宇宙のバリオン数生成

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Baryon Asymmetry of the Universe

$$\frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} = (0.2 \ - \ 0.9) \times 10^{-10}$$

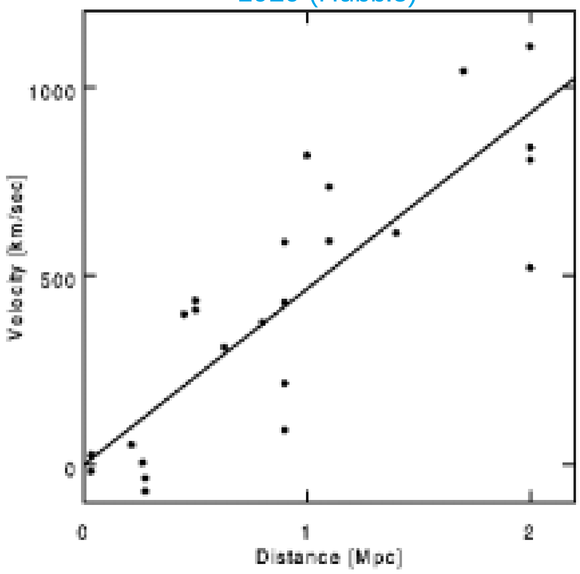
Big Bang Cosmology

[Kolb & Turner, The Early Universe]

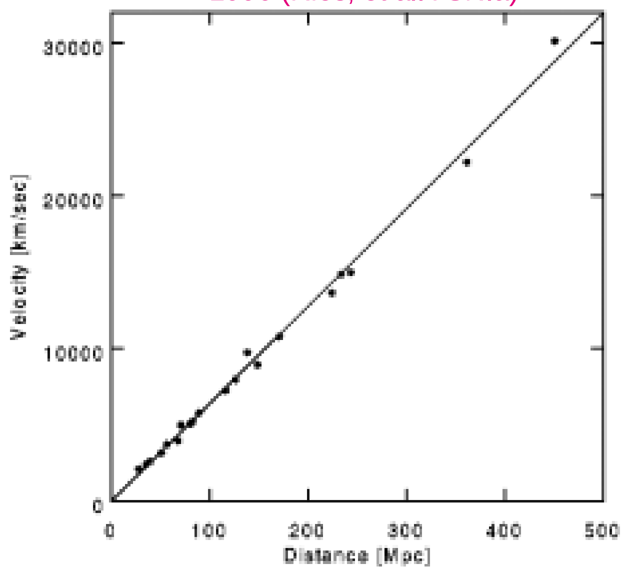
3 great successes

1. Expanding universe — Hubble's law

1929 (Hubble)

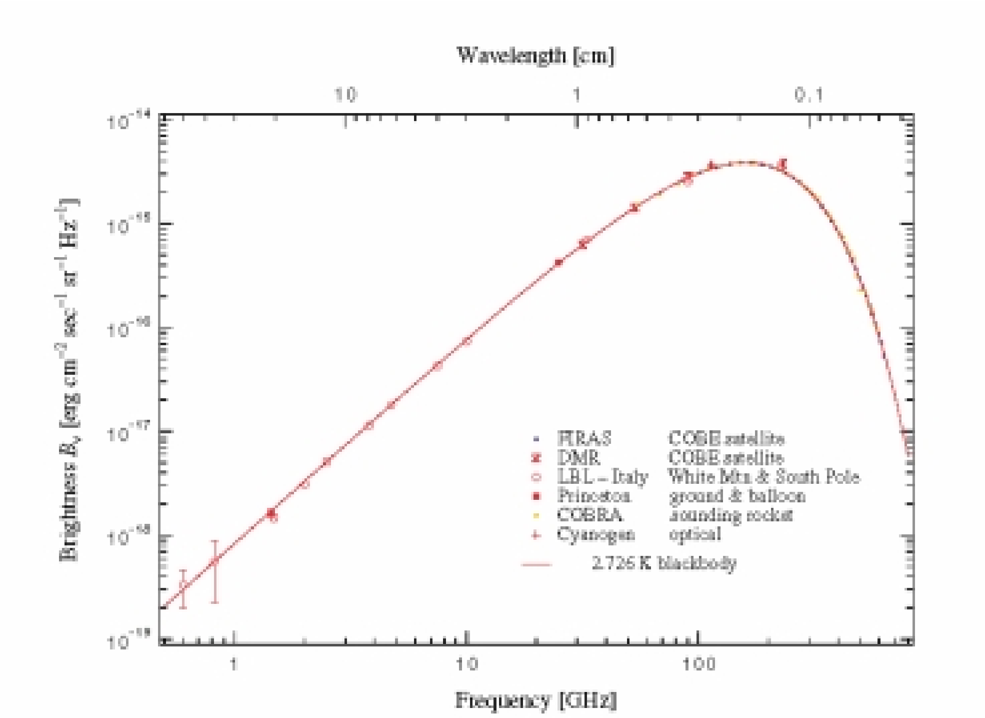


1996 (Ries, et al. : SNIa)



$$H = (71 \pm 7)\,\text{km s}^{-1}\text{Mpc}^{-1}$$

2. Cosmic Microwave Background



$$T = (2.725 \pm 0.005)\ \text{K}$$

Friedmann Universe

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

$R(t)$: scale factor in the comoving coordinate
$k = 1, 0, -1$: closed, flat, open space

Einstein eq. : $$\begin{cases} H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{R^2} + \frac{\Lambda}{3}, \\ \frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \end{cases}$$

energy cons. : $$R^3\frac{dp}{dt} = \frac{d}{dt}[R^3(\rho + p)] \Rightarrow \frac{d}{dt}\rho R^{3(1+\gamma)} = 0$$

ρ = energy density, p = isotropic pressure

$p = \gamma\rho$ with $\begin{cases} \gamma = 1/3 & \text{(RD universe)} \\ \gamma \ll 1 & \text{(MD universe)} \end{cases}$

For RD universe, the energy per degree of freedom is

$$\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\,|\boldsymbol{k}|\,\frac{1}{e^{|\boldsymbol{k}|/T} \mp 1} = \begin{cases} \frac{\pi^2}{30}T^4, \\ \frac{7}{8}\,\frac{\pi^2}{30}T^4, \end{cases}$$

$$\therefore \quad \rho(T) = \frac{\pi^2}{30}g_* T^4 \quad \text{with} \quad g_* \equiv \sum_B g_B + \frac{7}{8}\sum_F g_F$$

For the EW theory with N_f generations and m Higgs doublets,

$$g_* = 24 + 4m + \frac{7}{8} \times 30 N_f$$

so that $g_* = 106.75$ for the Minimal SM.

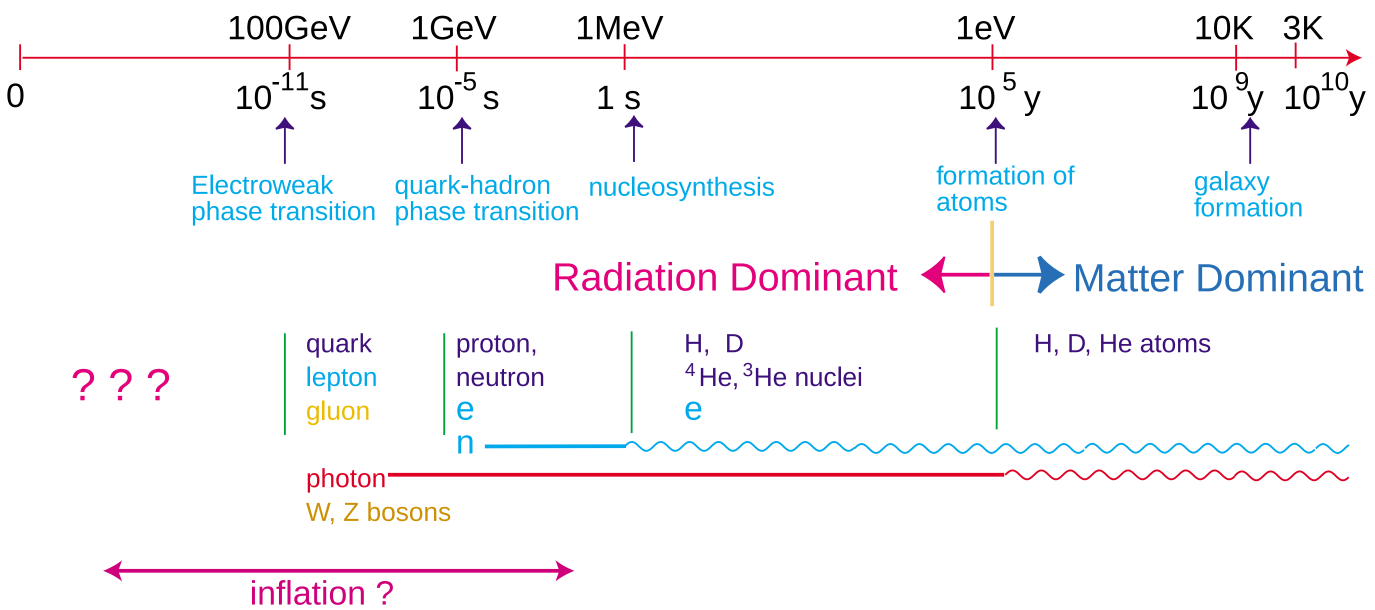
In RD universe, neglecting Λ,

$$\boxed{H \simeq \sqrt{\frac{8\pi G}{3}\rho} \simeq 1.66\sqrt{g_*}\frac{T^2}{m_{Pl}}}$$

$m_{Pl} = G^{-1/2} = 1.22 \times 10^{19}\text{GeV}$

Einstein eq.

$$\Longrightarrow \begin{cases} \text{RD}: & \rho \propto R^4 \quad \Longrightarrow \quad R \propto t^{1/2} \\ \text{MD}: & \rho \propto R^3 \quad \Longrightarrow \quad R \propto t^{2/3} \end{cases}$$



significance of Hubble constant

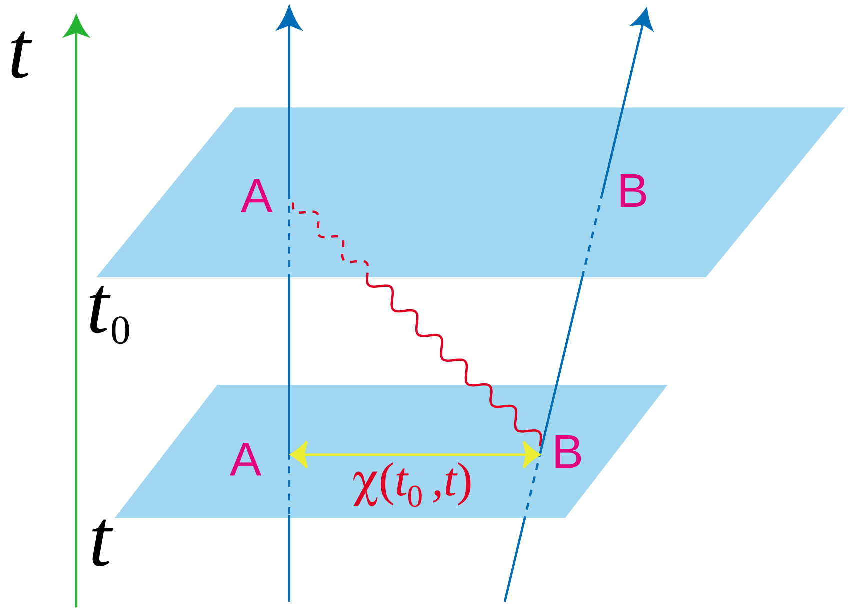
1. expansion rate of the universe

$$H(t) = \frac{\dot{R}(t)}{R(t)}$$

some process $A \leftrightarrow B$ is in equilibrium

$$\Longleftrightarrow \quad \boxed{\Gamma_{A\leftrightarrow B} > H(t)}$$

2. (particle) horizon — causal region



light in the comoving co.: $ds^2 = dt^2 - R^2(t)dr^2 = 0$

$\therefore$ causally related region: $\chi(t_0, t) = -\int_t^{t_0} \frac{dt'}{R(t')}$

$\longrightarrow$ proper distance at t_0: $\quad d = R(t_0)\chi(t_0, t)$

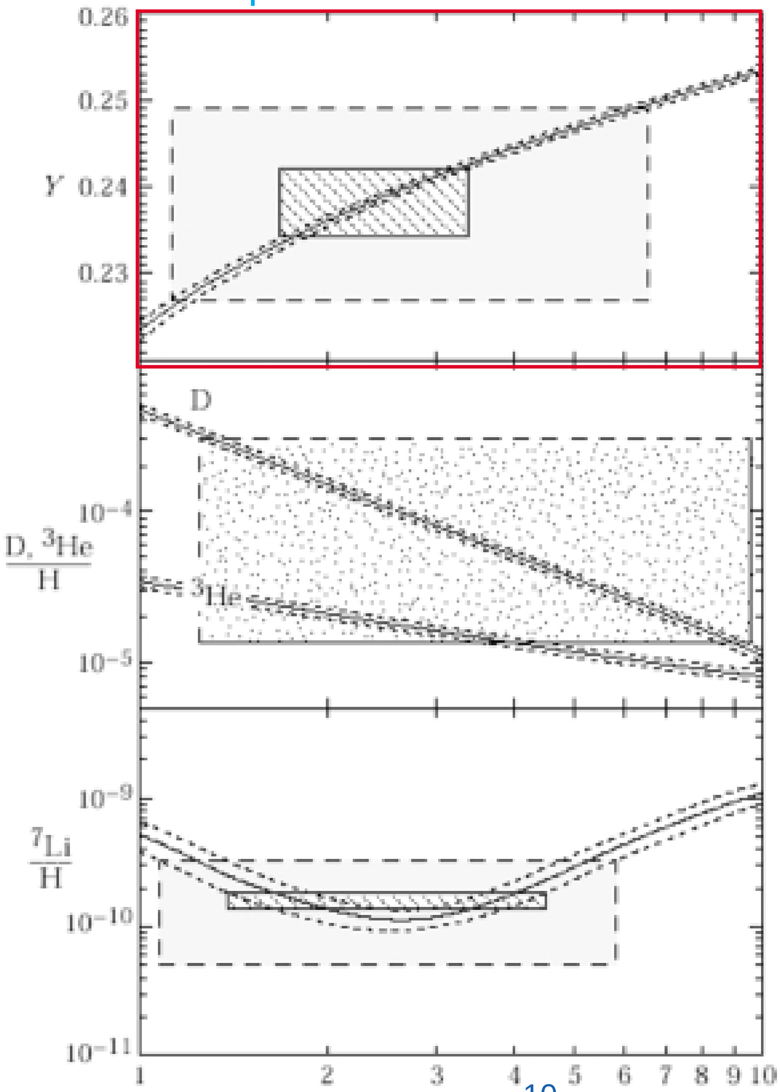
For $R(t) \propto t^\alpha$, taking $t \to 0$,

$$d_H \equiv R(t_0)\chi(t_0, 0) = \frac{t_0}{1-\alpha} \simeq \frac{t_0}{\alpha} = H^{-1}(t_0)$$

3. Nucleosynthesis

[http://ccwww.kek.jp/pdg/2000/bigbangnucrpp.pdf]

SM prediction vs Observation



$$Y = \frac{2\, n/p}{1 + n/p}$$

primordial mass fraction of ^{4}He

$Y = 0.25 \longleftrightarrow n/p = 1/7$

- $T \gg$ 1MeV : $n \leftrightarrow p + e + \bar{\nu}_e \Rightarrow n/p = 1$
- $T = T_F \simeq$ 1MeV $\quad \Gamma_{n\leftrightarrow p}(T_F) \simeq H$

$$\left(\frac{n}{p}\right)_{\text{freeze-out}} = e^{-(m_n - m_p)/T_F} \simeq \frac{1}{6}$$

- $T = 0.3 - 0.1$MeV $\quad E_B/A \simeq 1 - 8$MeV

$$\frac{n}{p} \longrightarrow \frac{1}{6} - \frac{1}{7} \quad \text{depending on } \frac{n_B}{n_\gamma}$$

cf. $s \simeq 7 n_\gamma$

plan of this lecture

1. What do we need for the BAU ?
2. Sphaleron process
3. Electroweak phase transition (EWPT)
4. Electroweak baryogenesis
5. Baryogensis in the MSSM
6. Discussions

1. What do we need for the BAU ?

$$\frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} = (0.2 \; - \; 0.9) \times 10^{-10}$$

— constant after the decoupling of $\Delta B \neq 0$ process

evidence of the BAU [Steigman, Ann.Rev.Astron.Astrop.**14**('76)]

1. no anti-matter in cosmic rays from our galaxy
   some anti-matter consistent as secondary products
2. nearby clusters of galaxies are stable
   a cluster: $(1 \sim 100) M_{\text{galaxy}} \simeq 10^{12\sim14} M_\odot$

Starting from a B-symmetric universe . . .

$$\frac{n_b}{s} \simeq \frac{n_{\bar{b}}}{s} \sim 8 \times 10^{-11} \quad \text{at } T = 38\text{MeV}$$

$$\sim 7 \times 10^{-20} \quad \text{at } T = 20\text{MeV}$$

$N\bar{N}$-annihilation decouple

At $T = 38\text{MeV}$,
mass within a causal region $= 10^{-7} M_\odot \ll 10^{12} M_\odot$.

$\Downarrow$

We must have the BAU $\frac{n_B}{s} = (0.2 - 0.9) \times 10^{-10}$
before the universe was cooled down to $T \simeq 38\text{MeV}$.

3 requirements for generation of BAU [Sakharov, '67]

(1)	baryon number violation
(2)	C and CP violation
(3)	departure from equilibrium

$\because$ (2) If C *or* CP is conserved, no B is generated:
This is because B is odd under C and CP.

indeed . . .

ρ_0 : baryon-symmetric initial state of the universe $s.t.$

$$\langle n_B \rangle_0 = \mathrm{Tr}[\rho_0 n_B] = 0$$

time evolution of $\rho \Leftrightarrow$ Liouville eq.: $i\hbar \frac{\partial \rho}{\partial t} + [\rho, H] = 0$

If H is C- *or* CP-invariant, $[\rho, \mathcal{C}] = 0$ *or* $[\rho, \mathcal{CP}] = 0$

[spontaneous CP viol. $\Longrightarrow [\rho, \mathcal{CP}] \neq 0$]

Since $\mathcal{C}B\mathcal{C}^{-1} = -B$ and $\mathcal{CP}B(\mathcal{CP})^{-1} = -B$

$$\langle n_B \rangle = \mathrm{Tr}[\rho n_B] = \mathrm{Tr}[\rho\, \mathcal{C} n_B \mathcal{C}^{-1}] = -\mathrm{Tr}[\rho n_B]$$

or

$$\langle n_B \rangle = \mathrm{Tr}[\rho\, \mathcal{CP} n_B (\mathcal{CP})^{-1}] = -\mathrm{Tr}[\rho n_B]$$

$\therefore$ Both C and CP must be violated to have $\langle n_B \rangle \neq 0$.

$\because$ (3):

If $\Delta B \neq 0$ processes are in equilibrium ($\mu_B = 0$),

$$n_b = n_{\bar{b}} = \frac{1}{e^{\sqrt{\boldsymbol{k}^2 + m_b^2}/T} + 1}$$

since $m_b = m_{\bar{b}}$ from the CPT invariance.

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possiblities ?

- B violation
  - explicit violation — GUTs
  - spontaneous viol. — $\langle$squark$\rangle \neq 0$
  - chiral anomaly — sphaleron process

  It must be suppressed at present for proton not to decay.

- C violation $\Longleftarrow$ chiral gauge interactions (EW, GUTs)

- CP violation
  - KM phase in the MSM
  - beyond the SM ?

- out of equilibrium
  - expansion of the universe
  - first-order phase transition
  - reheating after inflation

All these conditions must be satisfied at the same time.

the first example — GUTs [Yoshimura, PRL '78]

$SU(5)$ model:

matter: $\left\{ \begin{array}{lll} \mathbf{5}^* : \psi^i_L & \ni & d^c_R, l_L \\ \mathbf{10} : \chi_{[ij]L} & \ni & q_L, u^c_R, e^c_R \end{array} \right.$

$i = 1-5 \ \longrightarrow (\alpha = 1-3, a = 1, 2)$

gauge: $A_\mu = \begin{pmatrix} G_\mu, B_\mu & X^{a\alpha}_\mu \\ X^{a\alpha}_\mu & W_\mu, B_\mu \end{pmatrix}$

$$\mathcal{L}_{\rm int} \ni g\bar{\psi}\gamma^\mu A_\mu \psi + g{\rm Tr}\left[\bar{\chi}\gamma^\mu\{A_\mu, \chi\}\right]$$

$$\ni gX^a_{\alpha\mu}\left[\varepsilon^{\alpha\beta\gamma}\bar{u}^c_{R\gamma}\gamma^\mu q_{L\beta a} + \epsilon_{ab}\left(\bar{q}_{L\alpha b}\gamma^\mu e^c_R + \bar{l}_{Lb}\gamma^\mu d^c_{R\alpha}\right)\right]$$

$\Longrightarrow$

process	br. ratio	B
$X \longrightarrow qq$	r	$2/3$
$X \longrightarrow \bar{q}\bar{l}$	$1-r$	$-1/3$
$\bar{X} \longrightarrow \bar{q}\bar{q}$	$\bar{r}$	$-2/3$
$\bar{X} \longrightarrow q, l$	$1-\bar{r}$	$1/3$

in the decay of X-$\bar{X}$ pairs

$$\langle \Delta B \rangle = \frac{2}{3}r - \frac{1}{3}(1-r) - \frac{2}{3}\bar{r} + \frac{1}{3}(1-\bar{r}) = r - \bar{r}$$

$\therefore$ C *or* CP is conserved $(r = \bar{r}) \Longrightarrow \Delta B = 0$

If the inverse process is suppressed, $B \propto r - \bar{r}$ is generated.

$\Updownarrow$

$X\bar{X} \leftrightarrow qq, \bar{q}\bar{l}$: out of equilibrium

At $T \simeq m_X$, decay rate of X $= \Gamma_D \simeq \alpha m_X$

$\alpha \sim 1/40$ for gauge boson, $\alpha \sim 10^{-6\sim-3}$ for Higgs boson

Hubble parameter : $H \sim 1.7\sqrt{g_*}\,\dfrac{T^2}{m_{Pl}}$

$g_* \simeq 10^{2\sim3}$: massless degrees of freedom

$\therefore\ \Gamma_D \simeq H$ at $T \simeq m_X$

$\Longrightarrow$ decay and production of $X\bar{X}$ are out of equil.

N.B.

The $SU(5)$ GUT model conserves $B-L$.

$i.e.$ $B+L$-genesis

$\Downarrow$

washed-out by the sphaleron process, as we see later

$\downarrow$

leptogenesis $\Rightarrow$ BAU $\quad B=-L$

other candidates for generating BAU

- $^\exists$Majorana neutrino $\Rightarrow$ L-violating interaction

[Fukugita & Yanagida, PL '86]

$$\left.\begin{array}{c}\text{decoupling of heavy-}\nu\text{ decay}\\ CP\text{ violation in the lepton sector}\end{array}\right\} \Rightarrow \text{Leptogenesis}$$

$$\xrightarrow{\text{sphaleron}} \text{BAU}$$

[recent review: Buchmüller & Plümacher, hep-ph/0007176]

- Affleck-Dine mechanism in a supersymmetric model

[Affleck & Dine, NPB '86]

$\langle$squark$\rangle \neq 0$ or $\langle$slepton$\rangle \neq 0$ along (nearly) flat directions, at high temperature

coherent motion of complex $\langle \tilde{q} \rangle$, $\langle \tilde{l} \rangle \neq 0$ B, C, CP viol.

$\Longrightarrow$ B- and/or L-genesis

- Electroweak Baryogenesis

(1) $\Delta(B+L) \neq 0$ $\left\{\begin{array}{l}\text{enhanced by sphaleron at } T > T_C\\ \text{suppressed by instanton at } T = 0\end{array}\right.$

(2) C-violation (chiral gauge)

CP-violation: KM phase or extension of the MSM

(3) first-order EWPT with expanding bubble walls

- topological defects

EW string, domain wall $\sim$ EW baryogenesis

effective volume is too small, mass density of the universe

2. Sphaleron process

⋆ Anomalous fermion number nonconservation

axial anomaly in the standard model

$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}]$$
$$\partial_\mu j^\mu_{B-L} = 0$$

$N_f =$ number of the generations, $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

integrating these equations,

$$\begin{aligned} B(t_f) - B(t_i) &= \int_{t_i}^{t_f} d^4x \, \frac{1}{2}\left[\partial_\mu j^\mu_{B+L} + \partial_\mu j^\mu_{B-L}\right] \\ &= \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x \left[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right] \\ &= N_f\left[N_{CS}(t_f) - N_{CS}(t_i)\right] \end{aligned}$$

where N_{CS} is the Chern-Simons number:
in the $A_0 = 0$ gauge,

$$N_{CS}(t) = \frac{1}{32\pi^2}\int d^3x \, \epsilon_{ijk}\left[g^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g'^2 B_{ij}B_k\right]_t$$

— gauge-dependent

classical vacua of the gauge sector $\mathcal{E} = \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2) = 0$

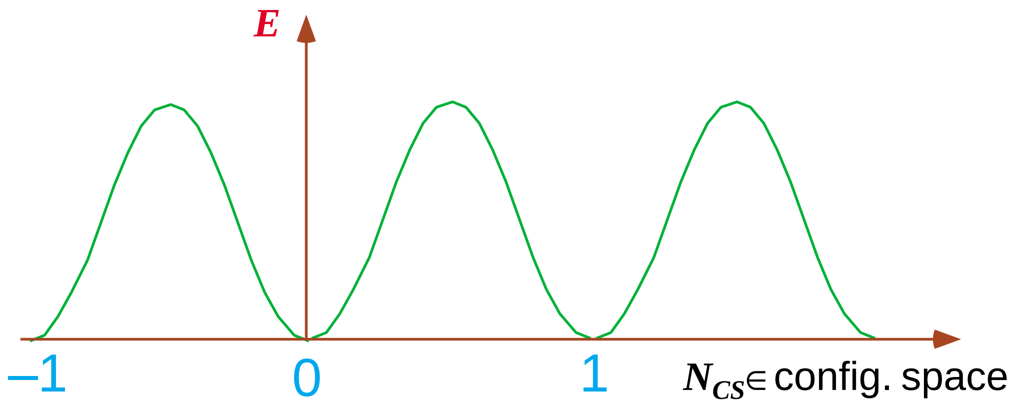
$\Longleftrightarrow F_{ij} = B_{ij} = 0$

$\Longleftrightarrow A = iU^{-1}dU$ and $B = dv$ with $U \in SU(2)$

$\therefore \quad U(\boldsymbol{x}) : S^3 \ni \boldsymbol{x} \longrightarrow U \in SU(2) \simeq S^3$

$\pi_3(S^3) \simeq \mathbf{Z} \Rightarrow U(\boldsymbol{x})$ is classified by an integer N_{CS}.

energy functional vs configuration space



background U changes with $\Delta N_{CS} = 1$

$\Longrightarrow \Delta B = 1$ $(\Delta L = 1)$ in each (left-) generation

$\Longleftrightarrow$
- level crossing
- index theorem

Transition of the field config. with $\Delta B \neq 0$?

▷ quantum tunneling	low temperature
▷ thermal activation	high temperature

transition rate with $\Delta N_{CS} = 1 \Longleftarrow$ WKB approx.

$T = 0$

(valley or constrained) instanton = *finite euclidean action*

tunneling probability $\sim e^{-2S_{\rm inst}} = e^{-8\pi^2/g^2}$

for EW theory, $e^{-2S_{\rm inst}} \simeq e^{-378} = 10^{-164}$

[cf. QCD — θ-vacuum]

$T \neq 0$ [Affleck, P.R.L.46('81)]

$^{\exists}$classical static saddle-point solution with *finite energy*

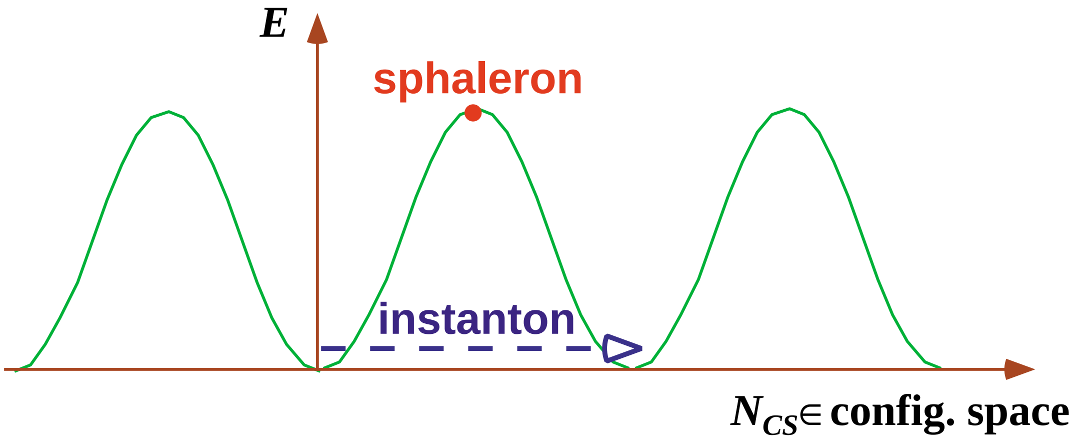
$\Updownarrow$

top of the energy barrier dividing two classical vacua

$\|$

sphaleron solution [Manton, P.R.D28('83)]

$\sigma\varphi\alpha\lambda\epsilon\rho o\sigma$ = 'ready to fall'



$$E_{\rm sph}(T=0) = \frac{2M_W}{\alpha_W} B\left(\frac{\lambda}{g^2}\right) \simeq 10\text{TeV}$$

λ:the Higgs self coupling, $\alpha_W = g^2/(4\pi)$

$1.5 \leq B \leq 2.7$ for $\lambda/g^2 \in [0, \infty)$

⋆ Transition rate

[Arnold and McLerran, P.R.D36('87)]

♣ $\omega_-/(2\pi) \lesssim T \lesssim T_C$

ω_-:negative-mode freq. around the sphaleron

$$\Gamma^{(b)}_{\rm sph} \simeq k \mathcal{N}_{\rm tr} \mathcal{N}_{\rm rot} \frac{\omega_-}{2\pi} \left(\frac{\alpha_W(T)T}{4\pi} \right)^3 {\rm e}^{-E_{\rm sph}/T}$$

zero modes $\rightarrow \begin{cases} \mathcal{N}_{\rm tr} = 26 \\ \mathcal{N}_{\rm rot} = 5.3 \times 10^3 \end{cases}$ for $\lambda = g^2$

$$\omega_-^2 \simeq (1.8 \sim 6.6) m_W^2 \quad \text{for } 10^{-2} \leq \lambda/g^2 \leq 10$$

$$k \simeq O(1)$$

♣ $T \gtrsim T_C$ symmetric phase — no mass scale

dimensional analysis :

$$\Gamma^{(s)}_{\rm sph} \simeq \kappa (\alpha_W T)^4$$

check by Monte Carlo simulation $\langle N^2_{CS}(t) \rangle = e^{-2\Gamma V t}$ as $t \rightarrow \infty$

$\kappa > 0.4$ $SU(2)$ gauge-Higgs system
[Ambjørn, et al. N.P.B353('91)]

$\kappa = 1.09 \pm 0.04$ $SU(2)$ pure gauge system
[Ambjørn and Krasnitz, P.L.B362('95)]

'sphaleron transition' even in the symmetric phase.

⋆ Washout of $B+L$ [Kuzmin, Rubakov, Shaposhnikov, PLB, '85]

sphaleron process is in equilibrium $\Longleftrightarrow \Gamma_{\rm sph} > H$

At $T = T_C \simeq 100$GeV,

$$H(T_C) = \frac{\dot{R}(t)}{R(t)} \simeq 1.7\sqrt{g_*}\frac{T_C^2}{m_{Pl}} \simeq 10^{-13}\text{GeV}$$

$g_* \sim 100$: effective massless degrees of freedom

At $T > T_C$,

$$\Gamma_{\rm sph} \simeq \Gamma_{\rm sph}^{(s)}/T^3 \simeq \kappa\alpha_W^4 T \sim 10^{-4}\text{GeV} \gg H(T_C)$$

$\Longrightarrow$ $B+L$-changing process in equilibrium

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relic baryon number after the washout

[Harvey & Turner, PRD, '90]

particle number density $[m/T \ll 1$ and $\mu/T \ll 1]$

$$\begin{aligned} n_+ - n_- &= \int \frac{d^3\boldsymbol{k}}{(2\pi)^2}\left[\frac{1}{e^{\beta(\omega_k-\mu)} \mp 1} - \frac{1}{e^{\beta(\omega_k+\mu)} \mp 1}\right] \\ &\simeq \begin{cases} \dfrac{T^3}{3}\dfrac{\mu}{T} & \text{for bosons} \\ \dfrac{T^3}{6}\dfrac{\mu}{T} & \text{for fermions,} \end{cases} \end{aligned}$$

chemical equilibrium — all the gauge int., Yukawa, sphaleron

W^-	$u_{L(R)}$	$d_{L(R)}$	$e_{iL(R)}$	ν_{iL}	ϕ^0	ϕ^-
μ_W	$\mu_{u_{L(R)}}$	$\mu_{d_{L(R)}}$	$\mu_{iL(R)}$	μ_i	μ_0	μ_-

gauge int. $\Leftrightarrow$ $\mu_W = \mu_{d_L} - \mu_{u_L} = \mu_{iL} - \mu_i = \mu_- + \mu_0$

$|0\rangle \leftrightarrow u_L d_L d_L \nu_L$ $\Leftrightarrow$ $N_f(\mu_{u_L} + 2\mu_{d_L}) + \sum_i \mu_i = 0$

Quantum number densities [in unit of $T^2/6$]

$$\begin{aligned}
B &= N_f(\mu_{u_L} + \mu_{u_R} + \mu_{d_L} + \mu_{d_R}) = 4N_f\mu_{u_L} + 2N_f\mu_W, \\
L &= \sum_i(\mu_i + \mu_{iL} + \mu_{iR}) = 3\mu + 2N_f\mu_W - N_f\mu_0 \\
Q &= \frac{2}{3}N_f(\mu_{u_L} + \mu_{u_R}) \cdot 3 - \frac{1}{3}N_f(\mu_{d_L} + \mu_{d_R}) \cdot 3 \\
&\quad - \sum_i(\mu_{iL} + \mu_{iR}) - 2 \cdot 2\mu_W - 2m\mu_- \\
&= 2N_f\mu_{u_L} - 2\mu - (4N_f + 4 + 2m)\mu_W + (4N_f + 2m)\mu_0 \\
I_3 &= \frac{1}{2}N_f(\mu_{u_L} - \mu_{d_L}) \cdot 3 + \frac{1}{2}\sum_i(\mu_i - \mu_{iL}) \\
&\quad -2 \cdot 2\mu_W - 2 \cdot \frac{1}{2}m(\mu_0 - \mu_-) \\
&= -(2N_f + m + 4)\mu_W
\end{aligned}$$

$\mu \equiv \sum_i \mu_i$, m : number of Higgs doublets

- symmetric phase $\Longrightarrow Q = I_3 = 0$

$$B = \frac{8N_f + 4m}{22N_f + 13m}(B - L), \quad L = -\frac{14N_f + 9m}{22N_f + 13m}(B - L)$$

- broken phase $\Longrightarrow Q = 0$ and $\mu_0 = 0$

$$B = \frac{8N_f + 4m + 8}{24N_f + 13m + 26}(B - L)$$

$$L = -\frac{16N_f + 9m + 18}{24N_f + 13m + 26}(B - L)$$

$\therefore$ If $(B - L)_{\mathrm{primordial}} = 0$, $B = L = 0$ at present !

To have nonzero BAU,

(i) we must have $B - L$ before the sphaleron process decouples, or

(ii) $B + L$ must be created at the first-order EWPT, and the sphaleron process must decouple immediately after that.

(i) $\Leftarrow$ GUTs, Majorana ν, Affleck-Dine

(ii) $=$ Electroweak Baryogenesis

3. Electroweak phase transition (EWPT)

rate of any interaction at T : $\Gamma(T) > H(T)$

$\Longrightarrow$ equilibrium thermodynamics can be applied to study static properties

- transition temperature T_C
- order of the phase transition
- latent heat and surface tension for 1st order PT

$\Uparrow$

free energy density = effective potential:

$$V_{\text{eff}}(v;T) = -\frac{1}{V} T \log Z = -\frac{1}{V} T \log \mathrm{Tr}\left[e^{-H/T}\right]_{\langle\phi\rangle = v}$$

where

H = hamiltonian of the QFT

$$e.g. \quad H = \int d^3\boldsymbol{x} \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)\right]$$

$v = \langle\phi\rangle$ = order parameter

when the symmetry of the theory is broken by $v \neq 0$

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thermodynamic quantities

$$E/V = \frac{1}{ZV}\mathrm{Tr}\left[He^{-H/T}\right] = V_{\text{eff}} - T\frac{\partial V_{\text{eff}}}{\partial T} = \sigma T^4$$

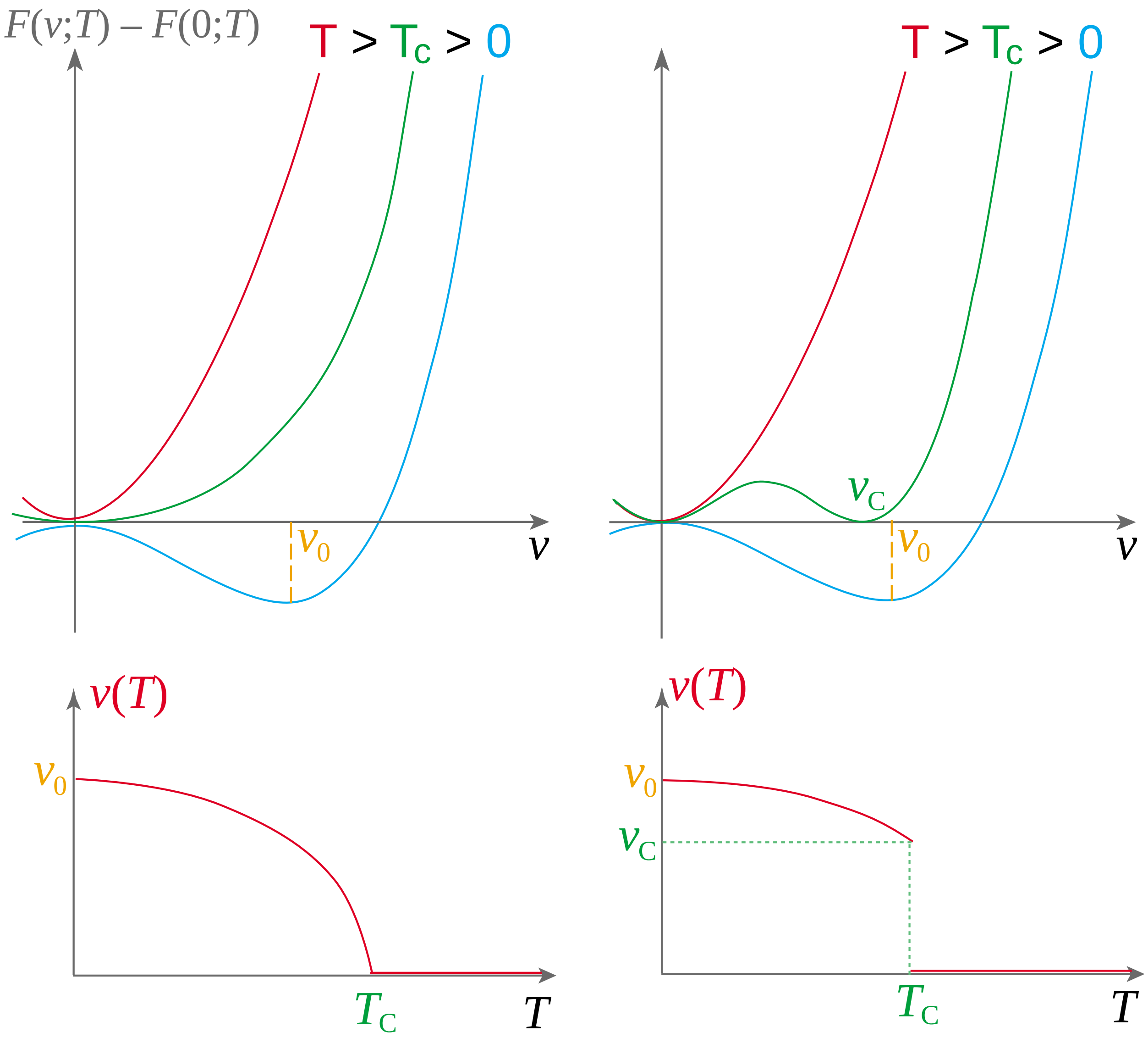
$$\therefore\ V_{\text{eff}}(v=0;T) = -\text{const.}T^4$$

$$s = S/V = -\frac{\partial V_{\text{eff}}}{\partial T} \propto T^3 \qquad (\because F = E - TS)$$

$V_{\rm eff}(v;T)$ $\Longleftarrow$ finite-temperature QFT

[review: Brandenberger, Rev.Mod.Phys. '85]

2nd order phase transition　　1st order phase transition



Minimal SM (MSM)

order parameter = Higgs VEV: $\langle\Phi\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \varphi \end{pmatrix}$

$\therefore$ 1st order EWPT $\Longleftrightarrow$ $\varphi_C \equiv \lim_{T\uparrow T_C} \varphi(T) \neq 0$

⋆ Perturbative calculation

[Dolan & Jackiw, PRD '74]

$$Z = \mathrm{Tr}\left[e^{-H/T}\right] = \int [d\phi] \exp\left\{-\int_0^{1/T} d\tau\, d^3\boldsymbol{x}\, \mathcal{L}_E(\phi)\right\}$$

where

$$\mathcal{L}_E(\phi) = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi), \quad \text{Euclidean: } \tau = -it$$

$\phi(0) = \phi(1/T)$ periodic b.c.

anti-periodic b.c. for fermions

$e.g.$ at the one-loop level (MSM),

$$V_{\text{eff}}(\varphi; T) = V_{\text{tree}}(\varphi) + V^{(1)}(\varphi; T),$$

where

$$V_{\text{tree}}(\varphi) = -\frac{1}{2}\mu_0^2\varphi^2 + \frac{\lambda_0}{4}\varphi^4$$

$$V^{(1)}(\varphi; T) = -\frac{i}{2}\sum_A c_A \int_k \log\det\left[i\mathcal{D}_A^{-1}(k;\varphi)\right]$$

with

μ_0^2, λ_0 : bare parameters $\Leftarrow$ renormlized at $T = 0$

A runs over all the partice species

$|c_A|$ counts the degrees of freedom $\begin{cases} c_A > 0 & \text{for bosons} \\ c_A < 0 & \text{for fermions} \end{cases}$

$$\int_k \equiv iT \sum_{n=-\infty}^{\infty} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}$$

with $k^0 = i\omega_n = i \begin{cases} 2n\pi T & \text{for bosons,} \\ (2n+1)\pi T & \text{for fermions.} \end{cases}$

$\mathcal{D}_A(k;\varphi)$: propagator in the background φ

i.e.

$$W\text{-boson}: \begin{cases} c_W = 2 \\ i\mathcal{D}^{-1\,\mu\nu}_{W}(k;\varphi) \\ \quad = (-k^2 + m_W^2(\varphi))\eta^{\mu\nu} + (1-\frac{1}{\xi})k^\mu k^\nu \\ \qquad m_W(\varphi) = \frac{1}{2}g\varphi \end{cases}$$

$$\text{Dirac fermion}: \begin{cases} c_f = -2 \\ i\mathcal{D}_f^{-1}(k;\varphi) = \not{k} - m_f(\varphi) \\ \qquad m_f(\varphi) = y_f\varphi/\sqrt{2} \end{cases}$$

formulas

$$\int_k \log(k^2 - m^2)$$

$$= \int \frac{d^4k}{(2\pi)^4} \log(k^2 - m^2) \pm 2iT \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \log\left(1 \mp e^{-\omega_k/T}\right),$$

$$\int_k \frac{1}{k^2 - m^2}$$

$$= \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \mp i \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{\omega_k} \frac{1}{e^{\omega_k/T} \mp 1},$$

etc. $\qquad \omega_k = \sqrt{\boldsymbol{k}^2 + m^2}$

For the MSM,

$$V_{\text{eff}}(\varphi;T) = V_0(\varphi) + \bar{V}(\varphi;T)$$

where

$$V_0(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2Bv_0^2\varphi^2 + B\varphi^4\left[\log\left(\frac{\varphi^2}{v_0^2}\right) - \frac{3}{2}\right]$$

$$\bar{V}(\varphi;T) = \frac{T^4}{2\pi^2}\left[6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)\right]$$

$$B = \frac{3}{64\pi^2 v_0^4}(2m_W^4 + m_Z^4 - 4m_t^4)$$

$$I_{B,F}(a^2) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right)$$

with

$v_0 = 246\text{GeV}$ is the minimum of $V_0(\varphi)$

$a_A = m_A(\varphi)/T$

high-temperature expansion $[m/T \ll 1]$

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right)\frac{a^4}{2} + O(a^6)$$

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6)$$

For $T > m_W, m_Z, m_t$,

$$V_{\rm eff}(\varphi;T) \simeq D(T^2 - T_0^2)\varphi^2 - E\,T\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2)$$

$$E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2)$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4}\left(2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right)$$

with $\log\alpha_B = 2\log 4\pi - 2\gamma_E$ and $\log\alpha_F = 2\log\pi - 2\gamma_E$

At T_C, $\quad \varphi_C = \dfrac{2E\,T_C}{\lambda_{T_C}}$

$$\Gamma^{(b)}_{\rm sph}/T_C^3 < H(T_C) \quad\Longleftrightarrow\quad \frac{\varphi_C}{T_C} \gtrsim 1$$

$\Longrightarrow$ upper bound on λ $\qquad [m_H = \sqrt{2}\lambda v_0]$

$$m_H \lesssim 46\text{GeV}$$

$\longleftrightarrow$ inconsistent with the lower bound $m_H > 95.3$GeV

⋆ Monte Carlo simulations [MSM]

effective fermion mass : $m_f(T) \sim O(T) \leftarrow$ nonzero modes

$\therefore$ simulation only with the bosons

QFT on the lattice $\begin{cases} \text{scalar fields:} & \phi(x) \text{ on the sites} \\ \text{gauge fields:} & U_\mu(x) \text{ on the links} \end{cases}$

$$Z = \int [d\phi\, dU_\mu] \exp\{-S_E[\phi, U_\mu]\}$$

- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet

  $\Uparrow$

  time-component of the gauge field

  only zero-freq. modes of the bosons survive as $T \to$ large

  matching finite-T Green's functions with 4-dim. theory

  $\Rightarrow$ T-dependent parameters

  [Laine & Rummukainen, hep-lat/9809045]

- 4-dim. $SU(2)$ system with a Higgs doublet

  [Csikor, hep-lat/9910354]

  EWPT is first order for $m_h < 66.5 \pm 1.4$GeV

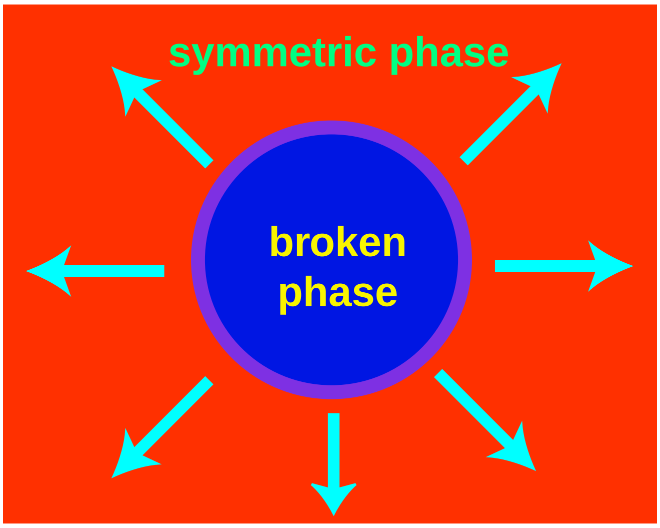
Both the simulations found end-point of EWPT at

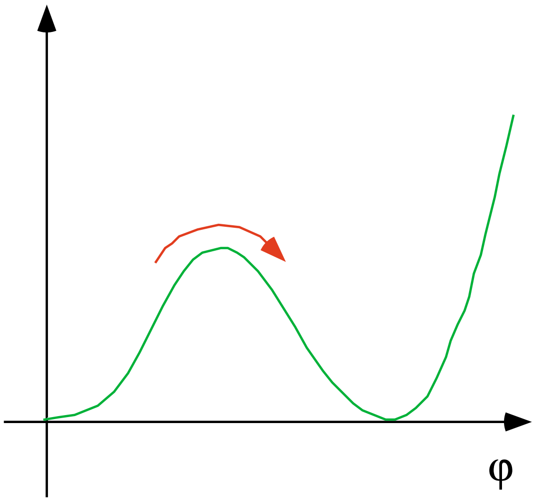
$$m_h = \begin{cases} 72.3 \pm 0.7 \text{ GeV} \\ 72.1 \pm 1.4 \text{ GeV} \end{cases} \Rightarrow \boxed{\text{no PT in the MSM !}}$$

no out-of-equilibrium state at the EWPT

⋆ Dynamics of the phase transition

first-order EWPT accompanying bubble nucleation/growth





Suppose that $V_{\rm eff}(\varphi; T_C)$ is known.

nucleation rate per unit time and unit volume:

$$I(T) = I_0 \mathrm{e}^{-\Delta F(T)/T}$$

where

$$\Delta F(T) = \frac{4\pi}{3} r^3 [p_s(T) - p_b(T)] + 4\pi r^2 \sigma$$

with

$$p_s(T) = -V_{\rm eff}(0; T), \qquad p_b(T) = -V_{\rm eff}(\varphi(T); T)$$

supercooling $\longrightarrow$ $p_s(T) < p_b(T)$

$\sigma \simeq \int dz (d\varphi/dz)^2$: surface energy density

radius of the critical bubble : $r_*(T) = \dfrac{2\sigma}{p_b(T) - p_s(T)}$

How the EWPT proceeds ? [Carrington and Kapsta, P.R.D47('93)]

$f(t)$: fraction of space converted to the broken phase

$$f(t) = \int_{t_C}^{t} dt' \, I(T(t'))[1 - f(t')]V(t', t)$$

where

$V(t', t)$: volume of a bubble at t which was nucleated at t'

$$V(t', t) = \frac{4\pi}{3} \left[r_*(T(t')) + \int_{t'}^{t} dt'' \, v(T(t'')) \right]^3$$

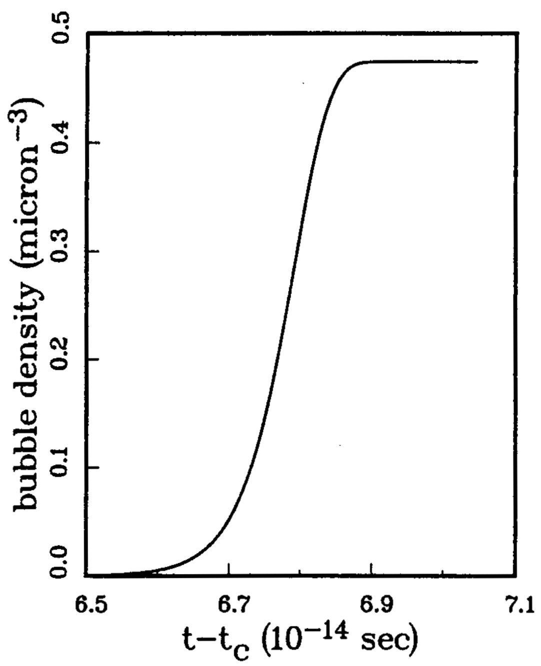
$T = T(t) \Leftarrow \rho = (\pi^2/30) g_* T^4 \propto R^{-4}$ for RD universe

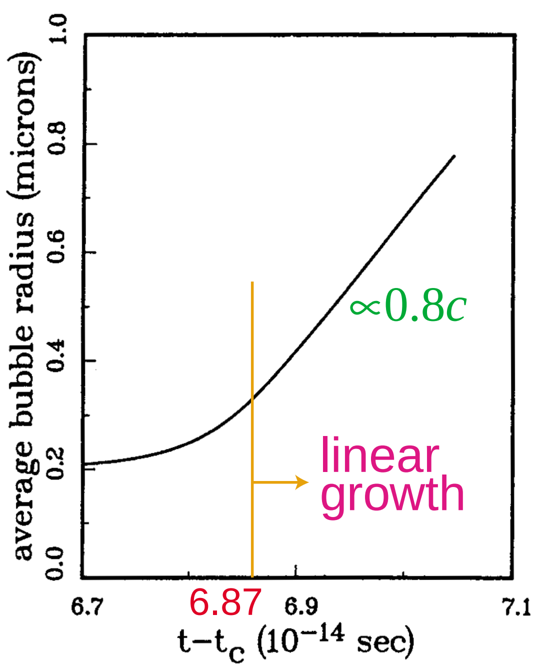
$v(T)$: wall velocity

- one-loop V_{eff} of MSM with $m_H = 60\text{GeV}$ and $m_t = 120\text{GeV}$

At $t = 6.5 \times 10^{-14}$ sec, bubbles began to nucleate.

[A characteristic time scale of the EW processes is $O(10^{-26})$sec.]

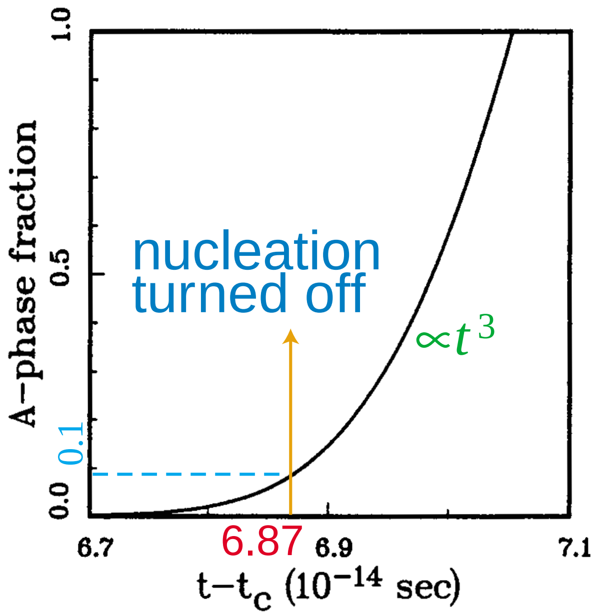


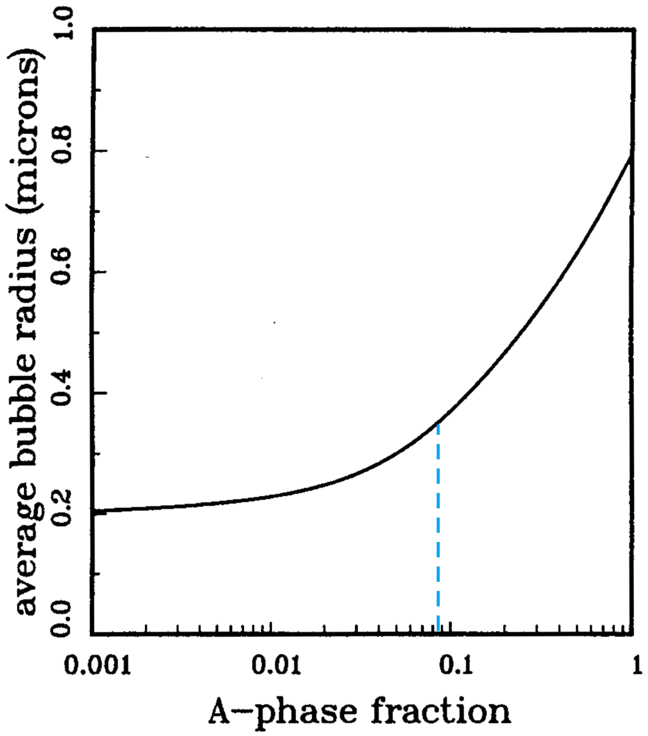


horizon size : $H^{-1} \simeq 7.1 \times 10^{12}$ GeV$^{-1} = 0.14$ cm

$r = 0.3\mu$m $\Rightarrow$ #(bubbles within a horizon) $\simeq 3 \times 10^{11}$

very small supercooling : $\dfrac{T_C - T_N}{T_C} \simeq 2.5 \times 10^{-4}$





90% of the universe is converted by bubble growth

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weakly first order $\Longleftrightarrow$ small φ_C and/or lower barrier height

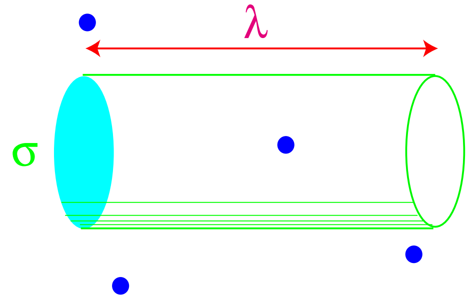
$\Longrightarrow$ { nucleation dominance over growth; thick bubble wall; large fluctuation between the two phases }

- nucleation dominance over growth
- thick bubble wall
- large fluctuation between the two phases

4. Electroweak baryogenesis

⋆ various time scales at $T \simeq T_C$

σ : total cross section of some interaction



mean free path : $\lambda \cdot \sigma = \dfrac{1}{n}$

where n is the density of the particles.

$$n = g \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{e^{|\boldsymbol{k}|/T} \mp 1} = \begin{cases} \dfrac{\zeta(3)}{\pi^2} g T^3, \\ \dfrac{3}{4} \dfrac{\zeta(3)}{\pi^2} g T^3, \end{cases}$$

$$\zeta(3) = 1.2020569 \cdots$$

$$\text{mean free time} = \bar{t} = \frac{\lambda}{v} = \frac{\lambda}{\sqrt{1 - m^2/E^2}} \simeq \lambda \quad \text{for } E \gg m$$

Since $\sigma \simeq \alpha^2/s$ and $\sqrt{s} \sim T$ at T,

$$\lambda \simeq \frac{10}{g_* T^3} \cdot \frac{T^2}{\alpha^2} \simeq \frac{1}{10\alpha^2 T}$$

At $T = 100\text{GeV}$,

$$\lambda_s \simeq \frac{1}{10^3 \alpha_s^2} \sim 0.1\text{GeV}^{-1} \quad \text{for strong interactions}$$

$$\lambda_{EW} \simeq \frac{1}{10^3 \alpha_W^2} \sim 1\text{GeV}^{-1} \quad \text{for electroweak interactions}$$

$$\lambda_Y \simeq \left(\frac{m_W}{m_f}\right)^4 \lambda_{EW} \quad \text{for Yukawa interactions}$$

$$\alpha_s(m_Z) = 0.117 \pm 0.005$$
$$\alpha_W = \alpha_{QED}/\sin^2\theta_W \simeq 1/30$$

the time scale of the universe expansion:

$$H^{-1}(T) \simeq 10^{14}\text{GeV}^{-1}$$

time scale of the sphaleron process:

$$\bar{t}_{\text{sph}} \simeq (\Gamma_{\text{sph}}/n)^{-1} \sim 10^5\text{GeV}^{-1}$$

EW bubble wall thickness and velocity:

$$l_w \simeq \frac{1 \sim 40}{T} \simeq 0.01 \sim 0.4\text{GeV}^{-1}$$
$$v_w \simeq 0.1 \sim 0.9$$

[Liu, McLerran and Turok, PRD,'92]

time scale of the EW bubble wall motion

$$t_{\text{wall}} = \frac{l_w}{v_w} \simeq 0.01 \sim 4\text{GeV}^{-1}.$$

From these we observe:

1. All the particles are in *kinetic equilibrium* at the same temperature, because of $H^{-1} \gg \bar{t}_{EW}$, far from the bubble wall.

2. The Yukawa interactions of the light fermions $(m_f < 0.1\text{GeV})$ are out of *chemical equilibrium*.

3. Some of the flavor-changing interactions are out of *chemical equilibrium* because of small KM matrix elements.

4. Since for the leptons $\lambda_Y > \lambda_{EW} \gg l_w$, the leptons propagate almost freely before and after the scattering off the bubble wall.

5. Because of $t_{\text{wall}} \ll \bar{t}_{\text{sph}}$, the sphaleron process is out of *chemical equilibrium* near the bubble wall.

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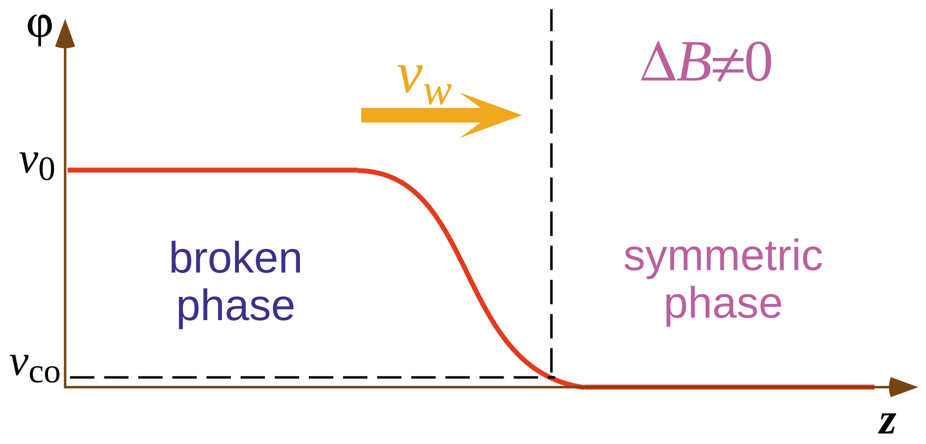
review articles on EW baryogenesis

K.F., Prog.Theor.Phys.**96** ('96) 475.

V.A. Rubakov and M.E. Shaposhnikov, hep-ph/9603208.

A.Cohen, et al., Ann.Rev.Nucl.Part.Sci. **33** ('93) 27.

⋆ Mechanism



$v_{co} \simeq 0.01 v_0 \Longleftarrow E_{\rm sph}/T_C \simeq 1$

bubble wall $\Leftarrow$ classical config. of the gauge-Higgs system

- interactions between the particles and the bubble wall
- accumulation of chiral charge in the symmetric phase

$\Downarrow$

generation of baryon number through sphaleron process

$\Downarrow$

decoupling of sphaleron process in the broken phase

2 scenarios:

- spontaneous baryogenesis + diffusion
  classical, adiabatic
- charge transport scenario
  quantum mechanical, nonlocal

Both need CP violation other than KM matrix

$\Longleftrightarrow$ extension of the MSM

two-Higgs-doublet model, MSSM, . . .

⋆ Charge transport mechanism

[Nelson, el al. NPB,'92]

CP violation in the Higgs sector [spacetime-dependent]

⇓

difference in reflections of chiral fermions and antifermions

⇓

net chiral charge flux into the symmetric phase

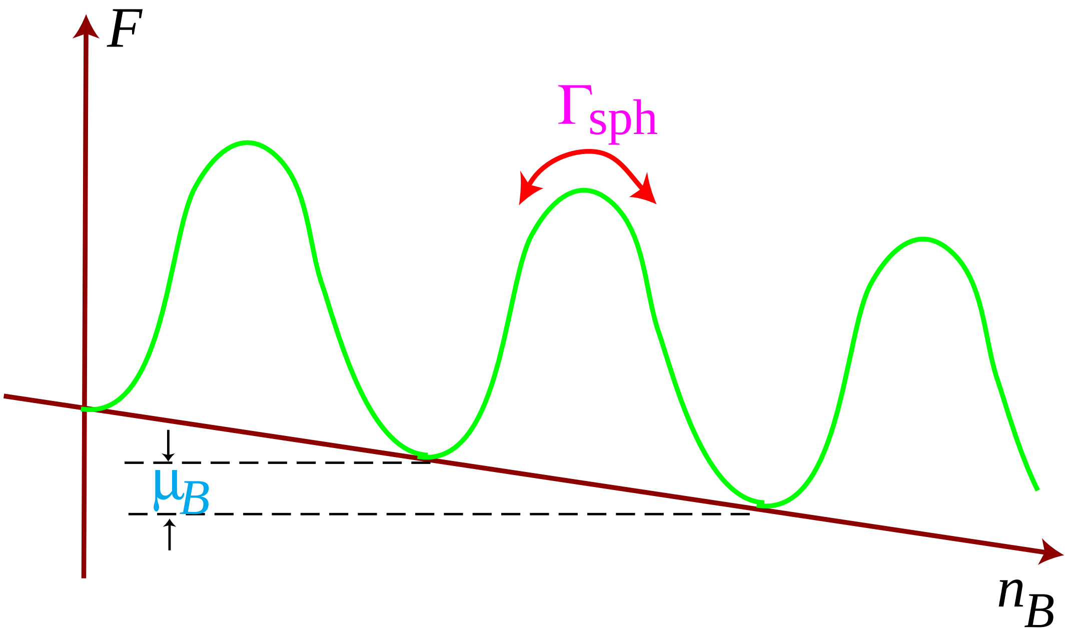
⇓

change of distribution functions by the chiral charge with the sphaleron process in equilibrium

⟸ Boltzmann equations

bubble wall velocity $\simeq$ const. $\Rightarrow$ constant chiral charge flux

$\Longrightarrow$ bias on free energy along B [stationary nonequilibrium]



$$\dot{n}_B \simeq -\frac{\mu_B \Gamma_{\rm sph}}{T}$$

⋆ Derivation of the B-changing rate

$P(i;t)$ = probability to find the system in state i at t

$\Gamma_{i\to j}$ = transition prob. for $i \to j$ per unit time

master equation:

$$P(i;t+\Delta t) - P(i;t) = -\sum_{j\neq i} P(i;t)\Gamma_{i\to j}\Delta t + \sum_{j\neq i} P(j;t)\Gamma_{j\to i}\Delta t$$

steady state: $P(i;t) \to P_{\rm eq}(B) \Rightarrow$ detailed balance

$$\sum_{n=1}^{\infty} P_{\rm eq}(B)\left(\Gamma_{B\to B+n} + \Gamma_{B\to B-n}\right)$$

$$= \sum_{n=1}^{\infty}\left[P_{\rm eq}(B+n)\Gamma_{B+n\to B} + P_{\rm eq}(B-n)\Gamma_{B-n\to B}\right],$$

$$\Gamma_{B\to B+n} \simeq \Gamma_+^n, \qquad \Gamma_{B\to B-n} \simeq \Gamma_-^n$$

$$P_{\rm eq}(B+n) \propto e^{-F_{B+n}/T} = e^{-(F_B + n\mu_B)/T}$$

Since $\Gamma_\pm \ll 1$, this reduces to

$$\Gamma_+ + \Gamma_- \simeq {\rm e}^{-\mu_B/T}\Gamma_- + {\rm e}^{\mu_B/T}\Gamma_+ \Rightarrow \frac{\Gamma_+}{\Gamma_-} \simeq {\rm e}^{-\mu_B/T}$$

$\Gamma_\pm$ = rate per unit volume $\Longrightarrow \dot{n}_B \equiv \Gamma_+ - \Gamma_-$

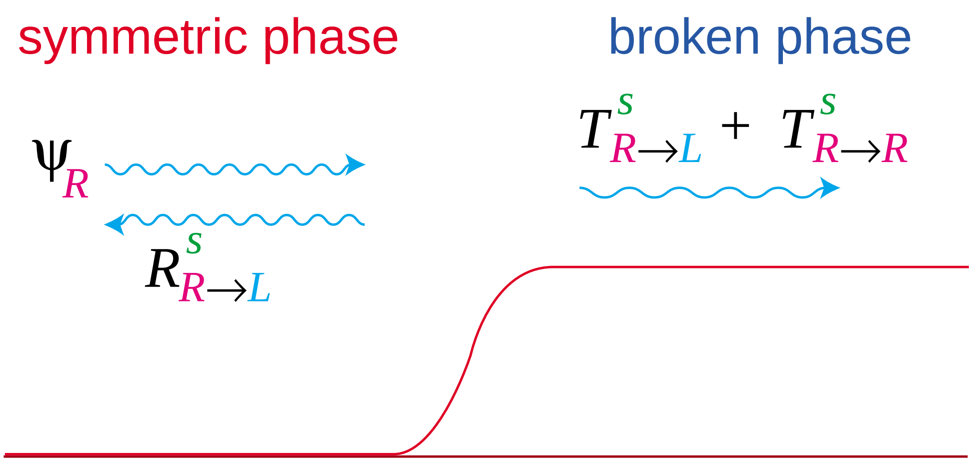
$\Gamma_+ \sim \Gamma_- \simeq \Gamma_{\rm sph}$

$$\dot{n}_B = \Gamma_-\left(\frac{\Gamma_+}{\Gamma_-} - 1\right) \simeq \Gamma_{\rm sph}({\rm e}^{-\mu_B/T} - 1) \simeq -\frac{\Gamma_{\rm sph}\mu_B}{T}$$

fermion scattering-off CP-violating bubble wall

$$i\partial\!\!\!/\psi(x) - m(x)P_R\,\psi(x) - m^*(x)P_L\,\psi(x) = 0$$

where $\quad -f\langle\phi(x)\rangle = m(x) \in \mathbf{C}$ $\quad$ through the Yukawa int.



$Q^i_{L(R)}$: charge of a L(R)-handed fermion of species i

$R^s{}_{R\to L}$: reflection coeff. for the R-handed fermion incident from the symmetric phase region

$\bar{R}^s{}_{R\to L}$: the same as above for the R-handed antifermion

⟨injected charge into symmetric phase⟩ brought by the fermions and antifermions in the symmetric phase :

$$\Delta Q_i{}^s$$
$$= [(Q_R{}^i - Q_L{}^i)R^s{}_{L\to R} + (-Q_L{}^i + Q_R{}^i)\bar{R}^s{}_{R\to L}$$
$$+(-Q_L{}^i)(T^s{}_{L\to L} + T^s{}_{L\to R}) - (-Q_R{}^i)(\bar{T}^s{}_{R\to L} + \bar{T}^s{}_{R\to R})]f^s{}_{Li}$$
$$+[(Q_L{}^i - Q_R{}^i)R^s{}_{R\to L} + (-Q_R{}^i + Q_L{}^i)\bar{R}^s{}_{L\to R}$$
$$+(-Q_R{}^i)(T^s{}_{R\to L} + T^s{}_{R\to R}) - (-Q_L{}^i)(\bar{T}^s{}_{L\to L} + \bar{T}^s{}_{L\to R})]f^s{}_{Ri}$$

the same brought by transmission from the broken phase :

$$\begin{aligned}\Delta Q_i{}^b &= Q_L{}^i ({T^b}_{L\to L} {f^b}_{Li} + {T^b}_{R\to L} {f^b}_{Ri}) \\ &+ Q_R{}^i ({T^b}_{L\to R} {f^b}_{Li} + {T^b}_{R\to R} {f^b}_{Ri}) \\ &+ (-Q_L{}^i)({\bar{T}^b}_{R\to L} {f^b}_{Li} + {\bar{T}^b}_{L\to L} {f^b}_{Ri}) \\ &+ (-Q_R{}^i)({\bar{T}^b}_{R\to R} {f^b}_{Li} + {\bar{T}^b}_{L\to R} {f^b}_{Ri})\end{aligned}$$

by use of

unitarity: $\quad {R^s}_{L\to R} + {T^s}_{L\to L} + {T^s}_{L\to R} = 1, \quad$ etc.

reciprocity: $\quad {T^s}_{R\to L} + {T^s}_{R\to R} = {T^b}_{L\to L} + {T^b}_{R\to L}, \quad$ etc.

$$f_{iL}^{s(b)} = f_{iR}^{s(b)} \equiv f_i^{s(b)}$$

we obatin

$$\boxed{\Delta Q^s{}_i + \Delta Q^b{}_i = (Q_L{}^i - Q_R{}^i)(f^s{}_i - f^b{}_i)\Delta R}$$

where

$$\boxed{\Delta R \equiv {R^s}_{R\to L} - {\bar{R}^s}_{R\to L}}$$

which depends on

- profile of the bubble wall

  wall thickness, height
  CP phase

- momentum of the incident particle

total flux injected into the *symmetric phase* region

$$F^i_Q = \frac{{Q_L}^i - {Q_R}^i}{4\pi^2\gamma} \int_{m_0}^{\infty} dp_L \int_0^{\infty} dp_T\, p_T$$
$$\times \left[{f_i}^s(p_L, p_T) - {f_i}^b(-p_L, p_T) \right] \Delta R(\frac{m_0}{a}, \frac{p_L}{a})$$

where

$$ {f_i}^s(p_L, p_T) = \frac{p_L}{E} \frac{1}{\exp[\gamma(E - v_w p_L)/T] + 1}$$
$$ {f_i}^b(-p_L, p_T) = \frac{p_L}{E} \frac{1}{\exp[\gamma(E + v_w \sqrt{p_L^2 - m_0^2})/T] + 1}$$

the fermion flux densities in the symmetric and broken phases.

m_0 : fermion mass in the broken pase
v_w : wall velocity
p_T : transverse momentum
$1/a$: wall width

$$\gamma = 1/\sqrt{1 - v_w^2}$$
$$E = \sqrt{p_L^2 + p_T^2}$$

available charge :

$Q_L - Q_R \neq 0$

conserved in the symmetric phase

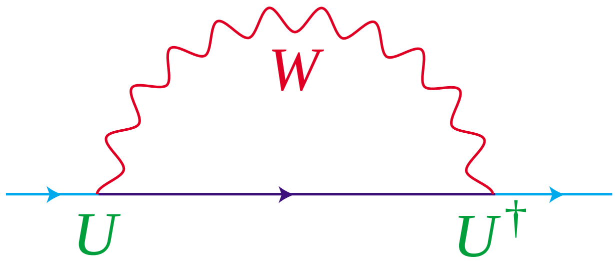
$\Bigg\} \Longrightarrow$ $Y, \quad I_3$

CP violation effective for ΔR

- MSM — KM matrix

  dispersion relation of the fermion $\sim O(\alpha_W)$

  [Farrar and Shaposhnikov, PRD,'94]



— decoherence by QCD effects (short range)

[Gavela, et al., NPB '94]

- CP violation in mass or mass matrix

  tree-level quantum scattering by the bubble wall

  relative phase of 2 Higgs doublets

  $\Rightarrow m(x) = -g\,|\phi(x)|\,e^{i\theta(x)}$

  relative phases of the complex parameters in the MSSM (Minimal SUSY SM)
  $\Rightarrow$ mass matrices of chargino, neutralino, sfermions

change of the state by the injection of the flux

Assume

- bubble is macroscopic and expand with const. velocity
- deep in the sym. phase, elementary processes are fast enough to realize a new stationary state
- the sphaleron process is out of equilibrium near the bubble wall

$\Longrightarrow$ chemical potential argument

μ_B in terms of the injected Y

charged-current interection :

$$\mu_W = \mu_0 + \mu_- = -\mu_{t_L} + \mu_{b_L} = -\mu_{\nu_\tau} + \mu_{\tau_L}$$

Yukawa interaction :

$$\mu_0 = -\mu_{t_L} + \mu_{t_R} = -\mu_{b_L} + \mu_{b_R} = -\mu_{\tau_L} + \mu_{\tau_R}$$

no further independent relations

chem. potentials of conserved and almost conserved quantum numbers :

$$\mu_{B-L}, \quad \mu_Y, \quad \mu_{I_3}; \quad \mu_B$$

$e.g.$, considering only the 3rd generation,

$$\begin{aligned}
\mu_{t_L(b_L)} &= \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} + \frac{1}{6}\mu_Y + (-)\frac{1}{2}\mu_{I_3}, \\
\mu_{t_R} &= \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} + \frac{2}{3}\mu_Y \\
\mu_{b_R} &= \frac{1}{3}\mu_B + \frac{1}{3}\mu_{B-L} - \frac{1}{3}\mu_Y \\
\mu_{\tau_L(\nu_\tau)} &= -\mu_{B-L} - \frac{1}{2}\mu_Y + (-)\frac{1}{2}\mu_{I_3} \\
\mu_{0(-)} &= +(-)\frac{1}{2}\mu_Y - \frac{1}{2}\mu_{I_3} \\
\mu_W &= -\mu_{I_3}
\end{aligned}$$

baryon and lepton number densities:

$$\begin{aligned}
n_B &= 3 \cdot \frac{1}{3} \cdot \frac{T^2}{6}(\mu_{t_L} + \mu_{t_R} + \mu_{b_L} + \mu_{b_R}) \\
&= \frac{T^2}{9}(2\mu_B + 2\mu_{B-L} + \mu_Y) \\
n_L &= \frac{T^2}{6}(\mu_{\nu_\tau} + \mu_{\tau_L} + \mu_{\tau_R}) = \frac{T^2}{6}(-3\mu_{B-L} - 2\mu_Y)
\end{aligned}$$

If $n_B = n_L = 0$ before the injection of the hypercharge flux,

$$\mu_{B-L} = -\frac{2}{3}\mu_Y, \qquad \mu_B = \frac{1}{6}\mu_Y$$

hypercharge density:

$$\frac{Y}{2} = \frac{T^2}{6}\left\{3\left[\frac{1}{6}(\mu_{t_L}+\mu_{b_L})+\frac{2}{3}\mu_{t_R}-\frac{1}{3}\mu_{b_R})\right]\right.$$
$$\left.-\frac{1}{2}(\mu_{\nu_\tau}+\mu_{\tau_L})-\mu_{\tau_R}\right\}+\frac{T^2}{3}\frac{1}{2}(\mu_0-\mu_-)m$$
$$= \frac{T^2}{6}(m+\frac{5}{3})\mu_Y \qquad [m = \text{\#(Higgs doublets)}]$$

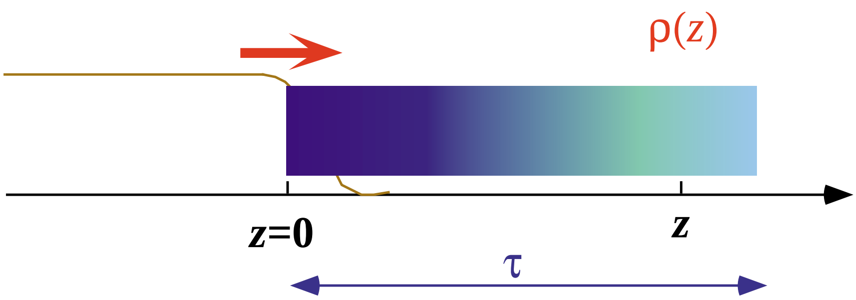
$$\therefore \quad \mu_B = \frac{Y}{2(m+5/3)T^2}$$

Integrating the equation for $\dot{n}_B$,

$$n_B = -\frac{\Gamma_{\rm sph}}{T}\int dt\,\mu_B = -\frac{\Gamma_{\rm sph}}{2(m+5/3)T^3}\int dt\,Y$$

where

$$\int dt\,Y = \int_{-\infty}^{z/v_w} dt\,\rho_Y(z-v_w t) = \frac{1}{v_w}\int_0^\infty dz\,\rho_Y(z).$$



$$\frac{1}{v_w}\int_0^\infty dz\,\rho_Y(z) \simeq \frac{F_Y\tau}{v_w}$$

τ = transport time within which the scattered fermions are captured by the wall generated BAU :

$$\frac{n_B}{s} \simeq \mathcal{N} \frac{100}{\pi^2 g_*} \cdot \kappa \alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T$$

$$\mathcal{N} \sim O(1)$$

$$\tau T \simeq \begin{cases} 1 & \text{for quarks} \\ 10^2 \sim 10^3 & \text{for leptons} \end{cases}$$

MC simulation $\Longrightarrow$ forward scattering enhanced :

for top quark

$\tau T \simeq 10 \sim 10^3$ max. at $v_w \simeq 1/\sqrt{3}$

for this optimal case [top quark]

$$\frac{n_B}{s} \simeq 10^{-3} \cdot \frac{F_Y}{v_w T^3}$$

$\Longrightarrow$ $F_Y/(v_w T^3) \sim O(10^{-7})$ would be sufficient to explain the BAU.

⋆ **Example** [Nelson et al., NPB, '92]

$$m(z) = m_0 \frac{1+\tanh(az)}{2} \exp\left(-i\pi \frac{1-\tanh(az)}{2} \right)$$

— no CP violation in the broken phase [$z \sim \infty$]

- Calculation of $\Delta R \longrightarrow$ chiral charge flux

  (i) perturbative method [FKOTT, PRD,'94]

  (ii) numerical method [CKN, NPB '92, FKOT, PTP,'96]

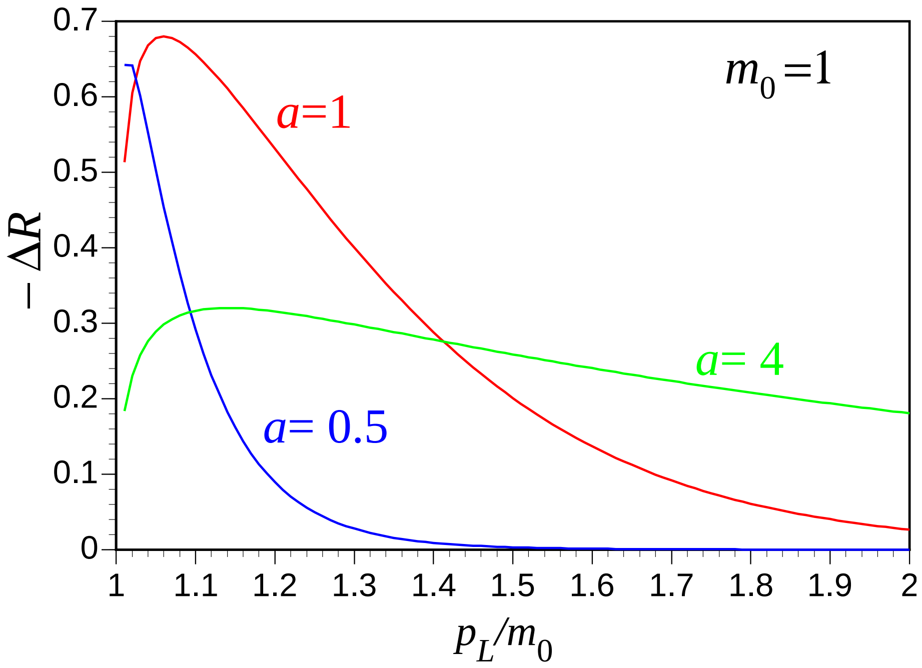
- $\Delta R \equiv {R^s}_{R\to L} - {\bar{R}^s}_{R\to L}$

  wall width $\simeq$ wave length of the carrier $\Rightarrow \Delta R \sim O(1)$

  $\Downarrow$

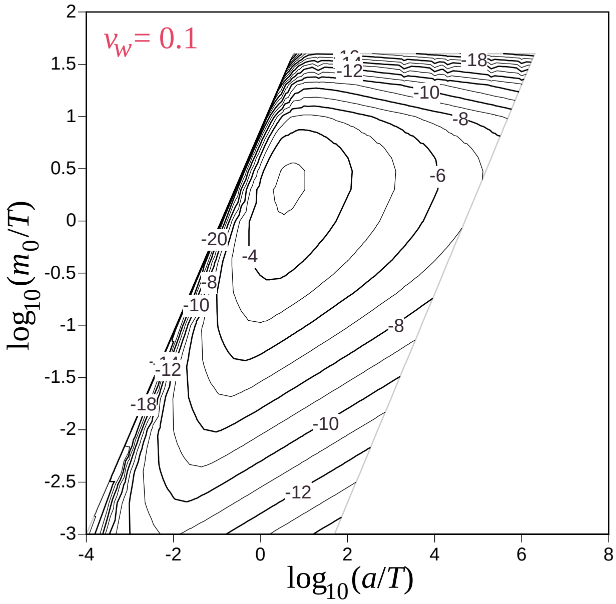
  stronger Yukawa coupling does *not* always implies larger flux

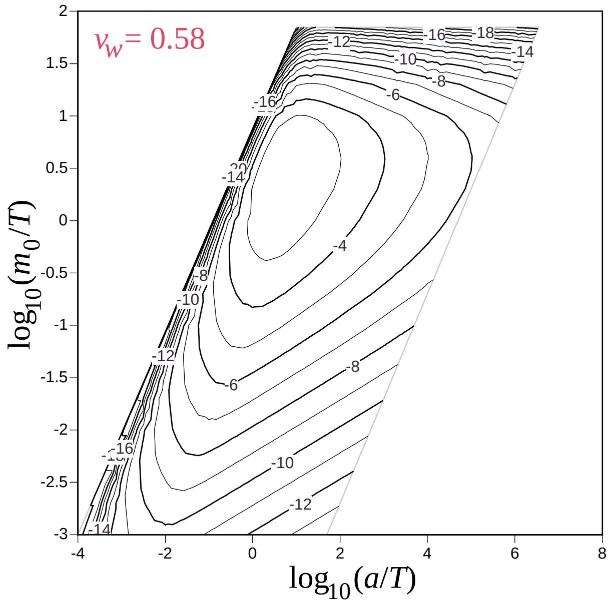
  for larger energy, ΔR decays exponentially

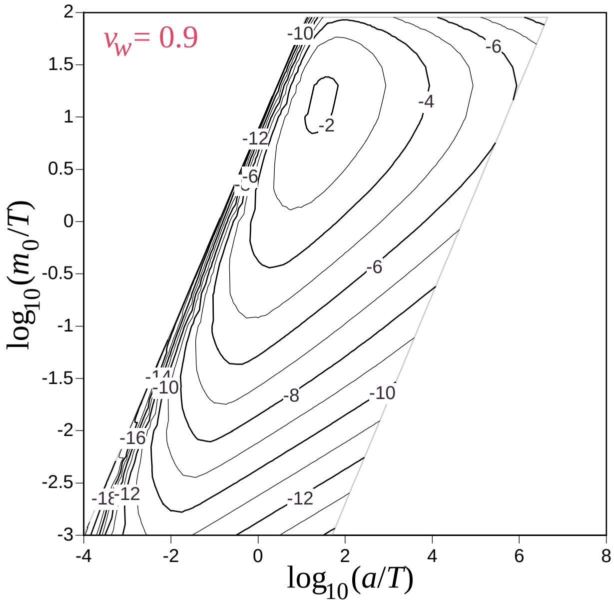


- chiral charge flux $\qquad T = 100$ GeV

normalized as $\dfrac{F_Q}{T^3(Q_L - Q_R)}$ [dimensionless]







$$\frac{n_B}{s} \simeq \mathcal{N} \frac{100}{\pi^2 g_*} \cdot \kappa \alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T$$

$$\simeq 10^{-3} \cdot \frac{F_Y}{v_w T^3}$$ for an optimal case (top quark)

⋆ Spontaneous baryogenesis

(i) in two-Higgs-doublet model [at $T = 0$]

$$\Delta\mathcal{L}_{\text{eff}} = -\frac{g^2 N_f}{24\pi^2}\theta(x)F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

—CP-even $\Leftarrow$ $\theta(x)$, $F\tilde{F}$: CP-odd

$\Longrightarrow$ $\dot{\theta} \sim$ chem.pot. for N_{CS}

At high-T, suppressed by $\left(\frac{m_t}{T}\right)^2$.

(ii) bias for the hypercharge instead of N_{CS} [CKN, PLB,'91]

neutral comp. of 2 Higgs scalars :

$$\phi_j^0(x) = \rho_j(x)\mathrm{e}^{i\theta_j}, \qquad (j = 1, 2)$$

Suppose only ϕ_1 couples to the fermions.

Eliminate θ_1 in Yukawa int. by anomaly-free $U(1)_Y$ trf.

fermion kinetic term induces:

$$2\partial_\mu\theta_1(x)\left[\frac{1}{6}\bar{q}_L(x)\gamma^\mu q_L(x) + \frac{2}{3}\bar{u}_R(x)\gamma^\mu u_R(x) - \frac{1}{3}\bar{d}_R(x)\gamma^\mu d_R(x) - \frac{1}{2}\bar{l}_L(x)\gamma^\mu l_L(x) - \bar{e}_R(x)\gamma^\mu e_R(x)\right]$$

$\langle\dot{\theta}_1\rangle \neq 0$ during EWPT $\Rightarrow$ *charge potential*

⋆ criticism by Dine-Thomas [PLB,'94]

▷ The current is not the conserved Y-current, but the fermionic part of it.
Nonconservation of Y in the broken phase leads to

$$\partial_\mu \theta_1 \cdot j_Y^\mu \propto \frac{{m_t}^2}{T^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

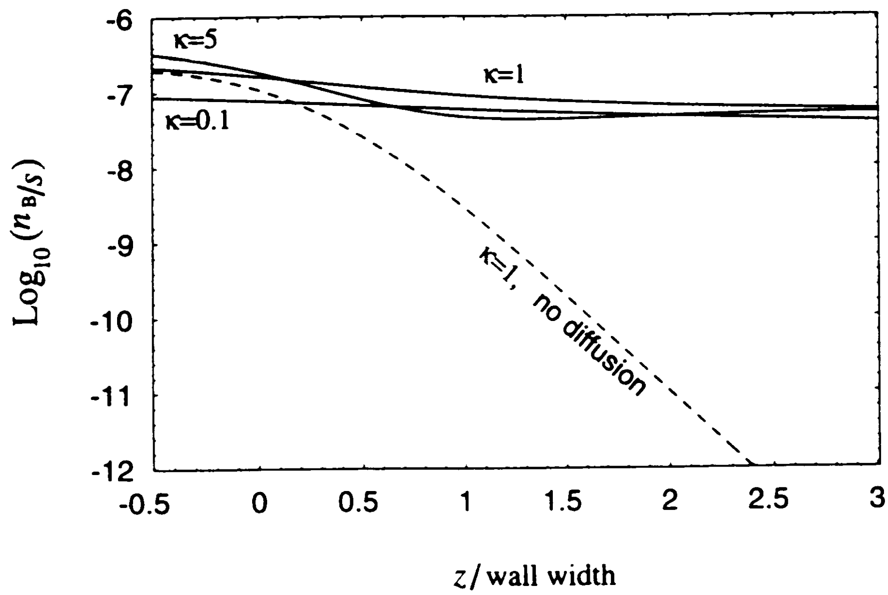
▷ The bias for Y exists where $(v/T)^2 > 0$.
The sphaleron process is effective for $v < v_{co}$
∴ The generated B is suppressed by $v_{co}/T^2 \sim O(10^{-6})$.

⋆ enhancement by diffusion [CKN, PLB,'94]

Diffusion carries Y into the symmetric phase.
⟶ nonlocal baryogenesis

for $\langle \phi(z) \rangle = v \frac{1-\tanh(az)}{2} \exp\left[-i \frac{\pi}{2} \frac{1-\tanh(az)}{2} \right]$

z_{co} vs $\log_{10}[(n_B/s)(g_*/100)]$ with $v_{co} = \varphi(z_{co})$



— almost independent of z_{co}

5. Baryogenesis in the MSSM

⋆ Minimal Supersymmetric Standard Model

chiral supermultiplet	$SU(3)_C \times SU(2)_L \times U(1)_Y$
$Q_A \ni q_{AL} = \begin{pmatrix} u_{AL} \\ d_{AL} \end{pmatrix}, \tilde{q}_{AL}$	$(3, 2, \frac{1}{6})$
$U_A \ni u^c_{AR}, \tilde{u}^c_{AR}$	$(3^*, 1, -\frac{2}{3})$
$D_A \ni d^c_{AR}, \tilde{d}^c_{AR}$	$(3^*, 1, \frac{1}{3})$
$L_A \ni l_{AL} = \begin{pmatrix} \nu_{AL} \\ e_{AL} \end{pmatrix}, \tilde{l}_{AL}$	$(1, 2, -\frac{1}{2})$
$E_A \ni e^c_{AR}, \tilde{e}^c_{AR}$	$(1, 1, 1)$
$H_d \ni \Phi_d = \begin{pmatrix} \phi^0_d \\ \phi^-_d \end{pmatrix}, \tilde{\Phi}_d$	$(1, 2, -\frac{1}{2})$
$H_u \ni \Phi_u = \begin{pmatrix} \phi^+_u \\ \phi^0_u \end{pmatrix}, \tilde{\Phi}_u$	$(1, 2, \frac{1}{2})$

vector supermultiplet	$SU(3)_C \times SU(2)_L \times U(1)_Y$
$V_3 \ni G^s_\mu, \tilde{G}^s$	$(8, 1, 0)$
$V_2 \ni A^a_\mu, \tilde{A}^a$	$(1, 3, 0)$
$V_1 \ni B_\mu, \tilde{B}$	$(1, 1, 0)$

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}$$

$\mathcal{L}_{\text{SUSY}}$:supersymmetric
$\mathcal{L}_{\text{soft}}$:soft-SUSY-breaking
} gauge invariant

$(\text{scalar})^2$, $(\text{scalar})^3$, $(\text{fermion})^2$

superpotential $\Leftarrow$ interaction other than the gauge int.

$$W = \epsilon_{ij}\big(f^{(e)}_{AB}H^i_dL^j_AE_B + f^{(d)}_{AB}H^i_dQ^j_AD_B - f^{(u)}_{AB}H^i_uQ^j_AU_B - \mu H^i_dH^j_u\big)$$

$\star$ new features (relevant to EWB-gensis)

1. more scalar fields $\Rightarrow$ { stronger (first-order) PT; 3-dim. order-parameter space }
2. many complex parameters $\Rightarrow$ explicit CP violation
   μ, A, B, gaugino masses
3. two Higgs doublets $\Rightarrow$ possibility of spontaneous CP viol.

Higgs potential $\Longleftarrow V_D$ & $\mathcal{L}_{\rm soft}$

$$V_0 = m_1^2\Phi_d^\dagger\Phi_d + m_2^2\Phi_u^\dagger\Phi_u + (m_3^2\Phi_u\Phi_d + \text{h.c.}) + \frac{g_2^2+g_1^2}{8}(\Phi_d^\dagger\Phi_d - \Phi_u^\dagger\Phi_u)^2 + \frac{g_2^2}{2}(\Phi_d^\dagger\Phi_d)(\Phi_u^\dagger\Phi_u)$$

vacuum:
$$\begin{cases} \varphi_d = \langle\Phi_d\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix} v_0\cos\beta_0 \\ 0 \end{pmatrix} \\ \varphi_u = \langle\Phi_u\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_0\sin\beta_0 \end{pmatrix} \end{cases}$$
CP symmetric

where

$$m_1^2 = m_3^2\cos\beta_0 - \frac{1}{2}m_Z^2\cos(2\beta_0)$$

$$m_2^2 = m_3^2\sin\beta_0 + \frac{1}{2}m_Z^2\cos(2\beta_0)$$

Higgs mass:

after SSB,

$\Phi_d,\ \Phi_u$ (8) $\Longrightarrow$ 3 neutral & 1 charged scalars $(3+2=8-3)$

$$m_h^2 = \frac{m_Z^2 + m_Z^2 - \sqrt{(m_Z^2 - m_A^2)^2 + 4m_Z^2 m_A^2 \sin^2(2\beta_0)}}{2}$$
$$\leq \min\{m_Z^2,\ m_A^2\},$$
$$m_H^2 = \frac{m_Z^2 + m_Z^2 + \sqrt{(m_Z^2 - m_A^2)^2 + 4m_Z^2 m_A^2 \sin^2(2\beta_0)}}{2}$$
$$\geq \max\{m_Z^2,\ m_A^2\},$$
$$m_A^2 = \frac{m_3^2}{\sin\beta_0 \cos\beta_0}$$

PDG2000: $m_h \geq 82.6\text{GeV}$, $m_A \geq 84.1\text{GeV}$

radiative corrections are significant [Okada et al. PLB '91]

mass eigenstates (after SSB)

charged Higgsino, Wino $\Longrightarrow$ chargino $\chi^{\pm}_{1,2}$

neutral Higgsino, Bino, Zino $\Longrightarrow$ neutralino $\chi^0_{1,2,3,4}$

L-squark (slepton), R-squark (slepton) $\Longrightarrow$ $\tilde{q}_{1,2}$ $(\tilde{l}_{1,2})$

⋆ Sphaleron

- 2-doublet Higgs model [Peccei, Zhang, Kastening, PLB '91]
- squarks vs sphaleron [Moreno, Oakini, Quirós, PLB '97]

⋆ Electroweak phase transition

3 order parametres:

$$\varphi_d = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \varphi_u = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 + i v_3 \end{pmatrix}$$

$v_3 \neq 0 \longrightarrow$ CP violation

light stop [de Carlos & Espinosa, NPB '97]

stop mass-squared matrix :

$$\begin{pmatrix} m^2_{\tilde{t}_L} + \left(\frac{g_1^2}{24} - \frac{g_2^2}{8}\right)(v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 & \frac{y_t}{\sqrt{2}}\left(\mu v_d + A(v_2 - i v_3)\right) \\ * & m^2_{\tilde{t}_R} - \frac{g_1^2}{6}(v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 \end{pmatrix}$$

$m^2_{\tilde{t}_L} = 0$ or $m^2_{\tilde{t}_L} = 0 \Longrightarrow$ smaller eigenvalue: $m^2_{\tilde{t}_1} \sim O(v^2)$

$\therefore$ high-T expansion

$$\bar{V}_{\tilde{t}}(\boldsymbol{v}; T) \Rightarrow -\frac{T}{6\pi}(m_-^2)^{3/2}$$

$\longrightarrow$ stronger 1st order PT

One-loop effective potential

[K.F., PTP, '99]

$$V_0 = m_1^2\varphi_d^\dagger\varphi_d + m_2^2\varphi_u^\dagger\varphi_u + (m_3^2\varphi_u\varphi_d + \text{h.c.}) + \frac{g_2^2+g_1^2}{8}(\varphi_d^\dagger\varphi_d - \varphi_u^\dagger\varphi_u)^2 + \frac{g_2^2}{2}(\varphi_d^\dagger\varphi_d)(\varphi_u^\dagger\varphi_u)$$

m_3^2: real positive

$$\begin{aligned} V_{\text{eff}}(\boldsymbol{v}; T=0) &= V_0(\boldsymbol{v}) + 6F\left(m_W^2(\boldsymbol{v})\right) + 3F\left(m_Z^2(\boldsymbol{v})\right) \\ &\quad -12\cdot F\left(m_t^2(\boldsymbol{v})\right) + 2\cdot 3\cdot \sum_{a=1,2} F\left(m_{\tilde{t}_a}^2(\boldsymbol{v})\right) \\ &\quad -4\sum_{a=1,2} F\left(m_{\chi_a^\pm}^2(\boldsymbol{v})\right) - 2\sum_{a=1,2,3,4} F\left(m_{\chi_a^0}^2(\boldsymbol{v})\right) \end{aligned}$$

where

$$F(m^2) \equiv \frac{m^4}{64\pi^2}\left(\log\frac{m^2}{M_{\text{ren}}^2} - \frac{3}{2}\right)$$

$$m_W^2 = \frac{g_2^2}{4}(v_1^2+v_2^2+v_3^2) \qquad m_Z^2 = \frac{g_2^2+g_1^2}{4}(v_1^2+v_2^2+v_3^2)$$

$$m_t^2 = \frac{y_t^2}{2}(v_2^2+v_3^2)$$

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & -\frac{i}{\sqrt{2}}g_2(v_2-iv_3) \\ -\frac{i}{\sqrt{2}}g_2 v_1 & -\mu \end{pmatrix}$$

$$M_{\chi^0} = \begin{pmatrix} M_2 & 0 & -\frac{i}{2}g_2v_1 & \frac{i}{2}g_2(v_2-iv_3) \\ 0 & M_1 & \frac{i}{2}g_1v_1 & -\frac{i}{2}g_1(v_2-iv_3) \\ -\frac{i}{2}g_2v_1 & \frac{i}{2}g_1v_1 & 0 & \mu \\ \frac{i}{2}g_2(v_2-iv_3) & -\frac{i}{2}g_1(v_2-iv_3) & \mu & 0 \end{pmatrix}$$

input:

$$v_0 = |\boldsymbol{v}| = 246\text{GeV},\ \tan\beta = \frac{\sqrt{v_2^2 + v_3^2}}{v_1}$$

$$\longrightarrow y_t = \sqrt{2} m_t / (v_0 \sin\beta)$$

M_1, M_2, $m^2_{\tilde{t}_L}$, $m^2_{\tilde{t}_R}$, m_3^2: soft-SUSY-br. parameters

$$m_1^2,\ m_2^2 \Longleftarrow \left.\frac{\partial V_{\text{eff}}}{\partial v_1}\right|_{\boldsymbol{v}} = 0, \quad \left.\frac{\partial V_{\text{eff}}}{\partial v_2}\right|_{\boldsymbol{v}} = 0$$

output:

masses of the neutral Higgs scalars

$$\Longleftarrow \text{eigenvalues of } \left.\frac{\partial^2 V_{\text{eff}}(\boldsymbol{v}; T=0)}{\partial v_i \partial v_j}\right|_{\boldsymbol{v}}$$

$m_{\tilde{t}_{1,2}}$, $m_{\chi^\pm_{1,2}}$, $m_{\chi^0_{1-4}}$

$m_{\tilde{t}_1} > 86.4\text{GeV}$,
$m_{\chi_1^0} > 32.5\text{GeV}$, $m_{\chi_1^\pm} > 67.7\text{GeV}$ for $\tan\beta > 0.7$

when $^\exists$explicit CP violation $(\mu, M_2, M_1, A_t \in \mathbf{C})$

θ = relative phase of the 2 Higgs = $\text{Arg}(v_2 + i v_3)$

$T \neq 0$

$v(T) = |\boldsymbol{v}(T)|$, $\tan\beta(T)$, $\theta(T)$

T_C : transition temperature

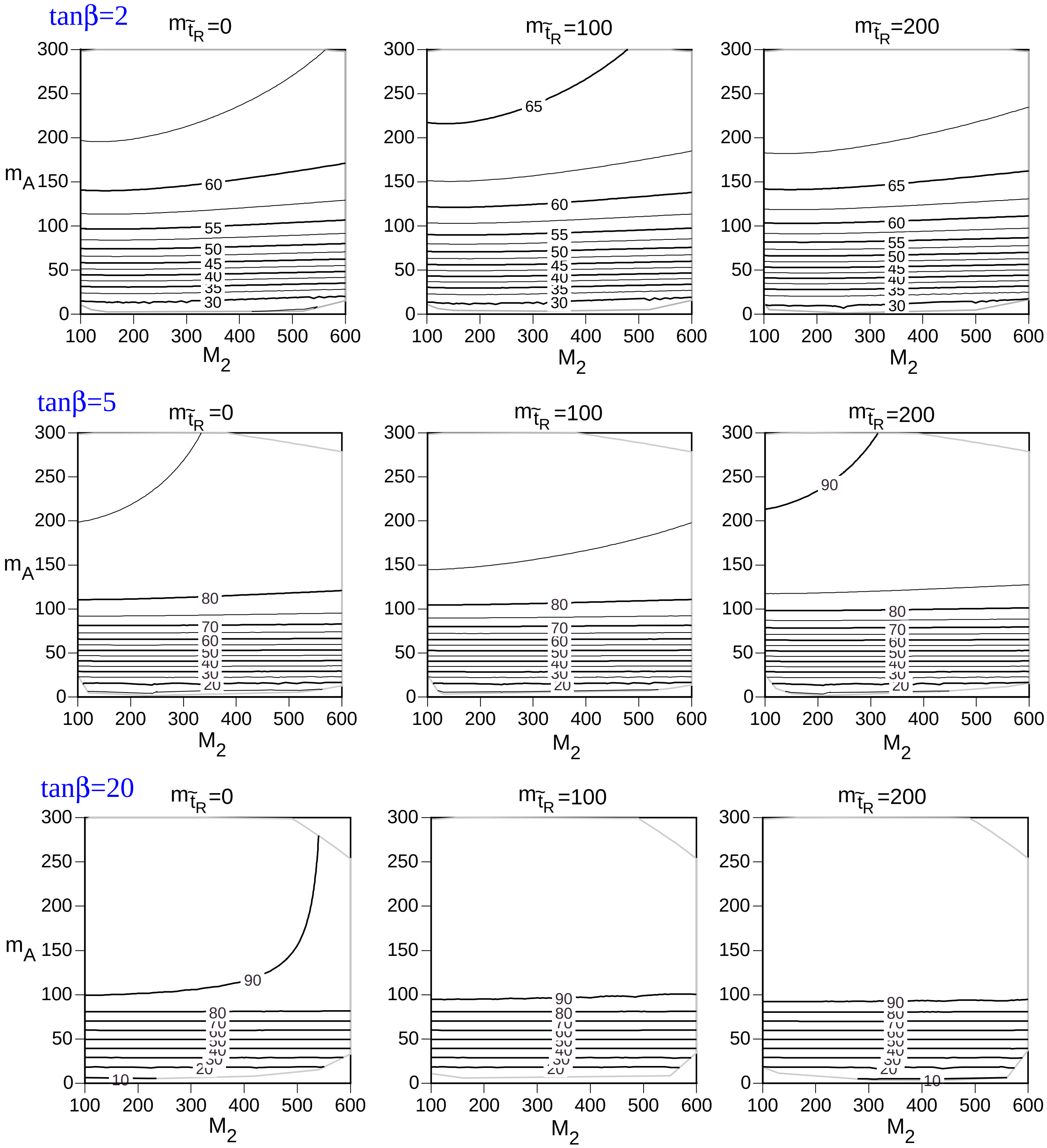
$\Longrightarrow$ crucial to estimate the BAU

numerical results $M_2 = M_1$

$m_t = 175$ GeV $m_{\tilde{t}_L} = 400$ GeV $\mu = -300$ GeV $A_t = 10$ GeV

without CP violation

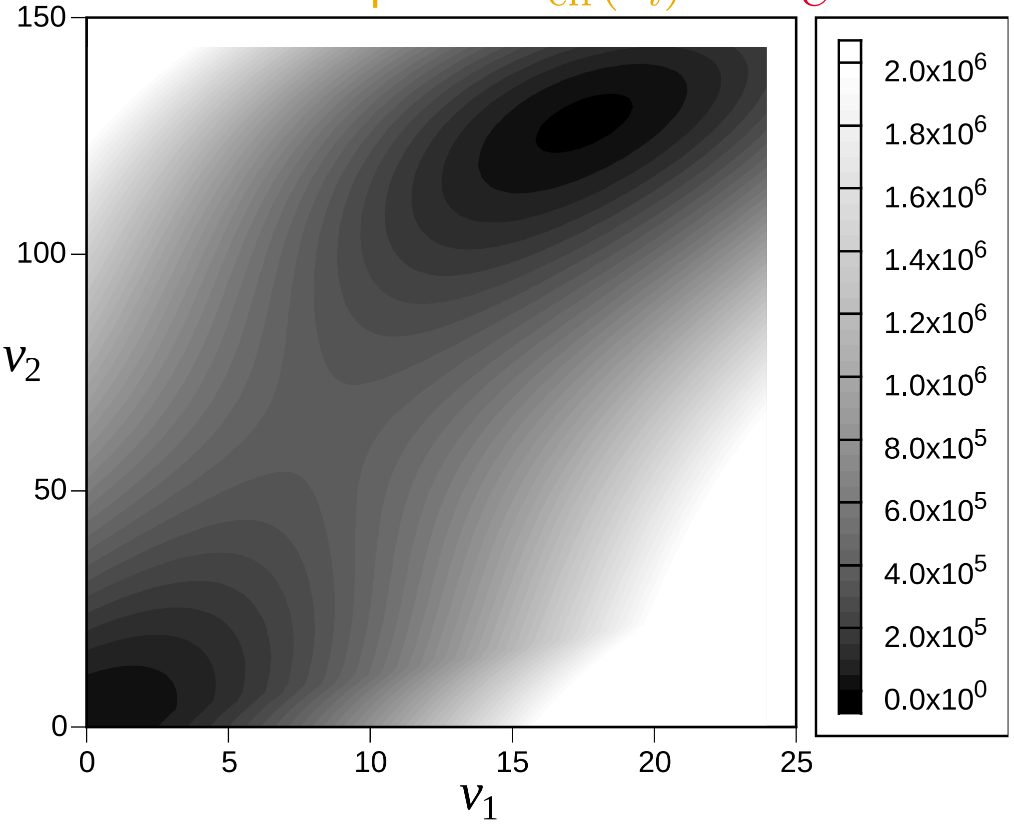
the lighter Higgs scalar mass : m_h (GeV)



at $T \neq 0$

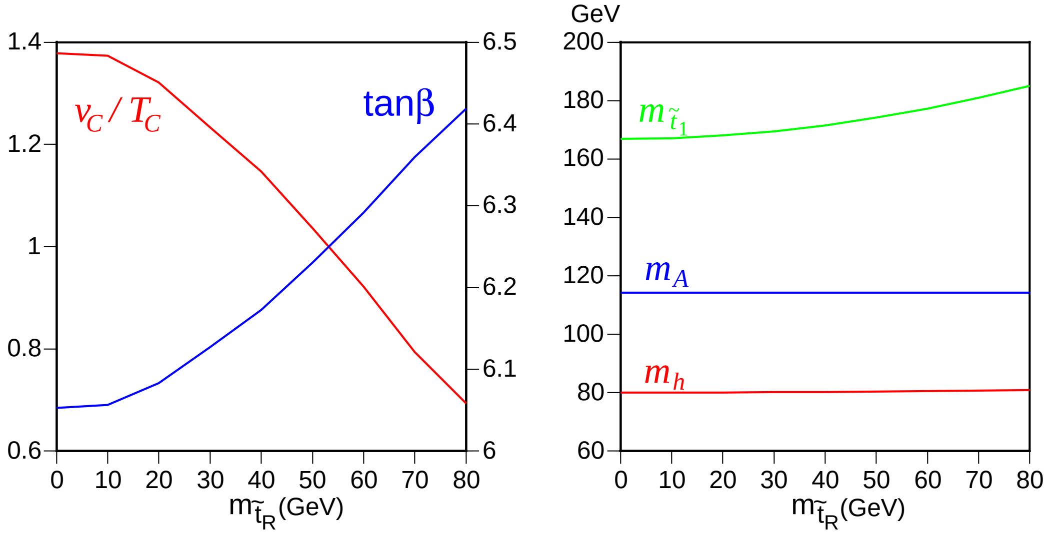
$$\left.\begin{array}{r} m_{\tilde{t}_1} \lesssim m_t \\ m_h \lesssim 100\text{GeV} \end{array}\right\} \Longrightarrow \frac{v_C}{T_C} > 1$$

an example of $V_{\rm eff}(v_i)$ at T_C



$\tan\beta = 6$, $m_h = 82.3\text{GeV}$, $m_A = 118\text{GeV}$, $m_{\tilde{t}_1} = 168\text{GeV}$

$T_C = 93.4\text{GeV}$, $v_C = 129\text{GeV}$



$\tan\beta = 5$, $m_3^2 = 4326\text{GeV}$

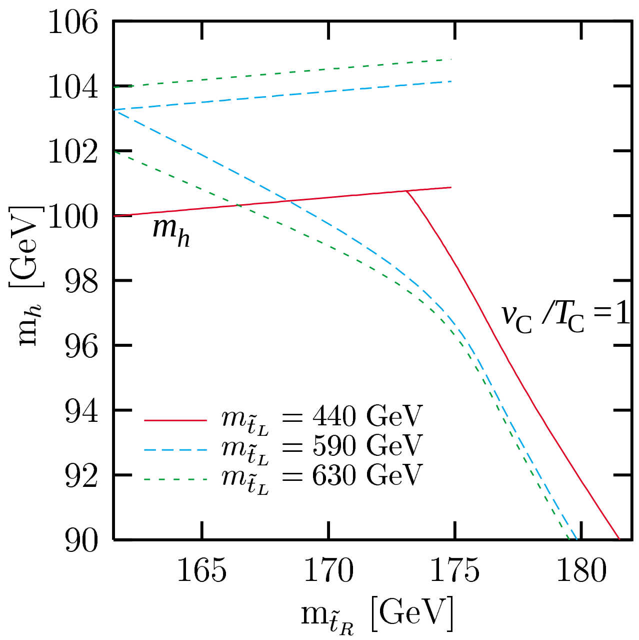
⋆ Lattice MC studies

- 3d reduced model [Laine et al. hep-lat/9809045]
  strong 1st order for $m_{\tilde{t}_1} \lesssim m_t$ and $m_h \leq 110\text{GeV}$

- 4d model [Csikor, et al. hep-lat/0001087]
  with $SU(3)$, $SU(2)$ gauge bosons, 2 Higgs doublets,
  L & R-stops, sbottoms

  no scalar trilinear (A) terms, $\tan\beta \simeq 6$

⟶ agreement with the perturbation theory within the errors

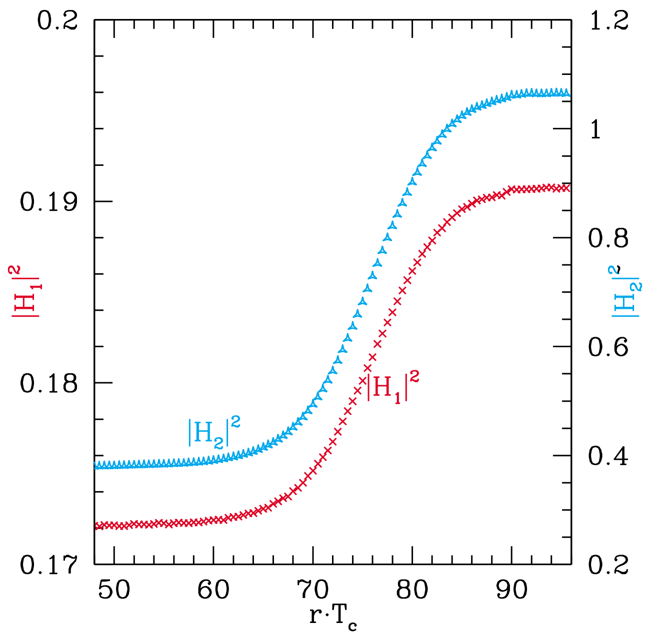


$m_A = 500$ GeV

$v_C/T_C > 1$
below the steeper lines

⇓

max. $m_h = 103 \pm 4$ GeV
for $m_{\tilde{t}_L} \simeq 560$ GeV



bubble-wall profile

$\Delta\beta = 0.0061 \pm 0.0003$

$\Rightarrow$ $\beta \simeq$ const.

wall width $\simeq \dfrac{11}{T_C}$

⋆ CP violation

⋆ relative phases of μ, M_2, M_1, A_t

chargino, neutralino, stop transport

[Huet & Nelson, PRD '96; Aoki, et al. PTP '97]

⋆ relative phase $\theta = \theta_1 - \theta_2$ of the two Higgs doublets

quarks and leptons $\Longleftarrow$ Yukawa coupl. $\propto \rho_i e^{i\theta_i}$

chargino, neutralino, stop mass matrix

[Nelson et al. NPB '92; FKOTT, PRD '94, PTP '96]

θ is induced by the loops of SUSY particle.

$\Uparrow \leftarrow \mathrm{Arg}(\mu M_2)$, $\mathrm{Arg}(\mu M_1)$, $\mathrm{Arg}(\mu A_t^*)$

minimum of $V_{\mathrm{eff}}(\rho_i, \theta; T)$

Some of the combinations of

$\delta_\mu = \mathrm{Arg}\mu$, $\delta_A = \mathrm{Arg}A_t$, $\delta_2 = \mathrm{Arg}M_2$, $\delta_1 = \mathrm{Arg}M_1$ and θ

are constrained by experiments.

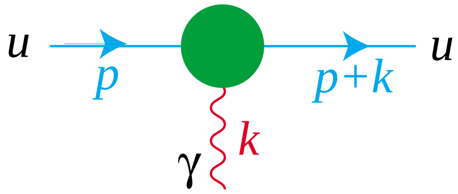
$e.g.$ chargino mass matrix

$$\begin{pmatrix} \tilde{W}^- & \tilde{\phi}_d^- \end{pmatrix} \begin{pmatrix} M_2 & -\frac{i}{\sqrt{2}} g_2 v_2 e^{-i\theta} \\ -\frac{i}{\sqrt{2}} g_2 v_1 & -\mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{\phi}_u^+ \end{pmatrix}$$

$$\xrightarrow{\text{rephasing}} \begin{pmatrix} |M_2| & -\frac{i}{\sqrt{2}} g_2 v_2 \\ -\frac{i}{\sqrt{2}} g_2 v_1 & -|\mu| e^{i(\theta+\delta_\mu+\delta_2)} \end{pmatrix}$$

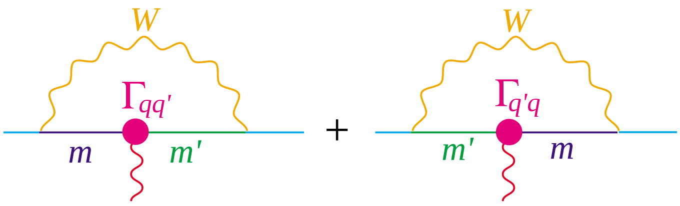
bounds from the EDM

[Kizukuri & Oshimo, PRD '92]

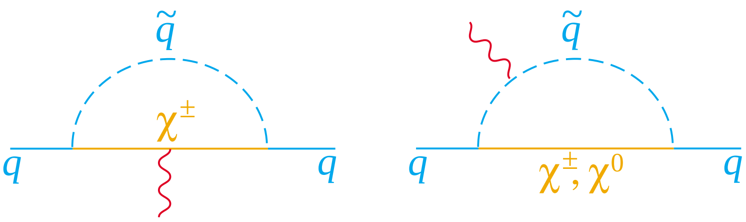
$e\, d_n(k^2)\, \bar{u}\sigma_{\mu\nu}k^\nu\gamma_5\, u\, A^\mu =$ 

present bound: $|d_n| < 0.63 \times 10^{-25} e \cdot \mathrm{cm}$

MSM contribution:

CP-odd of [] $< 10^{-33} e \cdot \mathrm{cm}$

MSSM contribution:



$\cdots$ etc.

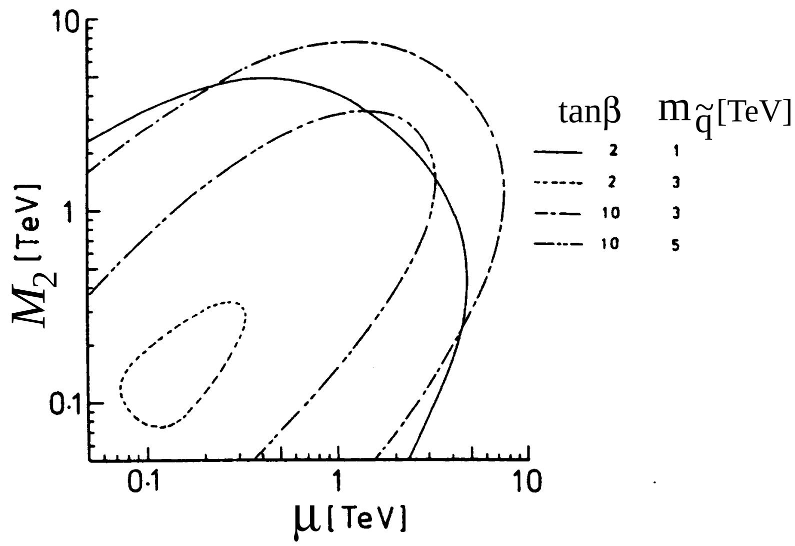


FIG. 4. The parameter regions compatible with the present experimental upper bound on the neutron EDM.

$\theta + \delta_\mu + \delta_2 = \pi/4$
$\mathrm{Arg}A = \pi/4$

inside is excluded

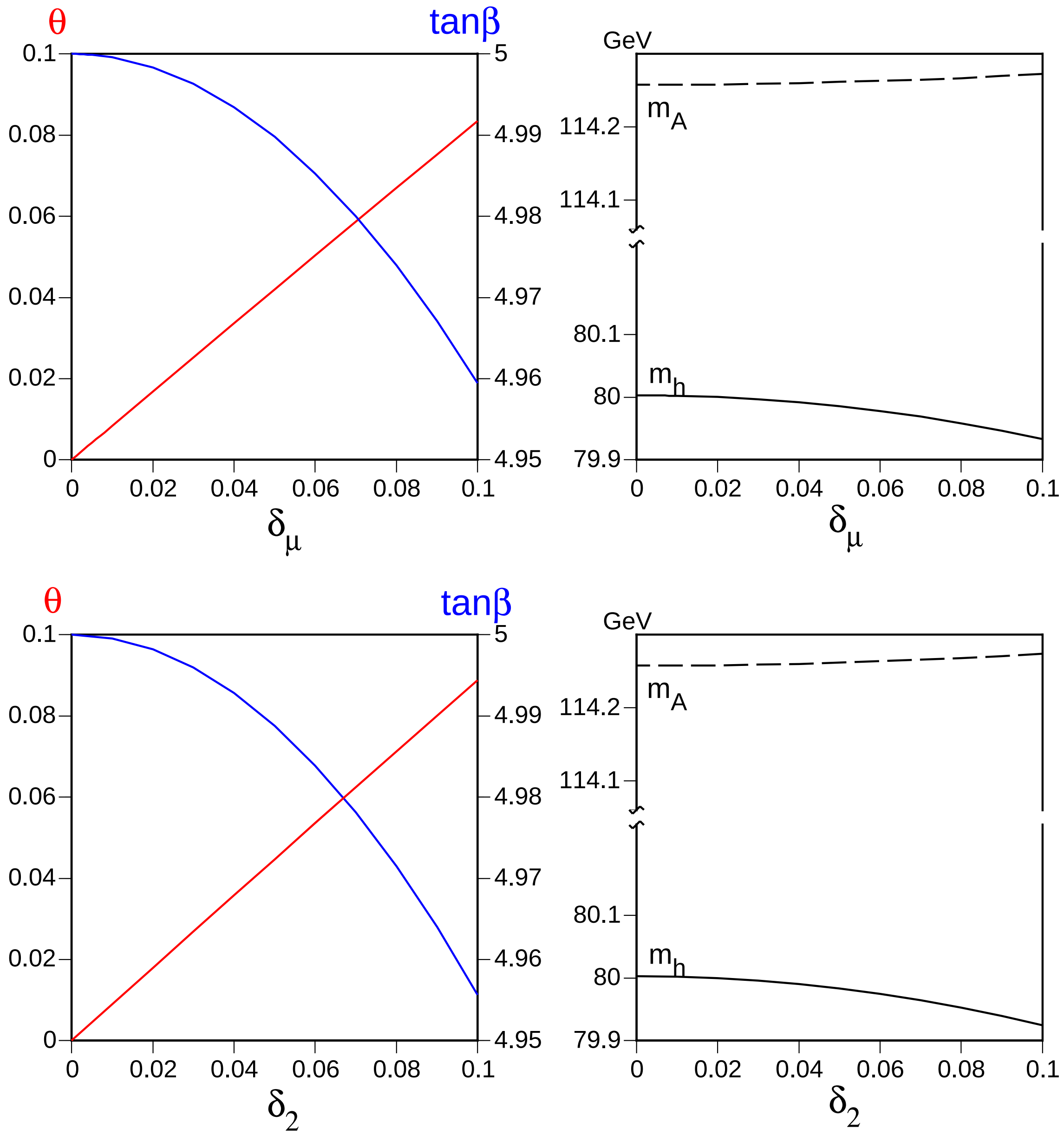
- $\theta + \delta_\mu + \delta_2 = O(1) \Longrightarrow m_{\tilde{q}}, m_{\tilde{l}} \gtrsim 10\mathrm{TeV}$
- $m_{\tilde{q}}, m_{\tilde{l}} \lesssim 1\mathrm{TeV} \Longrightarrow \theta + \delta_\mu + \delta_2 \lesssim 10^{-3}$

effects of $\delta_\mu = \mathrm{Arg}\mu$ and $\delta_2 = \mathrm{Arg}M_2$ on $\theta = \mathrm{Arg}(v_2 + iv_3)$

by minimizing $V_{\rm eff}(\boldsymbol{v}; T = 0)$

$m_3^2 = 4326$ GeV^2 and $\tan\beta = 5$ when $\theta = 0$

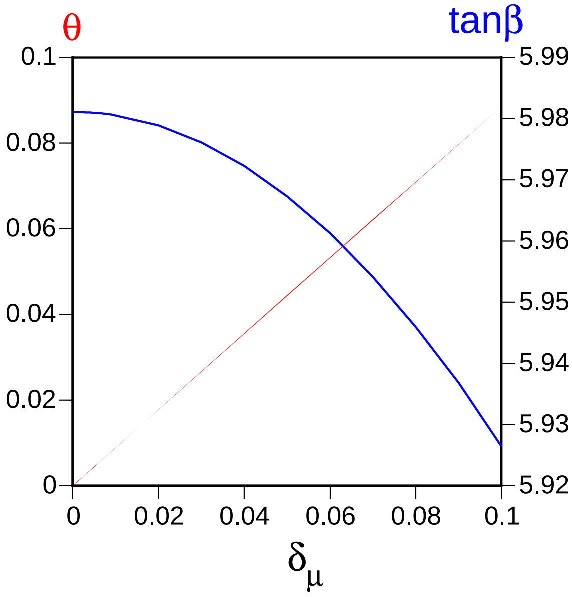
the other parameters are real



⋆ θ is the same order as δ_μ and δ_2

$\Longrightarrow$ more stringent bound on the explicit CP violation

θ at $T = T_C$ vs δ_μ



CP violation relevant to Baryogenesis

— $\theta(x)$ in the bubble wall

Eqs. of motion for $(\rho_i(x), \theta(x))$ with $V_{\rm eff}(\rho_i, \theta; T_C)$

with B.C. determined by the min. of $V_{\rm eff}(T_C)$

$\rho(x) \sim 1 + \tanh(ax)$: 0 (sym. phase) $\longrightarrow v_C$ (br. phase)

bubble wall $\sim$ macroscopic, static $\rightarrow$ 1d system

$$\frac{d^2\rho_i(z)}{dz^2} - \rho_i(z)\left(\frac{d\theta_i(z)}{dz}\right)^2 - \frac{\partial V_{\rm eff}}{\partial \rho_i} = 0,$$

$$\frac{d}{dz}\left(\rho_i^2(z)\frac{d\theta_i(z)}{dz}\right) - \frac{\partial V_{\rm eff}}{\partial \theta_i} = 0$$

with gauge-fixing condition

$$\rho_1^2(z)\frac{d\theta_1(z)}{dz} + \rho_2^2(z)\frac{d\theta_2(z)}{dz} = 0$$

Assume that

(i) $\tan\beta(z)$ be constant.

gauge-fixing $\Longrightarrow \begin{cases} \theta_1(z) = \theta(z)\sin^2\beta \\ \theta_2(z) = -\theta(z)\cos^2\beta \end{cases}$

(ii) V_{eff} can be approximated by a gauge-inv. polynomial of ρ_i up to 4th order

$\rightarrow$ if $\theta \equiv 0$, $\rho_i(z) \sim$ kink solution $\sim \tanh(az)$

$\therefore$ $^{\exists}$nontrivial solution $\mathcal{E} < \mathcal{E}_{\text{kink}} = av^2/3$

$\Uparrow$

minimum or saddle point of V_{eff} at $\theta \neq 0$

energy density per unit area

$$\mathcal{E} = \int_{-\infty}^{\infty} dz \left\{ \frac{1}{2} \sum_{i=1,2} \left[\left(\frac{d\rho_i}{dz}\right)^2 + \rho_i^2 \left(\frac{d\theta_i}{dz}\right)^2 \right] + V_{\text{eff}}(\rho_1, \rho_2, \theta) \right\}$$

Suppose that at $T \simeq T_C$, without explicit CP violation,

$$
\begin{aligned}
& V_{\rm eff}(\rho_i, \theta = \theta_1 - \theta_2) \\
= \ & \frac{1}{2}\bar{m}_1^2\rho_1^2 + \frac{1}{2}\bar{m}_2^2\rho_2^2 - \bar{m}_3^2\rho_1\rho_2\cos\theta + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\
+ \ & \frac{\lambda_3+\lambda_4}{4}\rho_1^2\rho_2^2 + \frac{\lambda_5}{4}\rho_1^2\rho_2^2\cos 2\theta - \frac{1}{2}(\lambda_6\rho_1^2 + \lambda_7\rho_2^2)\rho_1\rho_2\cos\theta \\
- \ & [A\rho_1^3 + \rho_1^2\rho_2(B_0 + B_1\cos\theta + B_2\cos 2\theta) \\
& \qquad + \rho_1\rho_2^2(C_0 + C_1\cos\theta + C_2\cos 2\theta) + D\rho_2^3] \\
= \ & \left[\frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2)\right] \\
& \quad \times \left[\cos\theta - \frac{2\bar{m}_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)}\right]^2 \\
& +\theta\text{-independent terms}
\end{aligned}
$$

where all the parameters are real

conditions for spontaneous CP violation for a given (ρ_1, ρ_2)

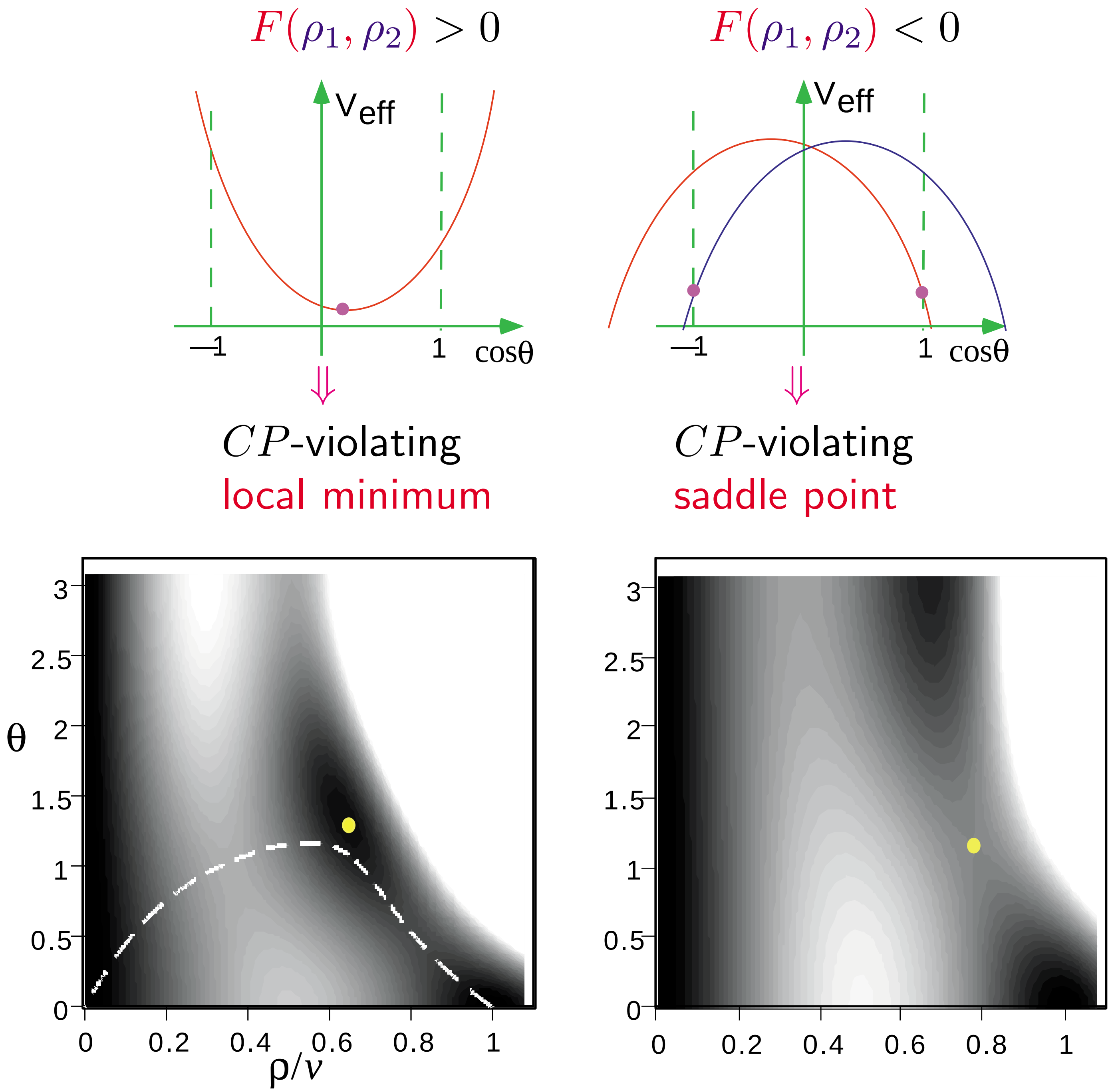
$$
F(\rho_1,\rho_2) \equiv \frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2) > 0,
$$

$$
-1 < G(\rho_1,\rho_2) \equiv \frac{2\bar{m}_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)} < 1
$$

At $T \simeq T_C$, around the EW bubble wall

$$(\rho_1, \rho_2) \ : \ (0,0) \longrightarrow (v_C \cos\beta_C, v_C \sin\beta_C)$$

There may be a chance to satisfy the conditions in the transient region.



Transitional CP Violation

N.B. no explicit CP violation $\Rightarrow$ no net BAU

[FKOT, PTP96 ('96)]

spontaneous CP violation in the transient region

$+$ small explicit CP violation

to resolve degeneracy between CP conjugate bubbles

net BAU

$$\frac{n_B}{s} = \frac{\sum_j \left(\frac{n_B}{s}\right)_j N_j}{\sum_j N_j}$$

with

$$N_j = \exp(-4\pi R_C^2 \mathcal{E}_j / T_C) \quad \text{nucleation rate}$$

$$\mathcal{E}_j = \text{energy density of the type-}j\text{ bubble}$$

Example

[K.F., Otsuki & Toyoda, PTP '99]

input parameters

$\tan\beta_0$	m_3^2	μ	A_t	$M_2 = M_1$	$m_{\tilde{t}_L}$	$m_{\tilde{t}_R}$
6	8110 GeV^2	-500 GeV	60 GeV	500 GeV	400 GeV	0

mass spectrum

m_h	m_A	m_H	$m_{\tilde{t}_1}$	$m_{\chi_1^\pm}$	$m_{\chi_1^0}$
82.28 GeV	117.9 GeV	124.0 GeV	167.8 GeV	457.6 GeV	449.8 GeV

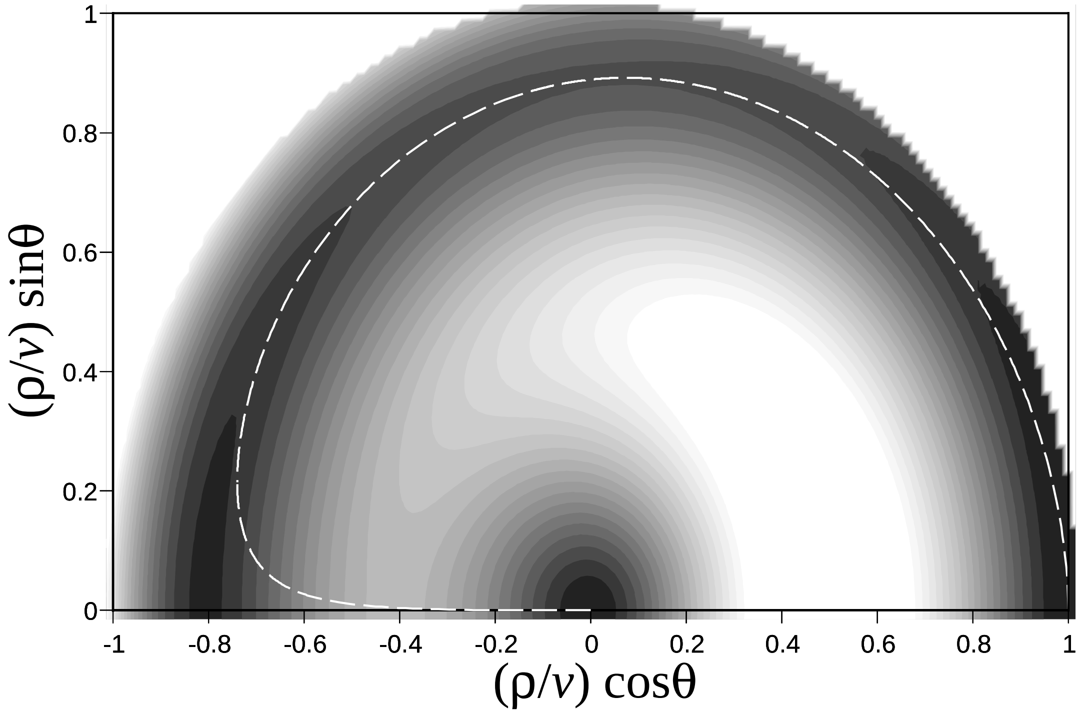
at the EWPT

$$T_C = 93.4 \text{ GeV}, \quad v_C = 129.17 \text{ GeV}, \quad \tan\beta = 7.292,$$

inverse wall thickness: $$a = \frac{\sqrt{8V_{\max}}}{v} = 13.23 \text{ GeV} \sim \frac{T_C}{7}$$

thinner than the MC result

lowest-$\mathcal{E}$ wall profile



Introduce an explicit CP violation by

$$\bar{m}_3^2 \cos\theta \rightarrow \frac{1}{2}\left(\bar{m}_3^2 e^{i(\theta+\delta)} + \text{h.c.}\right)$$

Net chiral charge flux

$$F_Q^{net} = \frac{N^+ F_Q^+ - N^- F_Q^-}{N^+ + N^-}.$$

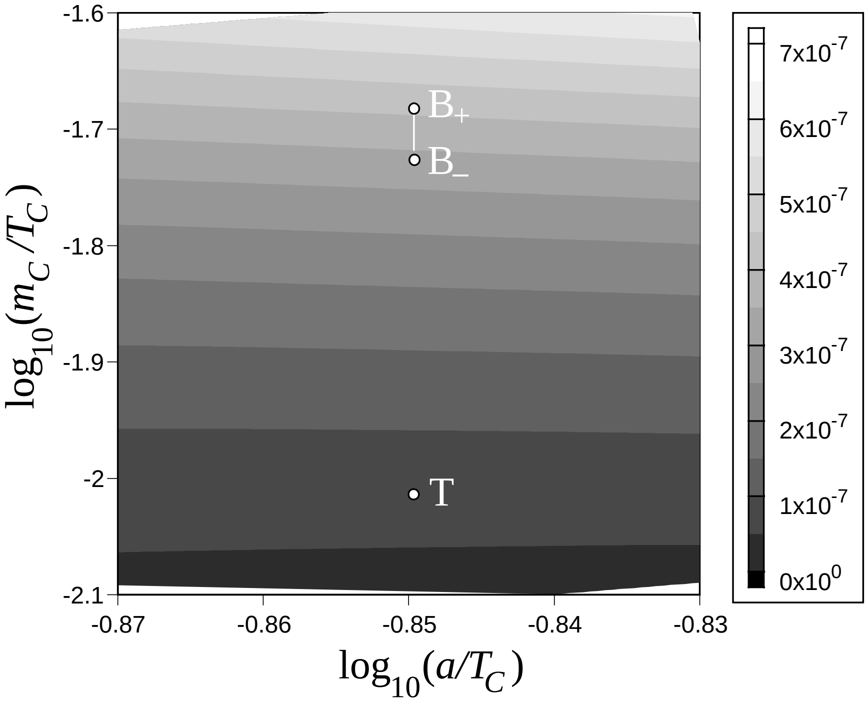
where

$$\frac{N^-}{N^+} = \begin{cases} 0.361 & \text{for } \delta = 0.001 \\ 0.601 & \text{for } \delta = 0.002 \end{cases}$$

by charge transport mechanism

$$\frac{n_B}{s} \sim 10^{-7} \times \frac{F_Q^{net}}{u} \times \frac{\tau}{T_C^2},$$

$$u = 0.1,\ \delta = 10^{-3} \Rightarrow \begin{cases} n_B/s < 10^{-12} & \text{for } b \text{ quark} \\ n_B/s \sim 10^{-(10-12)} & \text{for } \tau \text{ lepton} \end{cases}$$



$$\frac{F_Q^{net}}{(Q^L - Q^R) T_C^3 u}$$

$$|\theta_2| \propto \cos^2\beta \ll 1$$

$$m_C = m \frac{v_C \cos\beta_C}{v_0 \cos\beta_0}$$

♠ Enhancement of an explicit CP violation

$$\alpha = \mathrm{Arg}(\mu M_2) = \mathrm{Arg}(\mu M_1), \qquad \beta = \mathrm{Arg}(\mu A_t^*),$$

then

$$\begin{aligned} \bar{m}_3^2 &= m_3^2 + \Delta_{\phi^\pm}^{(0)} m_3^2 + e^{i\alpha}\Delta_{\chi}^{(0)} m_3^2 + e^{i\beta}\Delta_{\tilde{t}}^{(0)} m_3^2, \\ \lambda_5 &= \Delta_{\phi^\pm}^{(0)}\lambda_5 + e^{i2\alpha}\Delta_{\chi}^{(0)}\lambda_5 + e^{i2\beta}\Delta_{\tilde{t}}^{(0)}\lambda_5, \\ \lambda_{6,7} &= \Delta_{\phi^\pm}^{(0)}\lambda_{6,7} + e^{i\alpha}\Delta_{\chi}^{(0)}\lambda_{6,7} + e^{i\beta}\Delta_{\tilde{t}}^{(0)}\lambda_{6,7} \end{aligned}$$

$\Delta^{(0)} \equiv$ correction without explicit CP violation

If $\Delta_{\chi}^{(0)} \gg \Delta_{\tilde{t}}^{(0)}, \Delta_{\phi^\pm}^{(0)}$, by rephasing, $\lambda_{5,6,7} \in \mathbf{R}$ and

$$e^{-i\alpha}\bar{m}_3^2 = e^{-i\alpha} m_3^2 + \Delta_{\chi}^{(0)} m_3^2 \equiv e^{-i\delta}\left|\bar{m}_3^2\right|$$

with $$\tan\delta = -\frac{m_3^2 \sin\alpha}{m_3^2\cos\alpha + \Delta_{\chi}^{(0)} m_3^2}.$$

N.B $\left|m_3^2 + \Delta_{\chi}^{(0)} m_3^2\right| \ll m_3^2$ for transitional CP violation

for some parameter set, we have at $T \simeq T_C$

$$\Delta_{\tilde{t}}^{(0)} m_3^2 = 140.69, \qquad \Delta_{\chi}^{(0)} m_3^2 = -2356.73,$$

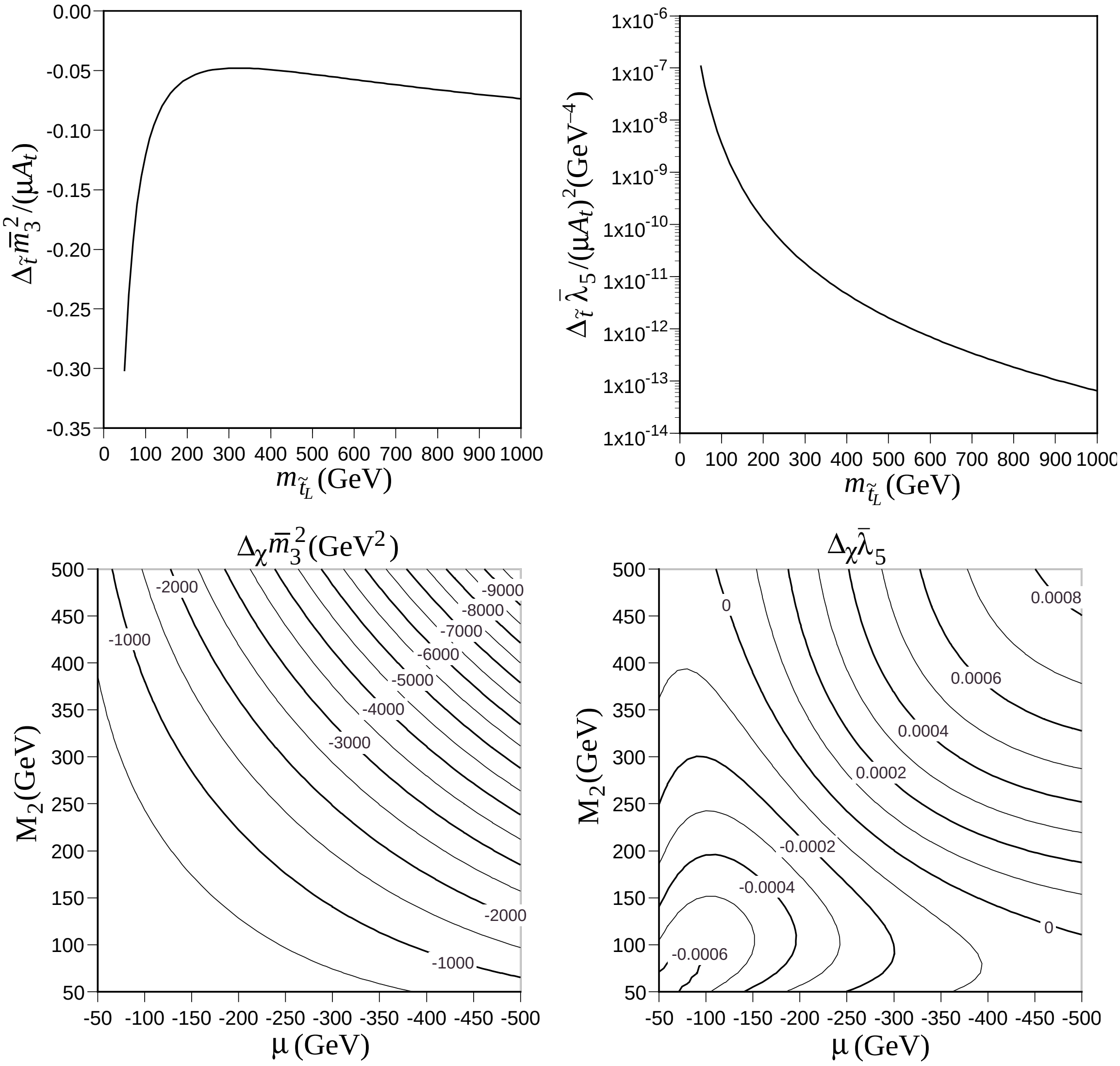
so that even for $\alpha = 10^{-3}$,

$$\begin{aligned} \tan\delta &= -\frac{(m_3^2 + \Delta_{\tilde{t}}^{(0)} m_3^2)\sin\alpha}{(m_3^2 + \Delta_{\tilde{t}}^{(0)} m_3^2)\cos\alpha + \Delta_{\chi}^{(0)} m_3^2} \\ &\simeq \frac{10^{-3}}{6.8\times 10^{-3}} = 0.147 \end{aligned}$$

$\Longrightarrow$ only the lowest-energy bubble survives

♠ Possibility of $F < 0$ $[\leftrightarrow \lambda_5 < 0]$

for $m_{\tilde{t}_R} = 0$, at $T = 95$ GeV,



⋆ $\lambda_5 < 0 \Longleftarrow \Delta_\chi \lambda_5 < 0$
$\longrightarrow \Delta_\chi \bar{m}_3^2 < -1500$ GeV^2

⋆ μA_t is restricted to have $\lambda_5 = \Delta_\chi\lambda_5 + \Delta_{\tilde{t}}\lambda_5 < 0$
$\longrightarrow \Delta_{\tilde{t}}\bar{m}_3^2$ is negative and bounded from below.

⋆ to have small $\bar{m}_3^2$, the tree-level $m_3^2 \lesssim 2500$ GeV^2
$\longrightarrow$ too small m_h and m_A $(< 67.5$ GeV$)$

∴ difficult to realize transitional CP violation with $F < 0$ in an acceptable MSSM

6. Discussions

BAU from B-symmetric universe:

1. baryon number violation
2. C and CP violation
3. departure from equilibrium

$$\parallel$$

combination of *rare processes*

B-violation :
- suppressed at present
- effective in early universe

$$\cup$$

anomalous $B+L$-violation — sphaleron at high-T

$\Longrightarrow$
- washout of $B+L$
- new possibilities of B-genesis
  - ⋆ EW baryogenesis
  - ⋆ L-genesis $\longrightarrow B$

conserved charge in the sym. phase $\rightarrow B$

observed CP violation → scenario model → the BAU $\frac{n_B}{s} = 0.2$–0.9×10^{-10}

scenario	scale (temperature)
GUTs	$M_{\rm GUTs} \simeq 10^{15}$GeV
L-genesis	$M_{\nu_R} \simeq 10^{10-12}$GeV
Affleck-Dine	$M_{\rm SUSYbr.} \simeq 10^{3-??}$GeV
EW B-genesis	$M_{\rm EW} \simeq 10^{2}$GeV

EW B-genesis by the MSM — rejected

× { strongly 1st-order EWPT (with acceptable m_h)
sufficient CP violation

EW B-genesis by the MSSM

⋆ $m_h \leq 110$GeV and $m_{\tilde{t}_1} \leq m_t$

$\Longrightarrow$ 1st-order EWPT with $v_C/T_C > 1$

⋆ many sources of CP violation

- complex parameters $\mu,\ M_2,\ M_1,\ A;\ \theta$
- transitional CP violation

We still need to know the dynamics of EWPT.

Other extensions of the MSM

$e.g.$ 2-Higgs-doublet model

many parameters $\longrightarrow$ broad allowed region ?

If EW baryogenesis could not work,...

▷ Leptogenesis $\xrightarrow{\text{sphaleron}}$ BAU

$$\begin{cases} L\text{-violation — } \nu\text{-Majorana mass} \\ \text{lepton sector } CP \text{ violation} \end{cases}$$

heavy neutrino production

▷ Affleck-Dine mechanism — B- and/or L-genesis

- ⋆ potential for $\tilde{q}$, $\tilde{l}$
- ⋆ initial condition for the coherent motion
- ⋆ explicit CP violation

▷ GUTs

- ⋆ $B - L$-violation
- ⋆ $M_X > 10^{16}$ GeV for $\tau_p > 10^{31-33}$ y

⋮

▷ Preheating or reheating after inflation