Classification of CP-Violating Electroweak Bubble Wall

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content

I. Introduction

Model $\cdots$ MSSM, 2-doublet Higgs, ...

(CP viol. in Higgs sector)

↓

finite-T perturbation

lattice simulation(?)

↓

⋆ effective potential at T_C: $V_{\text{eff}}(\Phi_i; T_C)$

↓

Equations of Motion for the gauge-Higgs system

↓

numerical solution

= dynamically realized

CP-violating profile of the bubble wall

↓

EW baryogenesis scenarios (spontaneous, charge transport)

↓

$$\frac{n_B}{s} = \mathcal{N}\frac{100}{\pi^2 g_*}\kappa\alpha_W^4 \cdot \tau T \cdot \frac{F_Y}{v_w T^3}$$

$$\sim 10^{-3}\frac{F_Y}{v_w T^3} \qquad \text{for optimal case}$$

⇑

CP violation arond the bubble wall

two-Higgs-doublet model (2HDM)

We assume that the bubble wall profile is determined by the classical EOM of the gauge-Higgs sector, with the effective potential at T_C.

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{i=1,2}(D_\mu\Phi_i)^\dagger D^\mu\Phi_i - V_{\rm eff}(\Phi_1,\Phi_2;T),$$

$$D_\mu\Phi_i(x) \equiv (\partial_\mu - ig\frac{\tau^a}{2}A^a_\mu(x) - i\frac{g'}{2}B_\mu(x))\Phi_i(x).$$

Boundary Conditions

$$\begin{aligned} |\Phi_i| &= v_i/\sqrt{2} && \text{in the broken phase} \\ \Phi_i &= 0 && \text{in the symmetric phase} \end{aligned}$$

The phase of Φ_i in the broken phase depends on $V_{\rm eff}$.

The phase of Φ_i around the EW bubble wall is dynamically determined by the EOM.

$\Longrightarrow$ CP-violating Dirac eq.

$\Longrightarrow$ net chiral charge flux

- What type of solutions exist ?

  enhancement of CP viol. near the bubble wall ?

- What model predicts such a solution ?

II. Ansatz and Equations of Motion

Assumptions

All the gauge fields are *pure gauge* type.

$$\Updownarrow$$

$$ig\frac{\tau^a}{2}A^a_\mu(x) = \partial_\mu U_2(x)U_2^{-1}(x), \quad i\frac{g'}{2}B_\mu(x) = \partial_\mu U_1(x)U_1^{-1}(x)$$

$\because$ spherical sym. or planar wall $\Rightarrow$ 1+1-dim. system
maybe the lowest energy configuration

$$\Downarrow$$

Gauge away all the gauge fields — *gauge fixing*
$\Longrightarrow$ No ambiguity in the Yukawa coupling $[\theta_1, \theta_2]$

Put

$$\Phi_i(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho_i(x)e^{i\theta_i(x)} \end{pmatrix}$$

Regarding the bubble as a static, planar object,

$$\begin{aligned} \frac{d^2\rho_i(z)}{dz^2} - \rho_i(z)\left(\frac{d\theta_i(z)}{dz}\right)^2 - \frac{\partial V_{\rm eff}}{\partial \rho_i} &= 0, \\ \frac{d}{dz}\left(\rho_i^2(z)\frac{d\theta_i(z)}{dz}\right) - \frac{\partial V_{\rm eff}}{\partial \theta_i} &= 0, \\ \rho_1^2(z)\frac{d\theta_1(z)}{dz} + \rho_2^2(z)\frac{d\theta_2(z)}{dz} &= 0 \end{aligned}$$

the last equation = sourcelessness condition or gauge fixing condition

$$V_{\rm eff} \Longleftarrow \begin{cases} \text{radiative \& finite-}T\text{ corrections} \\ \text{search for degenerate minima at } T_C \\ \text{in the 3-dim. order-parameter space} \end{cases}$$

Ansatz for V_{eff} at T_C

1) No explicit CP-violation   We shall introduce it later.
   Only spontaneous CP-violation is possible.

2) $V_{\text{eff}} = V_0 + \rho^3$-terms

$$
\begin{aligned}
&= \frac{1}{2}m_1^2\rho_1^2 + \frac{1}{2}m_2^2\rho_2^2 + m_3^2\rho_1\rho_2\cos\theta + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\
&+ \frac{\lambda_3 - \lambda_4 - \lambda_5\cos 2\theta}{4}\rho_1^2\rho_2^2 - \frac{1}{2}(\lambda_6\rho_1^3\rho_2 + \lambda_7\rho_1\rho_2^3)\cos\theta \\
&- [A\rho_1^3 + (B_0 + B_1\cos\theta + B_2\cos 2\theta)\rho_1^2\rho_2 \\
&\quad +(C_0 + C_1\cos\theta + C_2\cos 2\theta)\rho_1\rho_2^2 + D\rho_2^3]
\end{aligned}
$$

All the parameters are real.
$\theta \equiv \theta_1 - \theta_2 \leftrightarrow$ gauge invariance.

$\cos\theta$-dependent part of V_{eff}

$$
\begin{aligned}
&= -\left[\frac{\lambda_5}{2}\rho_1^2\rho_2^2 + 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2)\right] \\
&\times \left[\cos\theta + \frac{-2m_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 + 8(B_2\rho_1 + C_2\rho_2)}\right]^2 + \cdots
\end{aligned}
$$

N.B.

ρ_i changes from v_i (broken phase) to 0 (symmetric phase).

$\Downarrow$

(spontaneous) CP violation depends on z

The effective potential $V_{\rm eff}(\rho_i, \theta)$ must have two degenerate minima at $\rho_i = 0$ (symmetric phase) and $\rho_i \neq 0$ (broken phase).

For $(\rho_1, \rho_2) = (0, 0)$ to be a local minimum,

$$m_1^2 m_2^2 - m_3^4 > 0 \quad \text{and} \quad m_1^2 > 0 \quad \text{or} \quad m_2^2 > 0$$

For simplicity, we postulate the *"kink ansatz"* :

1. For $\theta(z) = 0$, the equations of motion have a kink-type solution which connects the two minima, $s.t.$

$$\rho_1(z) = v \cos\beta \frac{1 + \tanh(az)}{2},$$

$$\rho_2(z) = v \sin\beta \frac{1 + \tanh(az)}{2}.$$

2. The solution for $\rho_i(z)$ are the kinks above.

a : bubble-wall thickness
$z = -\infty \leftrightarrow$ symmetric phase
$z = +\infty \leftrightarrow$ broken phase

This assumption will be adequate for small $\theta(z)$.
For large θ, the kink is deformed for $\rho(z)$.

Some of the parameters are expressed in terms of the others:
$e.g.$

$$m_1^2 = 4a^2 - m_3^2 \tan\beta, \qquad m_2^2 = 4a^2 - m_3^2 \cot\beta,$$

$$B_1 + B_2 + B_3 = -2A \cot\beta + D \tan^2\beta + \frac{4a^2}{v \sin\beta}\left(3 - \frac{1}{\cos^2\beta}\right) ,$$

$$C_1 + C_2 + C_3 = A \cot^2\beta - 2D \tan^2\beta + \frac{4a^2}{v \cos\beta}\left(3 - \frac{1}{\sin 2\beta}\right) ,$$

etc.

Under the kink ansatz, the equations are reduced to that for $\theta(z)$ only :

$$\begin{aligned} & y^2(1-y)^2\frac{d^2\theta(y)}{dy^2} + y(1-y)(1-4y)\frac{d\theta(y)}{dy} \\ = \; & [b + c(1-y)^2 - e(1-y)] \sin\theta(y) \\ & + \left[\frac{d}{2}(1-y)^2 - 2f(1-y)\right]\sin(2\theta(y)) \end{aligned}$$

where

$$\begin{aligned} y &= \frac{1-\tanh(az)}{2}, \\ \theta(y) &= \theta_1(y)/\sin^2\beta = -\theta_2(y)/\cos^2\beta, \end{aligned}$$

$\Leftrightarrow$ sourcelessness condition

and

$$\begin{aligned} b &\equiv -\frac{m_3^2}{4a^2\sin\beta\cos\beta}, \\ c &\equiv \frac{v^2}{8a^2}(\lambda_6\cot\beta + \lambda_7\tan\beta), \\ d &\equiv \frac{\lambda_5 v^2}{4a^2}, \\ e &\equiv -\frac{v}{4a^2}\left(\frac{B_1}{\sin\beta} + \frac{C_1}{\cos\beta}\right), \\ f &\equiv -\frac{v}{4a^2}\left(\frac{B_2}{\sin\beta} + \frac{C_2}{\cos\beta}\right) \end{aligned}$$

Last year workshop and FKOTT, P.T.P.**94** ('95)

$(\rho_1, \rho_2) = (0,0)$ and $(v\cos\beta, v\sin\beta)$ = local minima

$$\Leftrightarrow \begin{cases} b > -1, \\ b - 2(e+f) + 3c > -1 + (\lambda_3 - \lambda_4 - \lambda_5)v^2/(4a^2) \end{cases}$$

Without the *kink ansatz*

To reduce the number of dynamical variables,
we impose the discrete symmetry under $(\rho_1, \theta_1) \longleftrightarrow (\rho_2, -\theta_2)$.

$$\begin{aligned} V_{\text{eff}}(\rho, \theta) &= \frac{1}{2}(m^2 + m_3^2 \cos\theta)\rho^2 \\ &+ \frac{\lambda_1 + \lambda_3 - \lambda_4 - \lambda_5 \cos 2\theta - 4\lambda_6 \cos\theta}{16}\rho^4 \\ &- \frac{1}{\sqrt{2}}[A + (B_0 + B_1 \cos\theta + B_2 \cos 2\theta]\rho^3 \end{aligned}$$

where

$$\begin{aligned} \rho_1(x) = \rho_2(x) &= \frac{1}{\sqrt{2}}\rho(x), \\ \theta_1(x) = -\theta_2(x) &= \frac{1}{2}\theta(x). \end{aligned}$$

Then the EOM is

$$\frac{d^2\rho(z)}{dz^2} - \frac{1}{4}\rho(z)\left(\frac{d\theta(z)}{dz}\right)^2 - \frac{\partial V_{\text{eff}}}{\partial \rho(z)} = 0,$$

$$\frac{1}{4}\frac{d}{dz}\left(\rho^2(z)\frac{d\theta(z)}{dz}\right) - \frac{\partial V_{\text{eff}}}{\partial \theta(z)} = 0$$

Without explicit CP violation and

$^{\exists}$solution violating CP

$\Longrightarrow$ CP conjugate pair of bubbles with the same probability

$\Longrightarrow$ cancellation of the generated baryon number

We assume an explicit CP violation in $V_{\rm eff}$ in the form of

$$m_3^2(\mathrm{e}^{-i\delta}\Phi_1^\dagger\Phi_2 + \text{h.c.}), \qquad (m_3^2 \in \mathbf{R})$$

which changes the EOM for θ with the kink ansatz as

$$\begin{aligned} & y^2(1-y)^2\frac{d^2\theta(y)}{dy^2} + y(1-y)(1-4y)\frac{d\theta(y)}{dy} \\ = \quad & b\sin(\theta(y)+\delta) + \left[c(1-y)^2 - e(1-y)\right]\sin\theta(y) \\ & + \left[\frac{d}{2}(1-y)^2 - 2f(1-y)\right]\sin(2\theta(y)) \end{aligned}$$

Net BAU if several sol. with the same b.c.

$$\frac{n_B}{s} = \frac{\sum_i \left(\frac{n_B}{s}\right)_i N_i}{\sum_i N_i}$$

where

$$N_i \simeq \exp\left(-\frac{4\pi {R_C}^2\mathcal{E}_i}{T_C}\right),$$

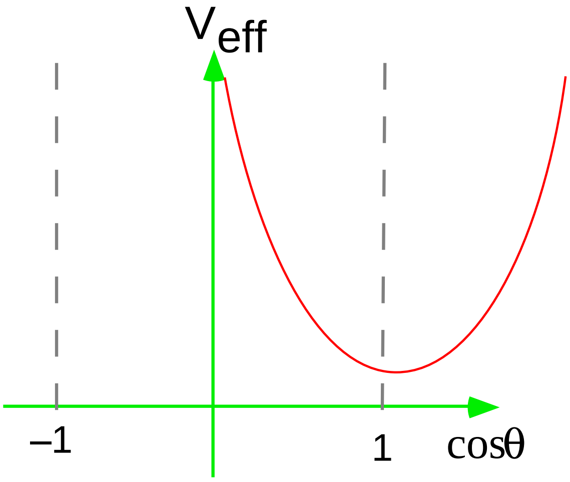
with $R_C \simeq \sqrt{3F_C/(4\pi a v^2)}$ and $F_C \simeq 145 T_C$

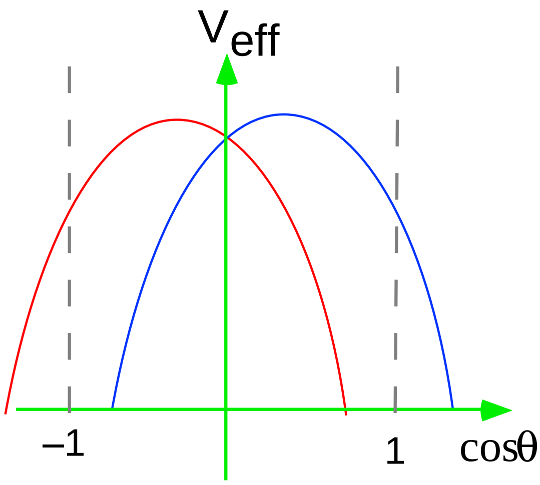
$$\begin{aligned} \mathcal{E} \quad = \quad & \int_0^1 dy\left\{ay(1-y)\left[\left(\frac{d\rho(y)}{dy}\right)^2 + \frac{1}{4}\rho^2(y)\left(\frac{d\theta(y)}{dy}\right)^2\right]\right. \\ & \left. + \frac{1}{2ay(1-y)}V_{\rm eff}(\rho,\theta)\right\}. \end{aligned}$$

Boundary conditions

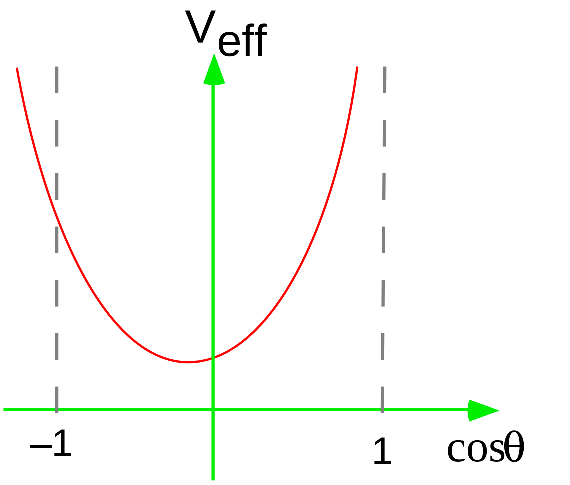
broken phase $[y=0]$ $\rho = v$		symmetric phase $[y=1]$ $\rho = 0$	
$\delta = 0$	$\delta \neq 0$	$\delta = 0$	$\delta \neq 0$
$\theta_0 = n\pi$ CP-conserving	$\theta_0 = O(\delta)$	$\theta_1 = m\pi$	$\theta_1 = m\pi - \delta$
$\theta_0 \neq n\pi$ spont CP viol	$\theta_0 \neq O(\delta)$		

finiteness of $\mathcal{E}$





$\Longrightarrow$ $\theta_0 = n\pi$



$\Longrightarrow$ $\theta_0 \neq n\pi$
spontaneous CP violation

III. Numerical Solutions

Assume the discrete symmetry : $(\rho_1, \theta_1) \longleftrightarrow (\rho_2, -\theta_2)$

$\rho(y)$ is not fixed to a kink.

Putting $\rho(y) = v\tilde{\rho}(y)$ and using the parameters (b, c, d, e, f),

$$
\begin{aligned}
V_{\rm eff} &= a^2 v^2 \tilde{\rho}^2 \{2(\tilde{\rho} - 1)^2 \\
&+ b[1 - \cos(\theta + \delta)] + [c(1 - \cos\theta) + \frac{d}{4}(1 - \cos 2\theta)]\tilde{\rho}^2 \\
&- [e(1 - \cos\theta) + f(1 - \cos 2\theta)]\tilde{\rho}\} \\
&= -\frac{(a\rho)^2}{2}(d\tilde{\rho}^2 - 4f\tilde{\rho})\left(\cos\theta + \frac{b + c\tilde{\rho}^2 - e\tilde{\rho}}{d\tilde{\rho}^2 - 4f\tilde{\rho}}\right)^2 + \cdots
\end{aligned}
$$

for $\delta = 0$

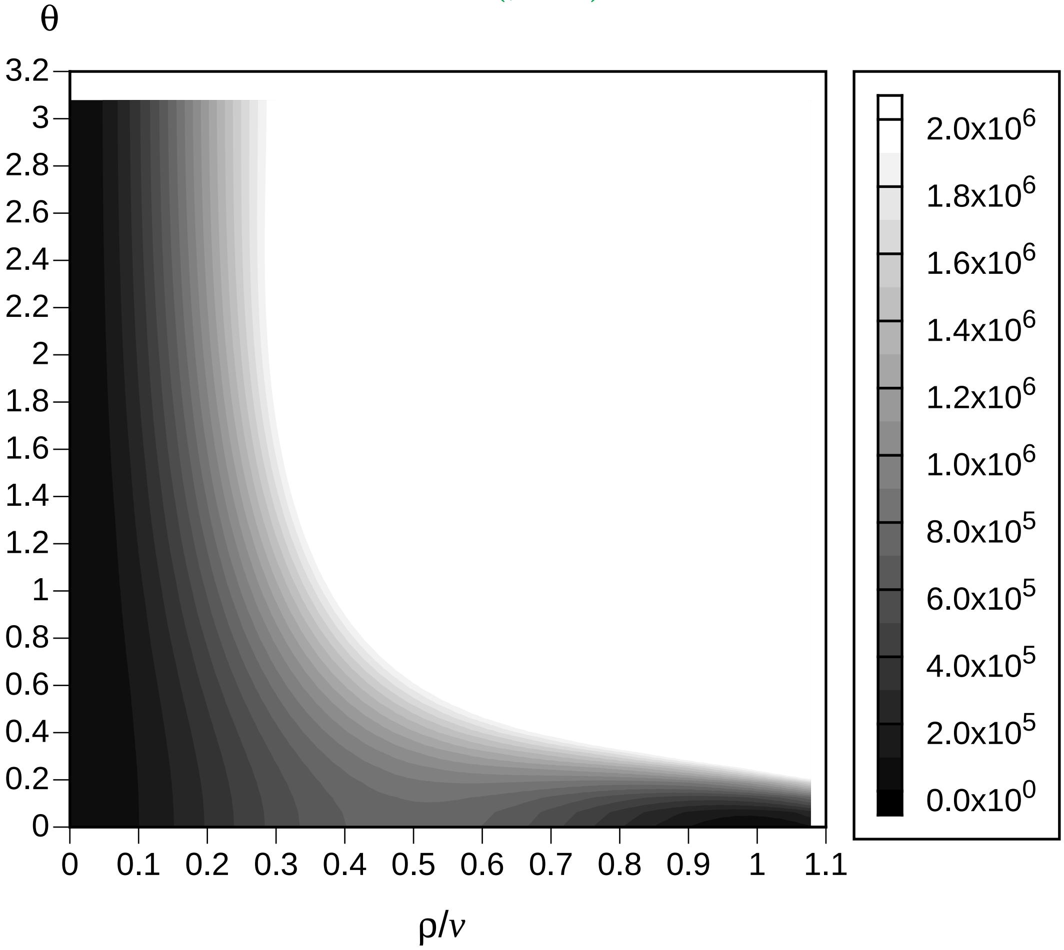
Possible Solutions

	$\delta = 0$	$\delta \neq 0$
(A)	$\theta(y) \equiv 0$ (trivial sol.)	$\theta(y) = O(\delta)$ for $^\forall y$
(B)	$\theta_0 = \theta_1 = 0$ spont. CP violation in the bubble wall	$\theta_0 = O(\delta)$, $\theta = -\delta$ $\lvert\theta(y)\rvert \gg \lvert\delta\rvert$
(C)	$\theta_0 \neq 0$, $\theta_1 = 0$ spont. CP violation in the broken phase	$\theta_0 > O(\delta)$, $\theta = -\delta$ $\lvert\theta(y)\rvert \gg \lvert\delta\rvert$
(D)	$\theta_0 = 0$, $\theta_1 = \pi$ maximal CP violion in the bubble wall	$\theta_0 = O(\delta)$, $\theta = -\delta$ maximal CP violation in the bubble wall

We have found solutions of type (A) $\sim$ (C) for $\delta = 0$ and $\delta \neq 0$.

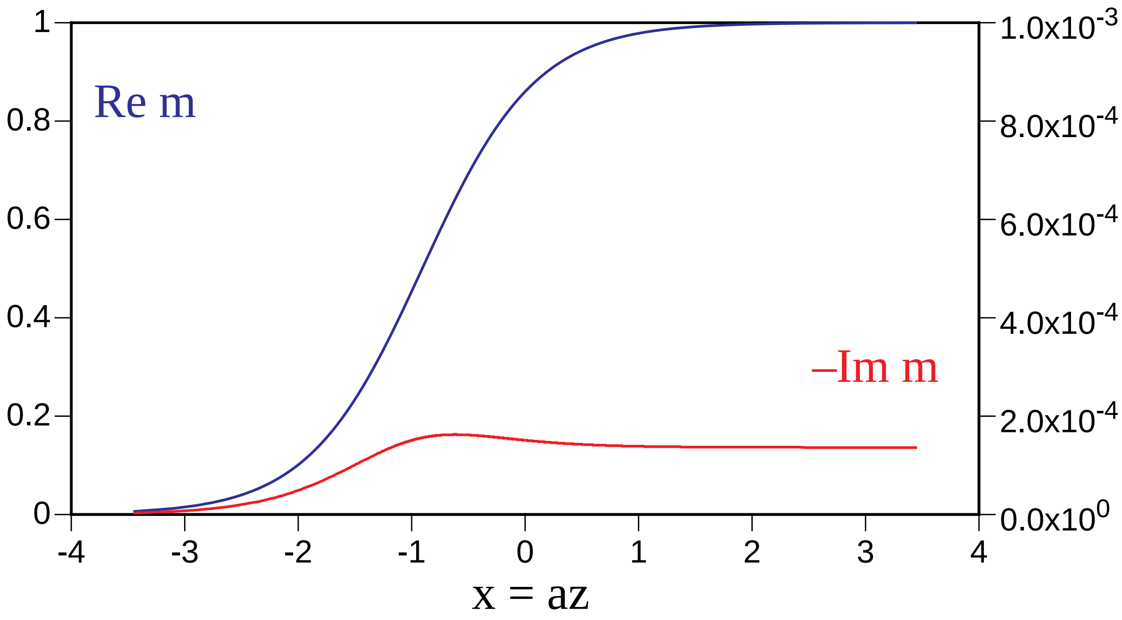
(A) $(b,c,d,e,f) = (3,5,5,7,-1.25)$ and $\delta = 10^{-3}$

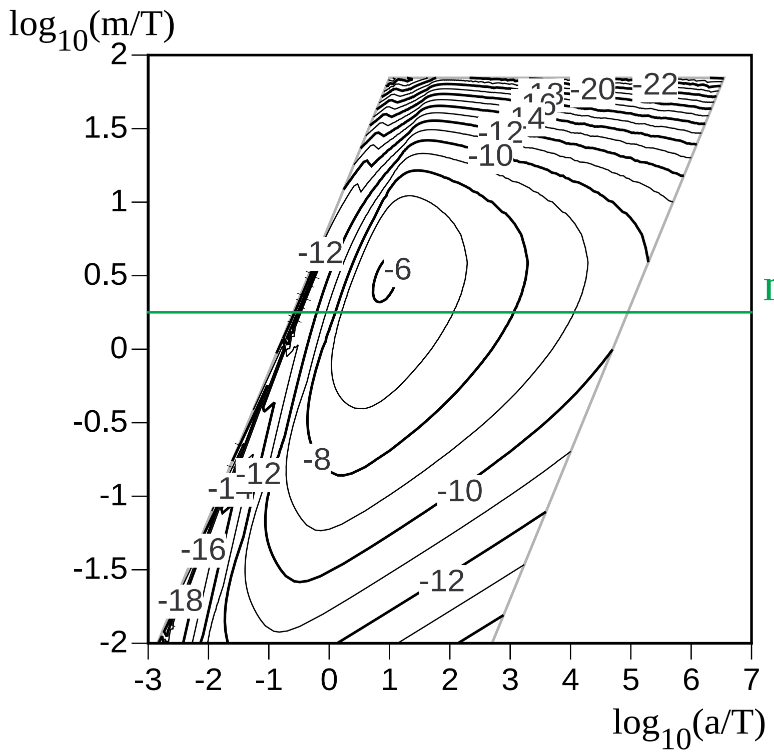
$V_{\rm eff}$ as a function of $(\tilde{\rho}, \theta)$:



profile of the bubble wall

Im m is CP-violating mass in the Dirac eq.



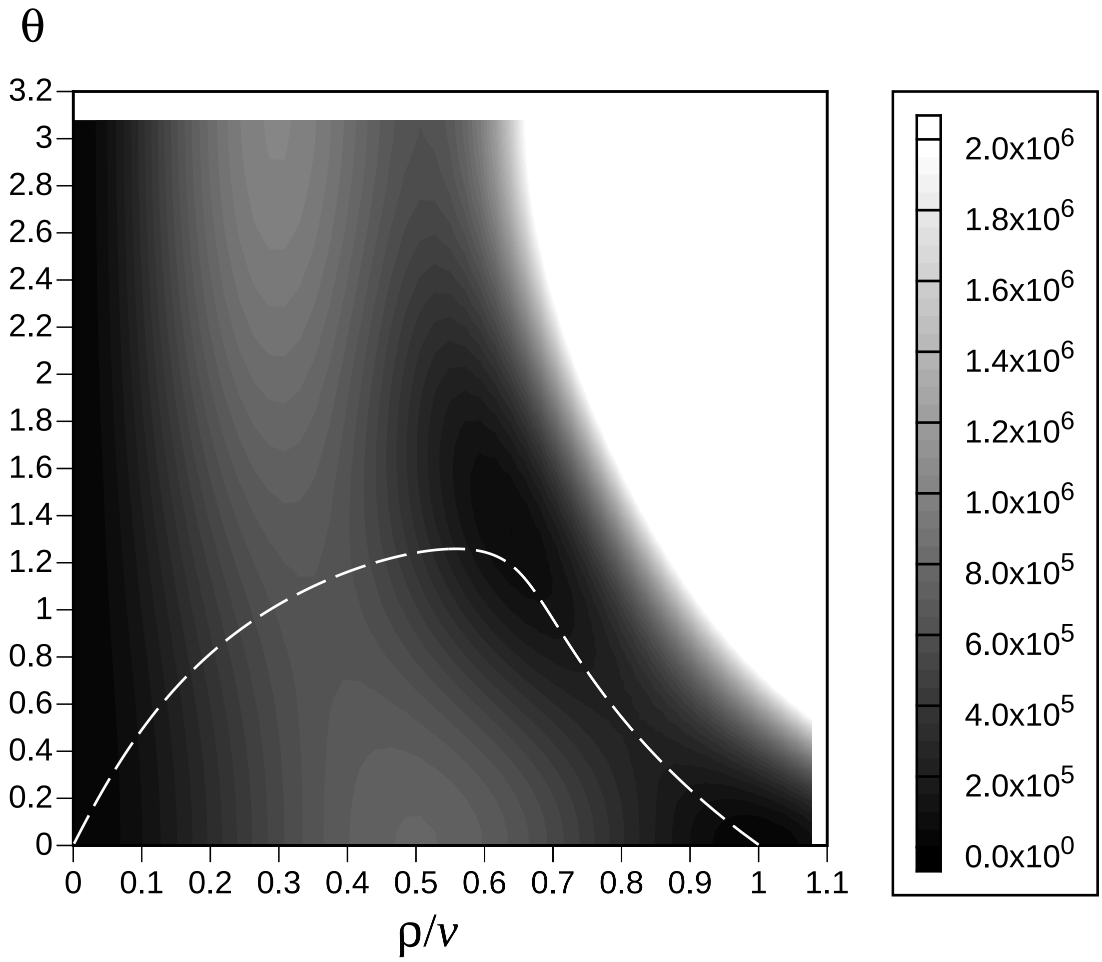


chiral charge flux

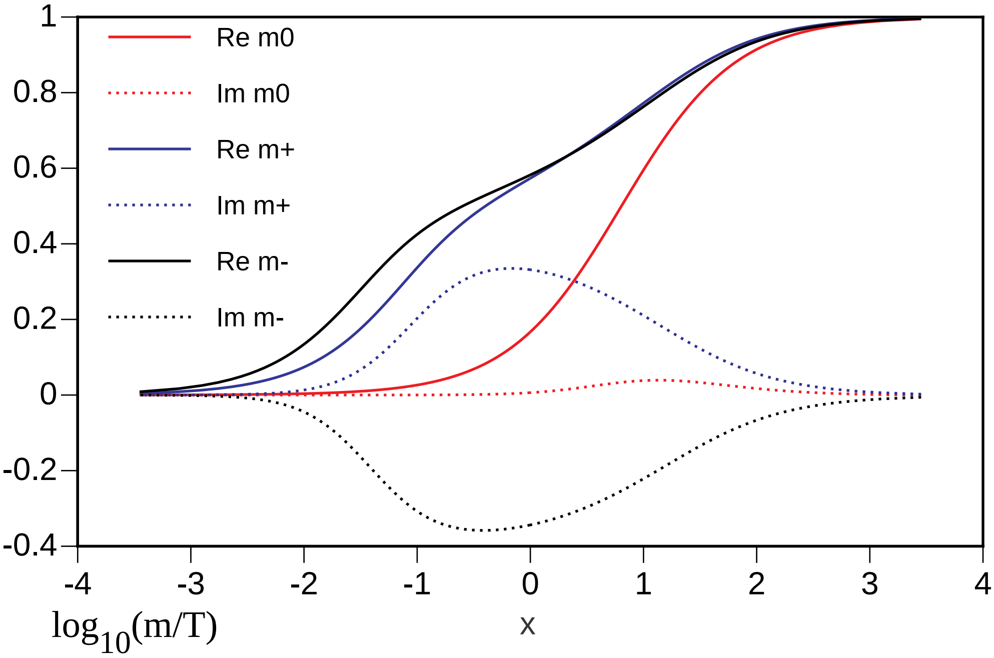
$$\log_{10} \frac{-F_Q}{uT^3(Q_L - Q_R)}$$

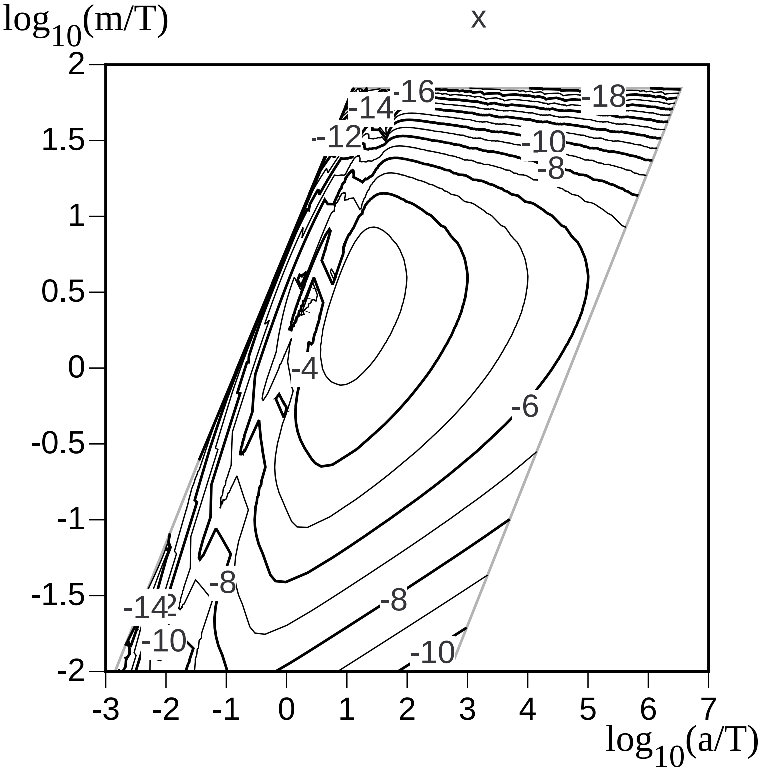
$T = 100$GeV
$u = 0.58$

(B) $(b, c, d, e, f) = (3, 12.2, -2, 12.2, 0)$ and $\delta = 10^{-3}$



profile of the bubble wall





chiral charge flux

$$\log_{10} \frac{-F_Q}{uT^3(Q_L - Q_R)}$$

$T = 100\mathrm{GeV}$

$u = 0.58$

Energy density

$$\delta\mathcal{E}[\rho,\theta] \equiv \mathcal{E}[\rho,\theta] - \mathcal{E}[\mathsf{kink}, 0]$$

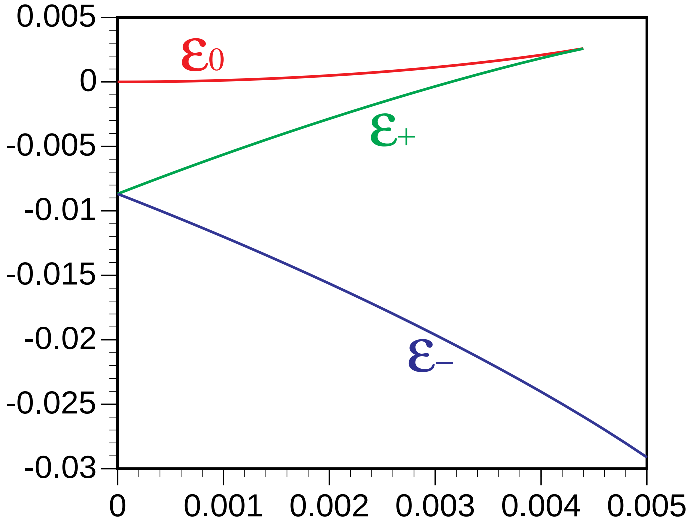
$$\delta\mathcal{E}[\rho^0,\theta^0] = 1.21567 \times 10^{-4} a v^2$$

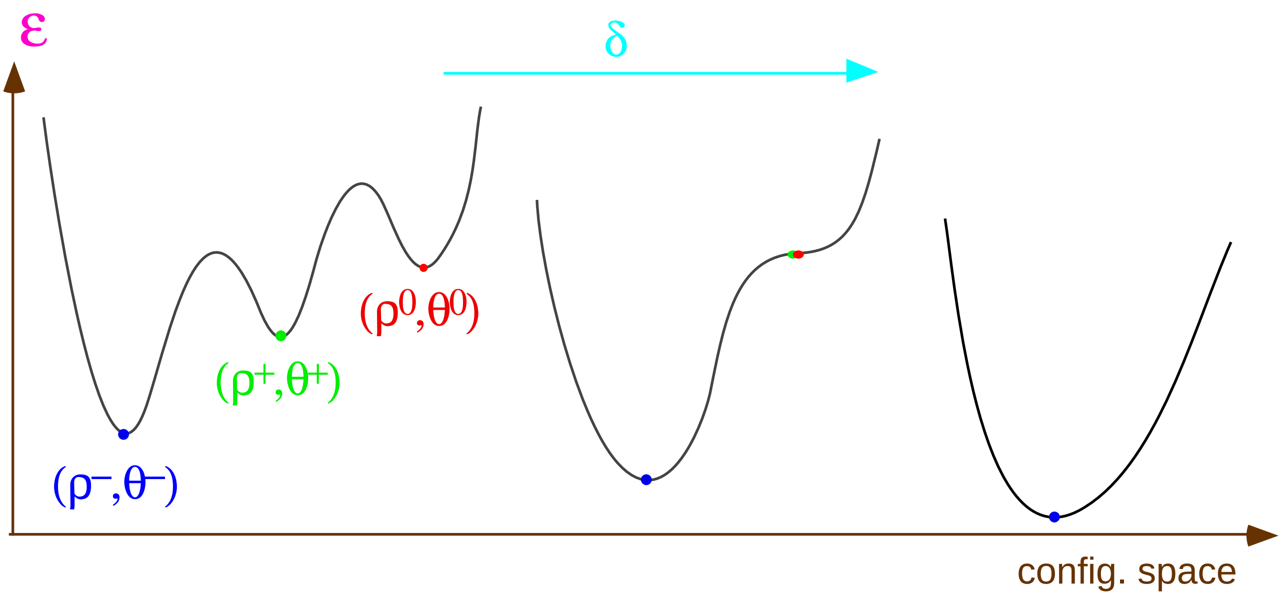
$$\delta\mathcal{E}[\rho^0,\theta^0] = -5.6350 \times 10^{-3} a v^2$$

$$\delta\mathcal{E}[\rho^-,\theta^-] = -1.2012 \times 10^{-2} a v^2$$

$$\therefore \quad \frac{N_+}{N_0} = 12.23, \qquad \frac{N_-}{N_0} = 196.0$$

energy density $(/av^2)$ vs explicit CP violation δ





Similar set of solutions for $(b, c, d, e, f) = (3, 5, 5, 7, 1.25)$

(C) $(b,c,d,e,f)=(3,7,7,-3/\cos(0.002),0)$

spont. CP violation in the broken phase for $\delta=0$:

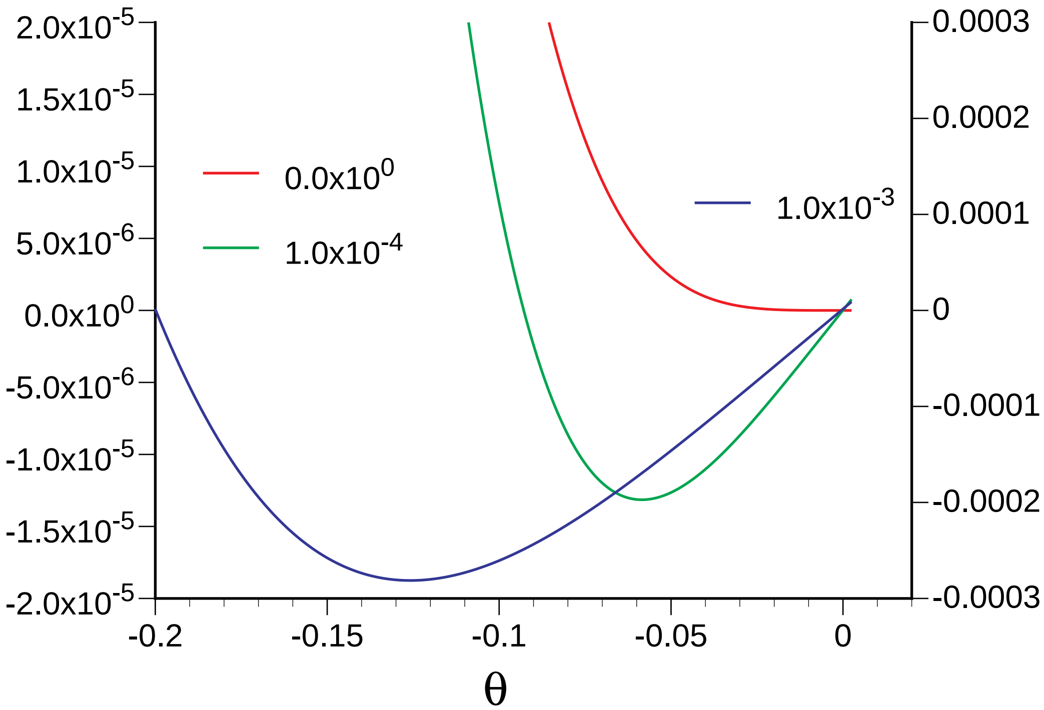
$$\left.\begin{aligned} d-4f &= -\frac{3}{\cos(0.002)}<0, \\ \left|\frac{b+c-e}{d}\right| &= \cos(0.002)<1 \end{aligned}\right\} \Longrightarrow \quad \theta_0=0.002$$

In the presence of δ, θ_0 is determined by

$$b\sin(\theta_0+\delta)+(c+d\cos\theta_0-e-4f\cos\theta_0)\sin\theta_0=0.$$

θ_0 heavily depends of δ.

$V_{\rm eff}(\rho=v,\theta)$ [in the broken phase]
for $\delta=0,10^{-4},10^{-3}$



Dangerous if θ_0 persists to $T=0$.
Favorable if $\theta_0\longrightarrow 0$ as $T\to 0$.

IV. Discussions

Ansatz for V_{eff} and the explicit CP violation

$\Downarrow$

numerical solutoins :

(A) $\theta(y) = O(\delta)$ for all y

(B) $\theta_0 = O(\delta)$ but $|\theta(y)| \gg |\delta|$ for some y

(C) $|\theta(y)| \gg |\delta|$ for y near 0

type (B), (C) needs $d < 0$ ($\lambda_5 < 0$) and/or $f > 0$.

general 2HDM with the discrete symmetry to avoid FCNC
$m_3^2 \neq 0$ softly breaks the symmetry and induce $\lambda_{6,7}$
$\lambda_5 =$ free parameter and $\lambda_6 = \lambda_7 = 0$.

MSSM

$$m_1^2, m_2^2 \longleftarrow |\mu|^2 + \text{soft SUSY-br. term}$$

$$m_3^2 \longleftarrow \text{soft SUSY-br. term} \quad m_{3/2}\mu B$$

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2),$$

$$\lambda_3 = \frac{1}{4}(g^2 - g'^2), \quad \lambda_4 = \frac{1}{2}g^2,$$

$$\lambda_5 = \lambda_6 = \lambda_7 = 0 \qquad \text{No } CP \text{ viol.}$$

with

$$\Phi_d \longleftarrow \tilde{\Phi}_1 \equiv i\sigma_2\Phi_1^*, \qquad \Phi_u \longleftarrow \Phi_2$$

explicit CP violation in the soft SUSY breaking terms
gaugino masses, scalar mass, scalar trilinear (A-parameter)

In principle, one can determine which bubble wall is realized, once one knows $V_{\rm eff}(\rho, \theta; T_C)$.

In MSSM, is $\lambda_5 < 0$ possible ?

- At T=0, yes, but accompanies a light scalar if CP is spontaneously violated. [Maekawa, P.L.**B282** ('92)]
- If $\lambda_5 < 0$ and other parameters satisfy the condition for the type (B) at $T \neq 0$, sufficient BAU will be generated.

chargino $(\chi^{\pm})$, stop $(\tilde{t})$ and charge Higgs $(\phi^{\pm})$ contributions

$$
\begin{aligned}
& \lambda_5 \\
= \; & \Delta_{\chi^{\pm}}\lambda_5 + \Delta_{\tilde{t}}\lambda_5 + \Delta_{\phi^{\pm}}\lambda_5 \\
= \; & -\frac{g_2^4}{8\pi^2}\left[K\left(\frac{M_2^2}{\mu^2}\right) - 8\left(f_+(a_\chi, b_\chi) + f_+(b_\chi, a_\chi)\right)\right] \\
& + \frac{N_c y_t^4}{32\pi^2}\frac{(\mu m_{3/2} A)^2}{m_{\tilde{q}}^2 m_{\tilde{t}}^2} \\
& \qquad \times \left[K\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{t}}^2}\right) + 8\left(f_-(a_{\tilde{t}}, b_{\tilde{t}}) + f_-(b_{\tilde{t}}, a_{\tilde{t}})\right)\right] \\
& + \frac{g_2^4}{128\pi^2}\frac{m_3^4}{\mu_1^2\mu_2^2}\left[K\left(\frac{\mu_1^2}{\mu_2^2}\right) + 8\left(f_-(a_\phi, b_\phi) + f_-(b_\phi, a_\phi)\right)\right],
\end{aligned}
$$

where

$$
\begin{aligned}
a_\chi &\equiv M_2/T, \qquad b_\chi \equiv \mu/T, \\
a_{\tilde{t}} &\equiv m_{\tilde{q}}/T, \qquad b_{\tilde{t}} \equiv m_{\tilde{t}}/T, \\
a_\phi &\equiv \mu_1/T, \qquad b_\phi \equiv \mu_2/T
\end{aligned}
$$

with

$$
\mu_{1,2}^2 \equiv \frac{m_1^2 + m_2^2 \pm \sqrt{(m_1^2 - m_2^2)^2 + 4m_3^4}}{2} > 0,
$$

and

$$K(\alpha) \equiv \frac{\alpha}{(\alpha-1)^2}\left(\frac{\alpha+1}{\alpha-1}\log\alpha - 2\right),$$

$$f_\pm(a,b) \equiv \frac{a^2b^2}{2(a^2-b^2)}\int_0^\infty \frac{dx}{\sqrt{x^2+a^2}}\left(1+\frac{4x^2}{a^2-b^2}\right) \times \frac{1}{e^{\sqrt{x^2+a^2}}\pm 1}$$

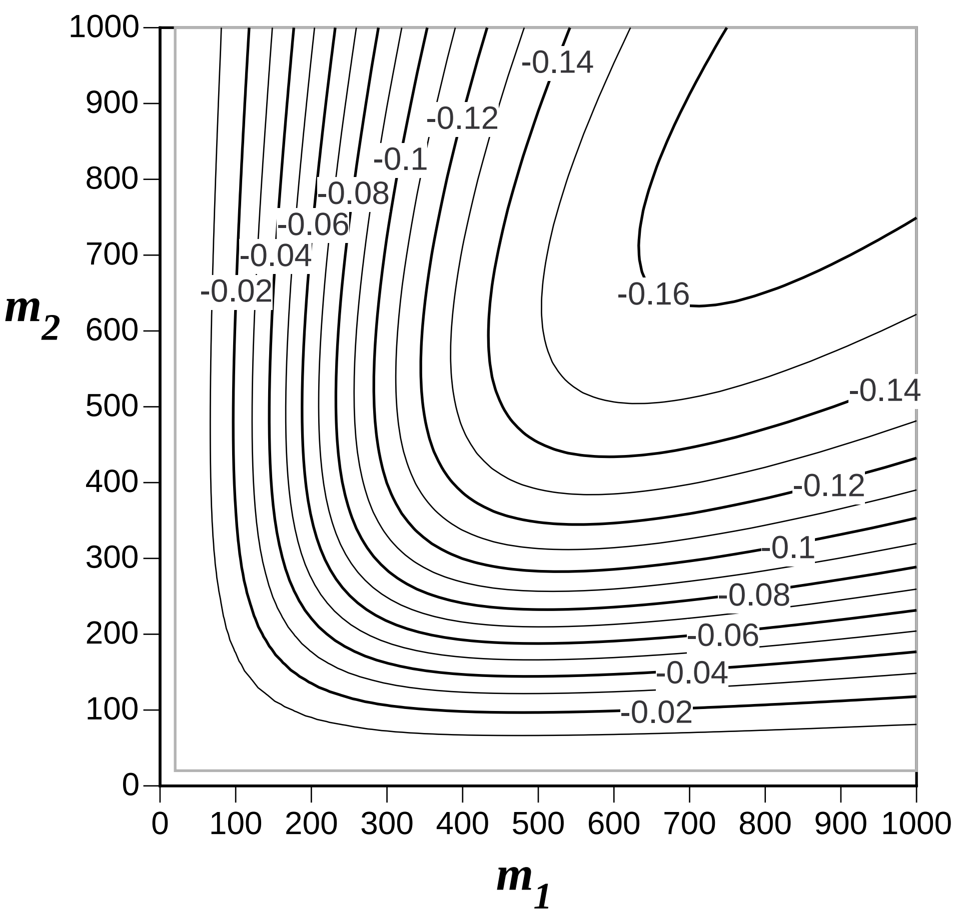
N.B.

⋆ $\Delta_{\chi^\pm}\lambda_5 < 0$ for any T.

⋆ $K(\alpha=1) = 1/6$ is the maximum.

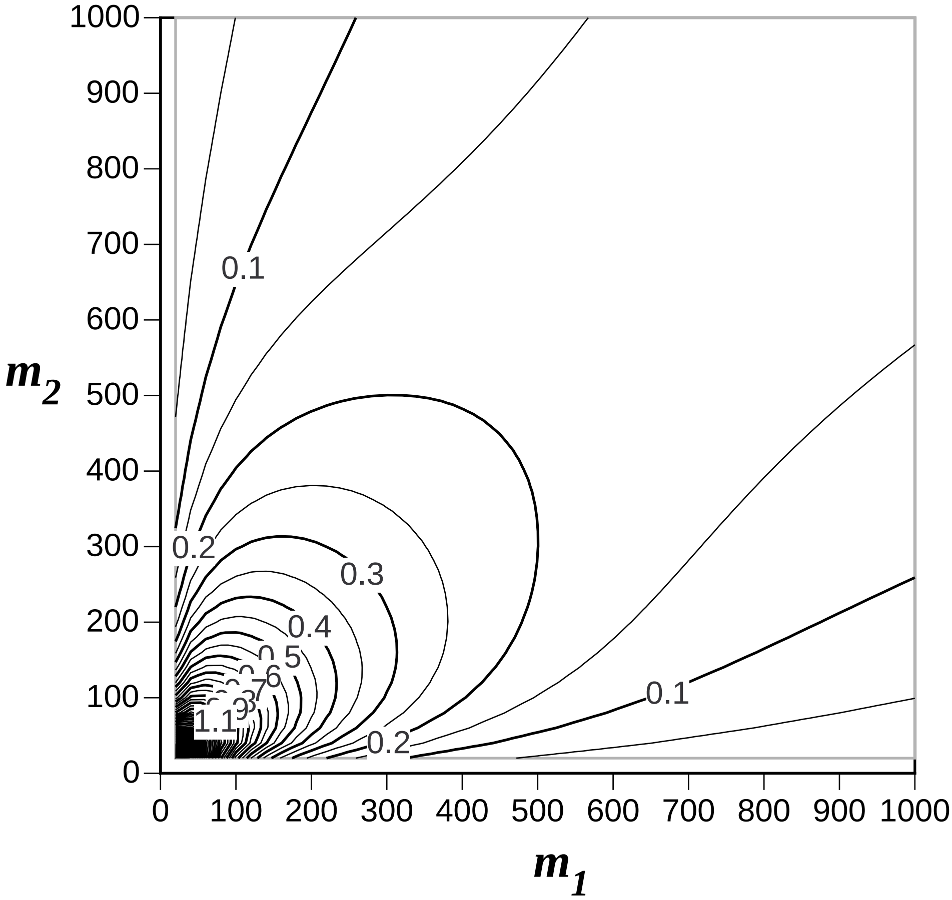
chargino contribution $[T = 100]$

$$-K\left(\frac{m_1}{m_2}\right) + 8\left(f_+(m_1/T, m_2/T) + f_+(m_2/T, m_1/T)\right)$$



bosonic contribution $[T = 100]$

$$K\left(\frac{m_1}{m_2}\right) + 8\left(f_-(m_1/T, m_2/T) + f_-(m_2/T, m_1/T)\right)$$



To have negative λ_5,

- $M_2 \simeq \mu$ and both are large compared to T_C.
  M_2 is the $SU(2)$ gaugino mass parameter.
- the factor in the bosonic corretions should be small;

$$\begin{aligned}(\mu m_{3/2} A)^2 &\ll m_{\tilde{q}}^2 m_{\tilde{t}}^2, \\ m_3^4 &\ll \mu_1^2 \mu_2^2\end{aligned}$$

- large discrepancy between $m_{\tilde{q}}^2$ and $m_{\tilde{t}}^2$ and that between μ_1^2 and μ_2^2 are favored to reduce the zero-temperature bosonic corrections.
  $m_{\tilde{t}}^2 \ll m_{\tilde{q}}^2 \Rightarrow$ first-order EWPT

For solutions of (B) or (C) to be realized,

$$\left.\begin{array}{l} d\tilde{\rho}^2 - 4f\tilde{\rho} < 0 \\ \left|\dfrac{b + c\tilde{\rho}^2 - e\tilde{\rho}}{d\tilde{\rho}^2 - 4f\tilde{\rho}}\right| < 1 \end{array}\right\} \quad \text{for} \quad {}^{\exists}\tilde{\rho} \in [0, 1)$$

where

$$\begin{aligned} b &= -\frac{m_3^2}{2a^2}, \\ c &= \frac{v^2}{4a^2}\lambda_6, \qquad d = \frac{v^2}{4a^2}\lambda_5, \\ e &= -\frac{v}{\sqrt{2}a^2}B_1, \qquad f = -\frac{v}{\sqrt{2}a^2}B_2 \end{aligned}$$

$\Longrightarrow$ same order of (b, c, d, e, f)

$v = 100 \sim 240\text{GeV}$ because $v > T_C \simeq 100\text{GeV}$ for the sphaleron decoupling after EWPT
$a \simeq T/(4 \sim 10) \simeq 10 \sim 25\text{GeV}$.

In MSSM, c, d, e and f are all zero at the tree level.
$\therefore$ $m_3^2(T)$ should be the same order as, *e.g.* $\lambda_5 v^2$.

m_3^2 receives $\log(T/m)$ corrections.

Calculation of $m_3^2(T)$, $\lambda_{6,7}(T)$, $B_{1,2}(T)$
— in progress