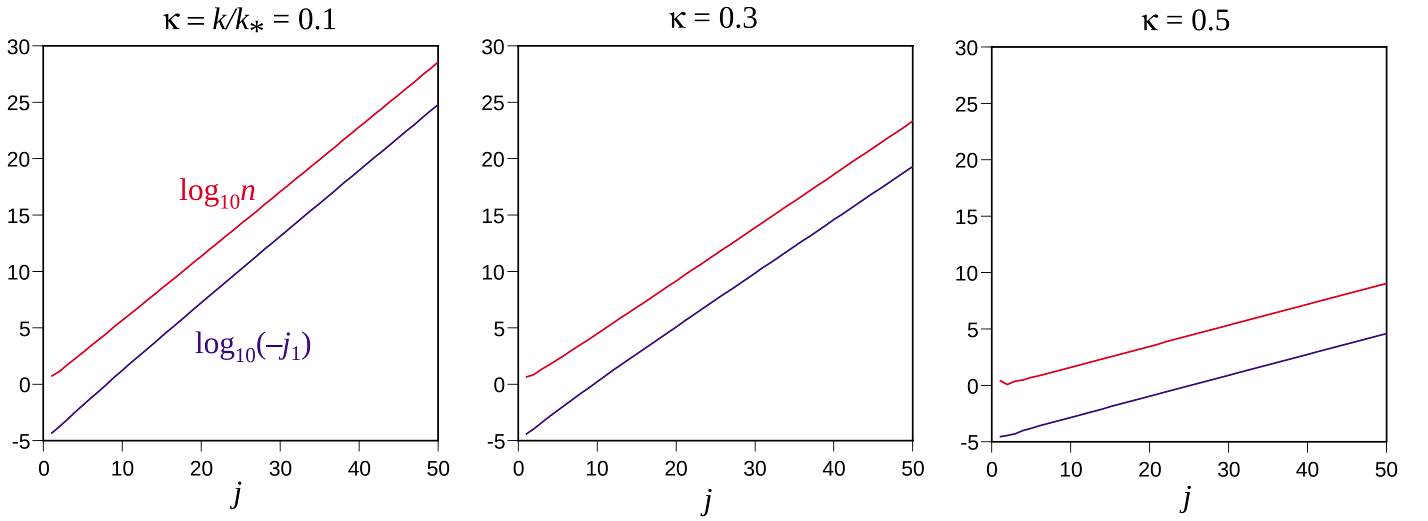
$$V_{11}(t) = -\frac{2g^2 l_1 \Phi^2}{\cosh^2[m(t-t_j)/\sqrt{2}]}$$

$$V_{12}(t) = -\frac{2g^2 l_2 \Phi^2 \, e^{i\theta(t)}}{\cosh^2[m(t-t_j)/\sqrt{2}]}$$

with
$$\theta(t) = \frac{\theta_0}{1 + e^{m(t-t_j)/\sqrt{2}}}$$

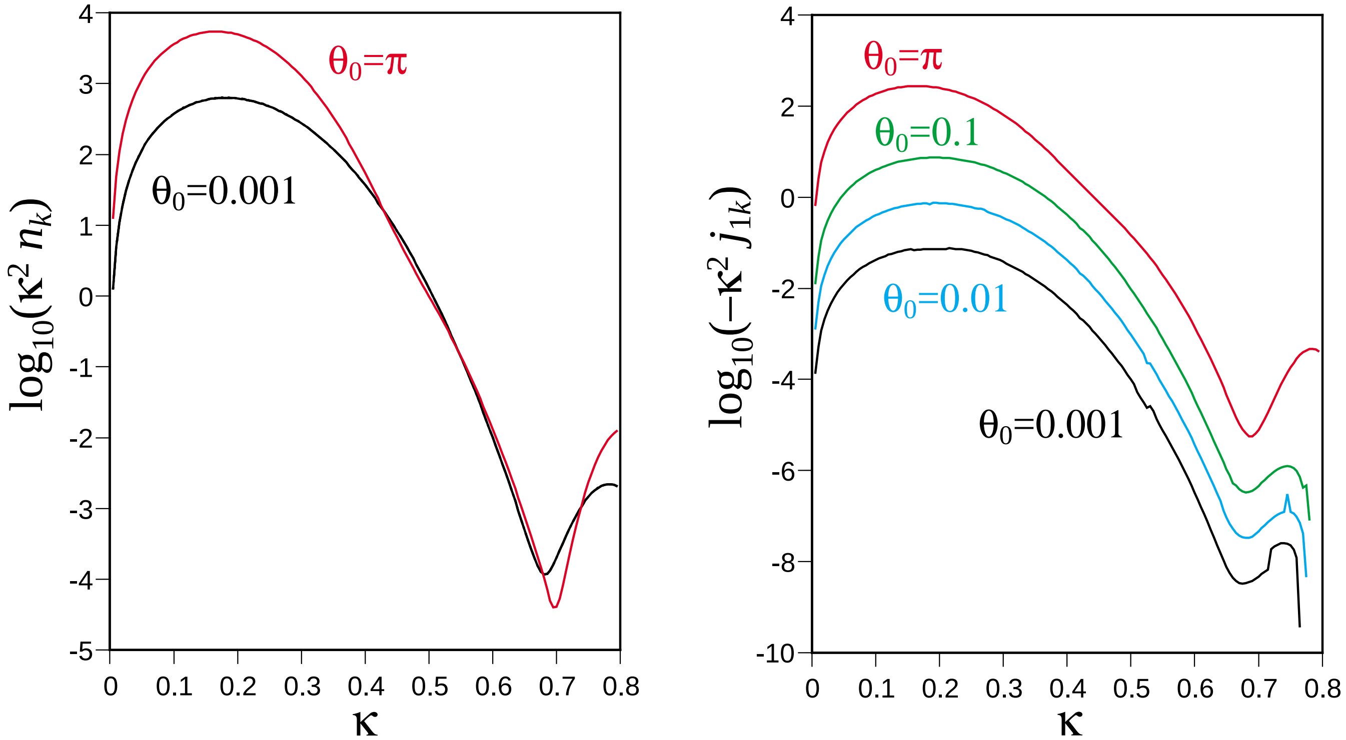
粒子数密度とcharge密度の時間発展

$$q = 200,\ l_1 = 0.01,\ l_2 = 0.02,\ \theta_0 = 10^{-3}$$



resonanceが終わる頃

densities at $j = 10$ for various θ_0



total number and charge densities

$$n = \int d^3\boldsymbol{k}\, n_k = 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, n_k,$$

$$j_1 = 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, j_{1k} = -j_2$$

θ_0	$\int d\kappa\, \kappa^2 n_k$	$\int d\kappa\, \kappa^2 j_{1k}$
10^{-3}	130.5096	-1.609334×10^{-2}
10^{-2}	130.5156	-1.544579×10^{-1}
10^{-1}	131.1163	-1.537716
π	990.7411	-50.84228

このメカニズムをbaryogenesisに利用できるか？

EW scaleの物理に依るとすれば、

▷ hybrid inflation — low $T_{\rm rh}$

▷ squarks & sleptons in the MSSM

$m^2 \longrightarrow$ soft-SUSY-breaking mass

$g^2 \longrightarrow$ gauge or Yukawa coupling (V_D or V_F)

source of CP violation
μ, A-term, B-term (relative phase)
corrections including gaugino loops, ...

flat directions in the MSSM [Gherghetta, NPB468 ('96)]

- $T_{\rm rh} < T_C$ of EW phase transition

  $B \neq 0$: udd, QdL, $QQQL$, $\ldots$

- $T_{\rm rh} > T_C$

  $B - L \neq 0$: LH_u, LLe, QdL, $QQQH_d$, $\ldots$

... work in progress

§ 4. Summary

BAU はいつ出来たのか？

- (inflation が起こったとしたら) inflation **より後**

  diluted by entropy production

- **元素合成より前** ($T > 1$MeV)

  実際には $T > 38$MeV

  $\because$ thermal fluctuation $\longrightarrow \frac{n_b}{s} \simeq \frac{n_{\bar{b}}}{s} \sim 10^{-10}$

viable scenarios of baryogenesis

⋆ **GUTs**

⋆ **Affleck-Dine mechanism**
with Q-ball formation [Enqvist & McDonald, PLB425 ('98)]

⋆ **Leptogenesis** + sphaleron

⋆ **Electroweak Baryogenesis**
1st order EWPT, CP violation

⋆ **Inflationary Baryogenesis**
[Nanopoulos & Rangarajan, hep-ph/0103348]
inflation **後の非平衡状態** (reheating or peheating)

preheating $=$ **振動するスカラー場**を背景とする場の理論

$\Downarrow$

指数関数的粒子生成

$+$ 指数関数的量子数生成 (if $^{\exists}$CP violation)

長波長モードの大きな量子揺らぎ

nonthermal symmetry restoration, ...

low-energy model の構成

▷ hyprid inflation coupled to a flat direction in the MSSM

▷ Affleck-Dine mechanism with $Q_{\mathrm{initial}} = 0$

Q is generated by the oscillation of AD scalar

$$V_{AD} = (m^2 - cH_I^2)\,|\phi|^2 + \left(\frac{A\lambda\phi^n}{M^{n-3}} + \text{h.c.} \right) + \frac{\lambda^2\,|\phi|^{2(n-1)}}{M^{2(n-3)}}$$