Higgs Physics and Cosmology

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This year will be the year of Higgs particle.

The discovery of 'Higgs-like' boson will be reported with higher statistics in this March.

To cofirm it is the Higgs boson, we must check

- relation between the couplings and the particle masses
  W, Z bosons, quarks, leptons
- 3- and 4-body self-interaction of the boson

The Higgs field in the Standard Model provides the masses of all the weak gauge boson and fermions.

$$\mathcal{L}_{\rm SM} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_Y + \mathcal{L}_H$$

$$\mathcal{L}_g = -\frac{1}{4}G^s_{\mu\nu}(x)G^{s\mu\nu}(x) - \frac{1}{4}F^a_{\mu\nu}(x)F^{a\mu\nu}(x) - \frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x)$$

$$G^s_{\mu\nu}(x) = \partial_\mu G^s_\mu(x) - \partial_\nu G^s_\mu(x) - g_3 f^{stu} G^t_\mu(x) G^u_\mu(x)$$

$$\mathcal{L}_f = \bar{q}_L(x) i\gamma^\mu \left(\partial_\mu - ig_3 \frac{\lambda^s}{2} G^s_\mu(x) - ig_2 \frac{\tau^a}{2} A^a_\mu(x) - \frac{i}{6} g_1 B_\mu(x) \right) q_L(x) + \cdots$$

$$\mathcal{L}_Y = \bar{q}_L(x) \boldsymbol{Y}_u u_R(x) \tilde{\Phi}(x) + \bar{q}_L(x) \boldsymbol{Y}_d d_R(x) \Phi(x) + \bar{l}_L(x) \boldsymbol{Y}_l e_R(x) + \text{h.c.}$$

Yukawa coupling matrix

Higgs field $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$ $\tilde{\Phi}(x) = i\tau^2 \Phi^*(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}$

No mass scale in the gauge and fermion sector.

The only mass scale arises from the Higgs sector.

$$\mathcal{L}_H = \left| \left(\partial_\mu - i g_2 \frac{\tau^a}{2} A_\mu^a(x) - \frac{i}{2} g_1 B_\mu(x) \right) \Phi(x) \right|^2 - V(\Phi)$$

$$\Phi \longrightarrow \langle \Phi \rangle = \begin{pmatrix} 0 \\ v_0/\sqrt{2} \end{pmatrix} \qquad m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + 0 A_\mu A^\mu$$

where

$$m_W^2 = \frac{1}{4} g_2^2 v_0^2, \qquad m_Z^2 = \frac{1}{4}(g_2^2 + g_1^2) v_0^2 = \frac{m_W^2}{\cos^2 \theta_W}$$

$$W_\mu^\pm(x) = \frac{1}{\sqrt{2}} \left(A_\mu^1(x) \mp i A_\mu^2(x) \right)$$

$$Z_\mu(x) = A_\mu^3(x) \cos \theta_W - B_\mu(x) \sin \theta_W$$

$$A_\mu(x) = A_\mu^3(x) \sin \theta_W + B_\mu(x) \cos \theta_W$$

Similarly for the fermions,

setting $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 + h(x) \end{pmatrix}$ unitary gauge

$$\mathcal{L}_Y = \frac{v_0 + h(x)}{\sqrt{2}} \left[\bar{u}_L \mathbf{Y}_u u_R + \bar{d}_L \mathbf{Y}_d d_R + \bar{e}_L \mathbf{Y}_e e_R + \text{h.c.} \right]$$

bi-unitary transformation by f_L and f_R

$$\left(1 + \frac{h(x)}{v_0} \right) \left(m_{u_A} \bar{u}_A u_A + m_{d_A} \bar{d}_A d_A + m_{e_A} \bar{e}_A e_A \right)$$

$A = 1, 2, 3:$ generation

Higgs boson

The couplings of the Higgs boson to the gauge bosons and fermions are proportional to their masses.

The effect of the unitary transformation resides only in the quark charged-current interaction.

CKM matrix

masses of the SM particles

except for the higgs boson

	electric charge	1st gen.	2nd gen.	3rd gen.
quark	$+\frac{2}{3}$	u	c	t
		2–3MeV	1.27GeV	174GeV
	$-\frac{1}{3}$	d	s	b
		4–6MeV	101MeV	4.2GeV
charged lepton	-1	e	μ	τ
		0.51MeV	106MeV	1.8GeV

Weak boson	
W^+, W^-	Z
80.4GeV	91.2GeV

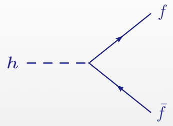
N.B.

$$m_u + m_u + m_d \sim \frac{1}{10} m_{\text{proton}}$$

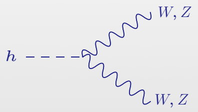
90% of the nucleon mass comes from QCD dynamics.

--- confirmed by lattice MC calculation

decay branching rate



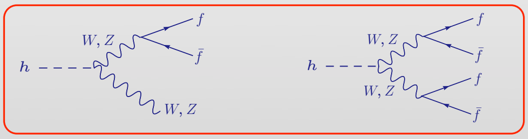
$$\Gamma(h \to f\bar{f}) = \frac{N_c m_f^2 m_h}{8\pi v_0^2}\left(1 - \frac{4m_f^2}{m_h^2}\right)^{\frac{3}{2}}$$



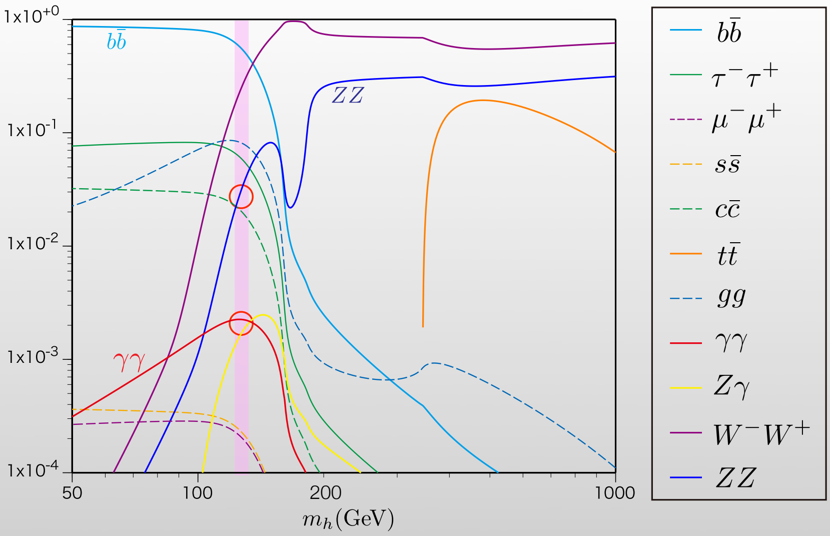
$$\Gamma(h \to VV) = \frac{C_V m_h^3}{8\pi v_0^2}\sqrt{1 - \frac{4m_V^2}{m_h^2}}\left(1 - \frac{4m_V^2}{m_h^2} + \frac{12 m_V^4}{m_h^4}\right)$$

$C_W = 2,\ C_Z = 1$

off-shell vector 3- or 4-body decay



one-loop processes



Made from the data generated by H-Decay package

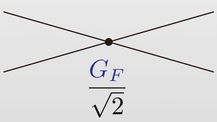
What determines the value of the Higgs VEV v_0 ?

4-Fermi effective theory vs W-exchange interaction

e.g. weak decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

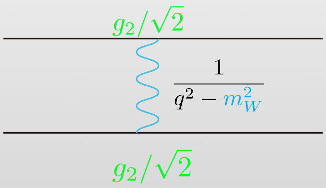
$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}}\bar{e}\gamma_\rho(1-\gamma_5)\nu_e\,\bar{\nu}_\mu\gamma^\rho(1-\gamma_5)\mu + \cdots$$

(V–A)-type current-current interaction



$$\mathcal{L}_{CC} = \frac{g_2}{\sqrt{2}}\left(J_\mu^- W^{+\mu} + J_\mu^+ W^{-\mu}\right)$$

$$J_\mu^- = \bar{\nu}_A\gamma_\mu\frac{1-\gamma_5}{2}e_A + \bar{u}_A\gamma_\mu\frac{1-\gamma_5}{2}U_{AB}d_B$$



$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_W^2} = \frac{1}{2v_0^2} \Rightarrow v_0 = \left(\frac{1}{\sqrt{2}\,G_F}\right)^{\frac{1}{2}} = 246.26\text{GeV}$$

Theoretically, it is the location of the minimum of the scalar potential of the Standard Model.

including quantum corrections

at the classical (tree) level

gauge-invariant
renormalizable

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2$$

$V(\Phi)$ takes its minimum at $\langle\Phi\rangle$. $\quad v_0 = \sqrt{\frac{\mu^2}{\lambda}}$

Even when $\mu^2 = 0$, Φ can acquire nonzero v_0.

Coleman-Weinberg mechanism

quantum correction to the potential

$$\Delta V(v) = \sum_{I=t,Z,W,\cdots} c_I \frac{m_I(v)^4}{64\pi^2}\left(\log\frac{m_I(v)^2}{M^2} - \frac{3}{2}\right)$$

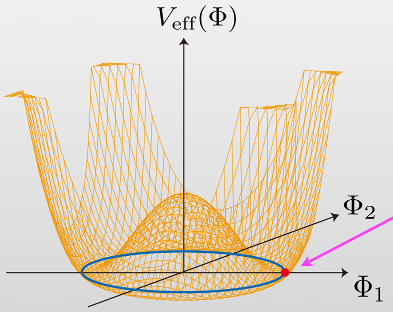
$c_t = -4N_c$
$c_Z = 3$
$c_W = 6$

renormalization scale

$V_{\text{eff}}(v) = V(v) + \Delta V(v)$ takes min. at v_0

A part of the gauge symmetry of the SM is *spontaneously* broken by $\langle\Phi\rangle \neq 0$

A symmetry of the lagrangian is broken by the ground state.



Picking up a point from the degenerate states breaks the symmetry.

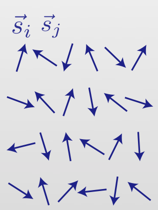
classically
degenerate ground state

Similarity to magnetism

Hamiltonian of the spin model

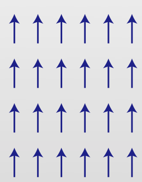
$$H = -\kappa \sum_{\langle i,j\rangle} \vec{s}_i \cdot \vec{s}_j$$

invariant under the spatial rotation

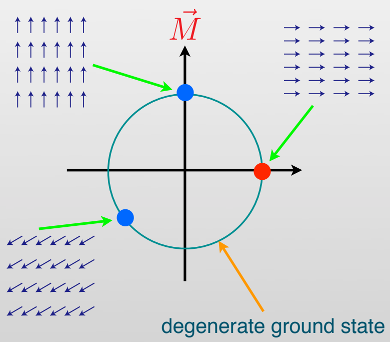


$M = \langle s_i \rangle = 0$

the lowest E

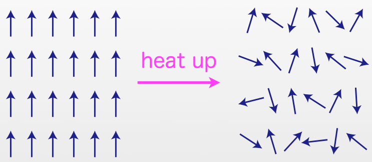


$M \neq 0$



breaks the rotational symmetry

Heating up a magnet looses its magnetism.



Symmetry restoring
phase transition

$T >$ Curie temperature

Fe: 1043K

One may expect a similar symmetry restoring
Phase Transition to occur in the Higgs sector.

maybe at 100GeV=10^{15}K

It's impossible to reach such a high-T on the Earth,
but is though to be realized in **the early Universe**.

Brief Review of the Big Bang Cosmology

Evolution of the space is described by **Einstein equation**:

$$R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}(x)R(x) + \Lambda g_{\mu\nu}(x) = 8\pi G T_{\mu\nu}(x)$$

curvature — 2nd order derive. of the metric $g_{\mu\nu}(x)$

cosmological constant (vacuum energy)

energy-momentum tensor

spatially uniform and isotropic Universe
= Friedmann-Robertson-Walker spacetime

so large scale as 10^8ly at present

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - R_0^2 a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

scale factor relative to the present

curvature parameter

$$k = \begin{cases} +1 & : \text{closed} \\ 0 & : \text{flat} \\ -1 & : \text{open} \end{cases}$$

Einstein eq. → diff. eq. for $a(t)$

$T_{\mu\nu}(x)$ uniform and isotropic $\begin{cases} \rho(t) & \text{energy density} \\ P(t) & \text{pressure} \end{cases}$

satisfies the conservation law $dE + PdV = 0$

$$\frac{d}{dt}\left(\rho(t)a(t)^3\right) + P(t)\frac{d}{dt}\left(a(t)^3\right) = 0$$

3 types of Equation Of State

		EM conservation
Matter (nonrelativistic)	$P_m(t) = 0$	$\rho_m(t) \propto a(t)^{-3}$
Radiation (relativistic)	$P_r(t) = \frac{1}{3}\rho_r(t)$	$\rho_r(t) \propto a(t)^{-4}$
Vacuum (or Dark Energy)	$P_\Lambda(t) = -\rho_\Lambda(t)$	$\rho_\Lambda(t) \propto a(t)^0$

Which species behaves as radiation depends on temperature.

$T \gg m \quad (k_B T \gg mc^2)$

Friedmann equation =Einstein equation for FRW metric

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\left(\rho_r(t)+\rho_m(t)+\rho_\Lambda\right) - \frac{k}{R_0^2 a(t)^2}$$

$$= \frac{8\pi G}{3}\left(\frac{\rho_r(t_0)}{a(t)^4}+\frac{\rho_m(t_0)}{a(t)^3}+\rho_\Lambda\right) - \frac{k}{R_0^2 a(t)^2}$$

observation : $k \simeq 0$ $\qquad \frac{\rho_r(t_0)}{\rho_m(t_0)} = \frac{\Omega_R}{\Omega_M} = \frac{5.0\times 10^{-5}}{0.27} \simeq 1.9\times 10^{-4}$

The early Universe of $a(t) < 2\times 10^{-4}$ is dominated by radiation.

entropy of radiation $\propto T^3 a^3$ =const.
temperature of radiation at present $T_0 = 2.73\mathrm{K}$

$$T = \frac{T_0}{a} \gtrsim 10^4\mathrm{K} \simeq 1\mathrm{eV}$$

At present, only the photons (and maybe the neutrinos) have the equilibrium distribution. [Cosmic Microwave Background]

In the radiation-dominated Universe $(T \gg 1\mathrm{eV})$
species tightly coupled to the plasma can be regarded as in equilib., even though the Universe was expanding.

criterion for coupling to the plasma

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{8\pi G}{3}\rho_r(t)} < \Gamma(t)$$

Expansion rate ($H(t)$)

interaction rate ($\Gamma(t)$)
determined by the cross section and number density

Then we can define the temperature T and apply the equilibrium statistical mechanics.

interaction rate

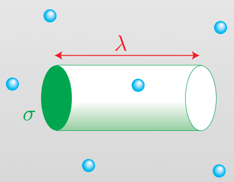
for relativistic species $m \lesssim T$ $\qquad \Gamma^{-1} = \bar{t} \simeq \lambda$ mean free path

total cross section of that species $\sigma \simeq \frac{\alpha^2}{s} \simeq \frac{\alpha^2}{T^2}$

for the weak interaction, $\alpha = \frac{g_2^2}{4\pi} = \frac{\alpha_{\mathrm{em}}}{\sin^2\theta_W}$

number density $n(T) \simeq g_{*n}\frac{\zeta(3)}{\pi^2}T^3$

$g_{*n} = \sum_B g_B + \frac{3}{4}\sum_F g_F$
effective degrees of freedom



$$\sigma\cdot\lambda = \frac{1}{n(T)}$$

$$\bar{t} = \lambda \simeq \frac{10}{g_{*n}T^3}\left(\frac{\alpha^2}{T^2}\right)^{-1} = \frac{10}{g_{*n}\alpha^2 T}$$

expansion rate $H(T) = \sqrt{\frac{8\pi G}{3}\rho_r(T)} \simeq 1.66\sqrt{g_*}\frac{T^2}{M_{\rm Pl}}$ $M_{\rm Pl} = 1.22 \times 10^{19}{\rm GeV}$

$$\rho_r(T) = g\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{|\boldsymbol{p}|}{e^{|\boldsymbol{p}|/T} \mp 1} = g\left\{\begin{matrix} 1 \\ 7/8 \end{matrix}\right\}\frac{\pi^2}{30}T^4$$

$g_* \equiv \sum_B g_B + \frac{7}{8}\sum_F g_F$ $g_* = 106.75$
when all the SM particles are relativistic

at $T = 100{\rm GeV}$

$$H(T) = 1.66\sqrt{106.75} \times \frac{10^4}{1.22\times 10^{19}}{\rm GeV} \simeq 10^{-14}{\rm GeV}$$

$$\Gamma(T) = g_{*n}\frac{\alpha(T)^2 T}{10} = 10^3\alpha(T)^2{\rm GeV} = (1-10){\rm GeV}$$

EW QCD

At temperatures of the weak scale, we can safely regard all the SM particles are in thermal equilibrium.

comments

If the Universe has been in equilibrium throughout its history, there would be no stars, galaxies, structures and creatures including ourselves.

nonequilibrium events due to $\Gamma(T) < H(T)$

- ☆ decoupling of photons (T=1eV)
- ☆ nucleosynthesis (T=1MeV)
- ☆ decoupling of the Dark Matter
- ☆ GUTs Baryogenesis/Leptogenesis

We need to treat time-dependent distribution functions.

We apply the equilibrium statistical mechanics to study *static features* of the phase transition of the EW symmetry breaking.

Quantum Field Theory at finite temperatures

Kapusta and Gale, 'Finite-temperature field theory' (2006) 2nd ed.
Le Bellac, 'Thermal Field Theory' (2000)
Landsman and van Weert, Phys. Rep. 145 (1987) 141
Dolan and Jackiw, Phys. Rev. D9 (1974) 3320

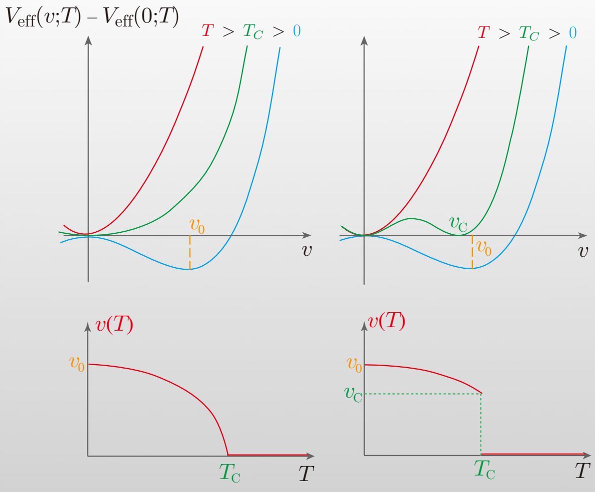
free energy density as a function of the order parameters
=effective potential at finite temperatures

$V_{\rm eff}(v;T) = -\Gamma[\varphi(x) = v]/\int d^4x$ $\Gamma[\varphi] =$ effective action

$${\rm Tr}(e^{-H/T}) = N(T)\int_{\rm pbc}[d\phi]\,\exp\left(-\int_0^{1/T} d^4x_E\,\mathcal{L}_E(\phi)\right)$$ euclidean path integral

$$\begin{cases} \phi(0,\boldsymbol{x}) = \phi(1/T,\boldsymbol{x}) & \text{boson} \quad k^0 = i\omega_n = i\pi 2nT \\ \psi(0,\boldsymbol{x}) = -\psi(1/T,\boldsymbol{x}) & \text{fermion} \quad k^0 = i\tilde{\omega}_n = i\pi(2n+1)T \end{cases}$$

free energy vs order parameter (Higgs VEV) at finite T



2nd order PT   1st order PT

order parameter

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(T) \end{pmatrix}$$

$$v_C \equiv \lim_{T \uparrow T_C} v(T) \neq 0$$

⇕

1st order phase transition

Standard Model

$$V_{\rm eff}(v;T) = -\frac{1}{2}\mu^2 v^2 + \frac{\lambda}{4} v^4 + 2Bv^4 \left[\log\left(\frac{v^2}{v_0^2}\right) - \frac{3}{2}\right] + \bar{V}(v;T)$$

1-loop corrections

$$B = \frac{3}{64\pi^2 v_0^4}\left(2m_W^4 + m_Z^4 - 4m_t^4\right)$$

$$\bar{V}(v;T) = \frac{T^4}{2\pi^2}\left(6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_W)\right) \qquad a_A = \frac{m_A(v)}{T}$$

$$I_{B,F}(a) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right)$$

High-T expansion

$a = m/T \ll 1$

IR nonanalyticity

$$I_B(a) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6)$$

$$I_F(a) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6)$$

$+T^4 a^2 \sim +T^2 v^2$ ⟶ symmetry restoration at high-T

Assuming $T > m_W, m_Z, m_t$

$$V_{\rm eff}(v;T) \simeq D(T^2 - T_0^2)v^2 - ETv^3 + \frac{\lambda_T}{4}v^4$$

$$D = \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v_0^2} \qquad E = \frac{2m_W^3 + m_Z^3}{4\pi v_0^3} \sim 10^{-2}$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4}\left(2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right)$$

$$T_0^2 = \frac{\mu^2 - 4Bv_0^2}{2D} \qquad \log\alpha_{F(B)} = 2\log(4)\pi - 2\gamma_E$$

At T_C, the local min. at v_C degenerates with that at $v = 0$.

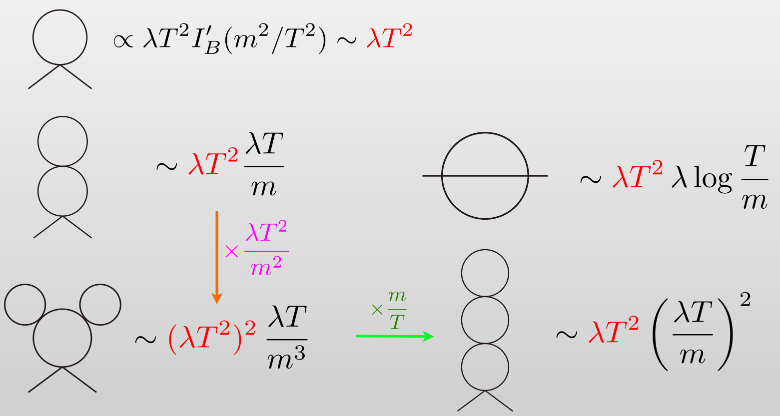
$$V_{\rm eff}(v_C;T_C) = V_{\rm eff}(0;T_C)$$

⟶ $$v_C = \frac{2ET_C}{\lambda_T}$$ 1st order PT

In most cases, the perturbative expansion at high temperatures is *not a good approximation*.

e.g. ϕ^4 theory — Dolan and Jackiw, Phys. Rev. D9 (1974)

corrections to 2-point function (High-T exp.) $a = \frac{m}{T} \ll 1$



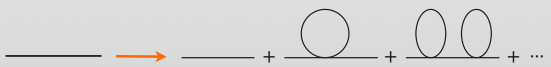
☆ the leading correction to $m^2 \sim \lambda T^2$

☆ the **bubble** subdiagram yields the largest corrections

 a factor of $\frac{\lambda T^2}{m^2}$ from a bubble

$\therefore T \gtrsim \frac{m}{\sqrt{\lambda}}$ ⟶ loop expansion is invalidated

The leading correction$(\sim \lambda T^2)$ to m^2 can be incorporated by 'resummation'



$$m^2 \rightarrow m^2 + \Delta_T m^2 = m^2 + \frac{\lambda T^2}{24}$$ in the propagator

thermal mass ⟶ weakens the PT

A nonperturbative analysis: Lattice MC calculation

$$Z(T) = \mathrm{Tr}\left(e^{-H/T}\right) = \int_{\phi(1/T)=\phi(0)} [d\Phi \, dU_\mu] \exp\left(-S_E[\Phi, U]\right)$$

$$U_\mu(x) = e^{igA_\mu(x)}$$ link variable

Standard Model (1 Higgs doublet) [Csikor, hep-lat/9910354]

1st order Phase Transition for $m_h < 66.5 \pm 1.4\,\mathrm{GeV}$

$T_C \simeq 90 - 100\,\mathrm{GeV}$

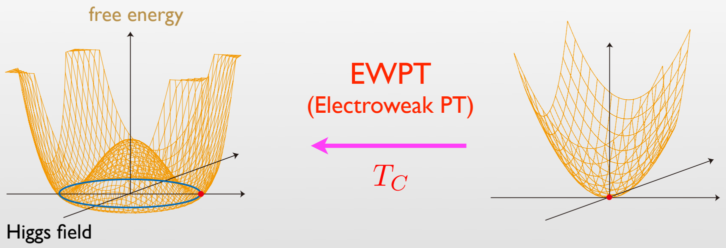
End point of the Phase Transition at $m_h = 72.1 \pm 1.4\,\mathrm{GeV}$

$m_h = 125\mathrm{GeV}$ ⟶ Cross Over

$v(T)$ continuously changes from 0 to v_0 as the Universe cooled down

As expected, we have seen that
the broken gauge symmetry was restored at high temperatures.

$T=0$ $\qquad\qquad$ $T > T_C \simeq 100\mathrm{GeV}$



$m_h > 72\mathrm{GeV}$ → no dramatic event

may be for $m_h > 67\mathrm{GeV}$

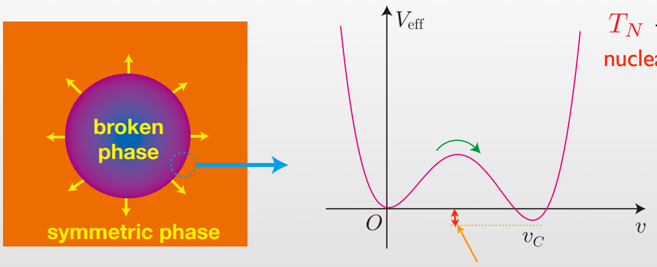
Recall
$H(T) \simeq 10^{-14}\mathrm{GeV} \ll \Gamma_{EW}(T) \simeq 1\mathrm{GeV}$

If the EWPT is 1st order, some events might occur
which affect the present Universe.

Why 1st order PT?

e.g. evaporation of water

PT proceeds through nucleation and growth of bubbles



$T_N < T_C$
nucleation temp.

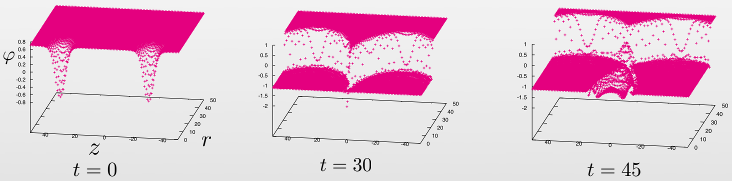
$$V_{\mathrm{eff}}(0;T_N) - V_{\mathrm{eff}}(v_C;T_N)$$
||
latent heat

- non-equilibrium state near expanding bubble walls
- release of the latent heat

If the EWPT is 1st order,

- **Gravitational wave** might be generated

  by bubble collisions and/or by turbulence of the plasma



EM tensor of the scalar field

$T[\varphi](x)_{\mu\nu}$ → $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$

The energy density of GW may be marginally observed by LISA.

Laser Interferometer Space Antenna
http://lisa.nasa.gov/

For a review, see M. Maggiore, Phys. Rep. 331 (2000) 283

the Baryon Asymmetry of the Universe might be generated at PT

= scenario of the Electroweak Baryogenesis (EWBG)

Both the baryon and lepton numbers are conserved in the SM at the classicl level (=symmetry of the Lagrangian).

However, (B+L) is not conserved *at quantum level.*

$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}\left[g_2^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu}B^{\mu\nu}\right] \qquad \partial_\mu j^\mu_{B-L} = 0$$

$$B(t_f) - B(t_i) = \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} dt \int d^3\boldsymbol{x} \left[g_2^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right]$$
$$= N_f\left[N_{CS}(t_f) - N_{CS}(t_i)\right]$$

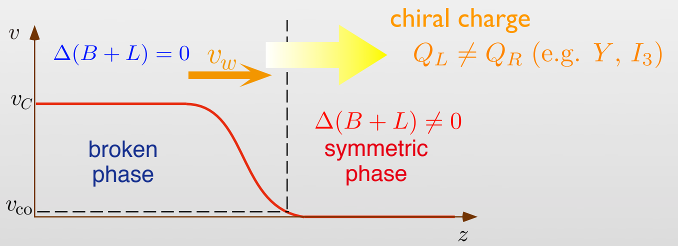
Chern-Simons number $(A_0 = 0\text{-gauge})$

$$N_{CS}(t) = \frac{1}{32\pi^2}\int d^3\boldsymbol{x}\, \epsilon_{ijk}\left[g_2^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}g_2 A_i A_j A_k\right) - g_1^2 B_{ij}B_k\right]_t$$

$N_{CS} \in \boldsymbol{Z}$ for classical vacua

$\Delta(B+L)$-changing process is suppressed in the broken phase by $e^{-E_{\mathrm{sph}}/T}$ sphaleron energy

If the interaction of the plasma particles with the bubble wall violates CP-symmetry, some chiral charge is injected in the symmetric phase.



The chiral charge biases the (B+L)-changing process.

Generated (B+L) is frozen in the broken phase.

CP-violating bubble walls are transparent to the ***vectorlike*** quantum numbers such as B and L.

Review articles on EWBG

KF, Prog. Theor. Phys. 96 (1996) 475

Rubakov and Shaposhnikov, Phys. Usp. 39 (1996) 461

Riotto and Trodden, Ann. Rev. Nucl. Part. Sci. 49 (1999) 35

Bernreuther, Lect. Notes Phys. 591 (2002) 237

Even if the EWPT in the SM is of first order, the KM phase is insufficient to generate the BAU.

For the EWPT to be of fist order, we must extend the SM.

Which extension makes the EWPT of first order ?

bosonic loop correction

$$V_{\rm eff}(v;T) \sim -T\left(m(v)^2\right)^{3/2} \quad \longleftarrow a^3\text{-term of } I_B(a^2)$$

bosons interacting with the Higgs whose mass behaves as

$$m(v)^2 \sim g^2 v^2 \quad (\text{for } v \sim 0)$$

e.g. extra scalars in the two-Higgs-double Model (2HDM), Supersymmetric SM's

$$m(v)^2 = m_0^2 + g^2 v^2 \quad (m_0^2 \ll g^2 v_0^2)$$

2HDM with the discrete symmetry to avoid FCNC

⟶ vast allowed region of parameters

KF, Kakuto, Takenaga, Prog. Theor. Phys. 91(1994)

MSSM Minimal Supersymmetric Standard Model

order parameters: $\langle\Phi_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle\Phi_u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 + i v_3 \end{pmatrix}$

8–3 = 5 physical Higgs particles 3 neutral, 1 charged

$$V_0 = m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u - \left(m_3^2 \epsilon_{ij} \Phi_d^i \Phi_u^j + \text{h.c.}\right)$$
$$+\frac{g_2^2+g_1^2}{8}\left(\Phi_d^\dagger\Phi_d - \Phi_u^\dagger\Phi_u\right)^2 + \frac{g_2^2}{2}\left|\Phi_d^\dagger\Phi_u\right|^2$$

Higgs self-coupling $\sim g_2^2, g_1^2$ ⟶ small Higgs mass

radiative corr. from top/stop loops ⟶ $m_h \lesssim 135\text{GeV}$

$m_{H^\pm}, m_A, m_H \longrightarrow \infty$ ⟶ SM with relatively light Higgs

extra Higgs bosons

$m_{H^\pm} > 200\text{GeV} \longrightarrow$ EWPT becomes SM-like

We expect PT unlike the SM when the extra Higgs are light.

the stop mass matrix

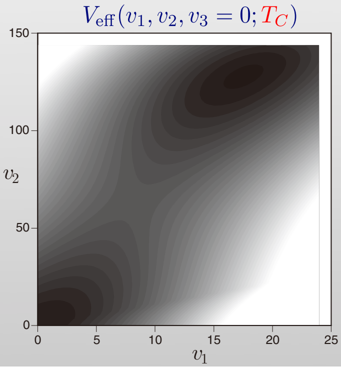
$$\mathcal{M}_{\tilde t}^2 = \begin{pmatrix} m_{\tilde t_L}^2 + \left(\frac{g_2^2}{8} - \frac{g_1^2}{24}\right)(v_1^2 - v_2^2) + \frac{y_t^2}{2}v_2^2 & \frac{y_t}{\sqrt{2}}(\mu v_1 + A_t v_2) \\ * & m_{\tilde t_R}^2 + \frac{g_1^2}{6}(v_1^2 - v_2^2) + \frac{y_t^2}{2}v_2^2 \end{pmatrix}$$

$m_{\tilde t_R}^2 = 0$ ⟶ smaller eigenvalue

$m_{\tilde t_1}^2 \sim y_t^2 O(v^2)$

makes the EWPT of first order

$V_{\rm eff}(v_1, v_2, v_3 = 0; T_C)$



With m_h=125GeV, it's difficult to leave the BAU.

KF and Senaha, Phys. Rev. D79 (2009)

new class of PT's in the model with a gauge singlet

e.g. Next-to-MSSM (NMSSM) = MSSM with a singlet superfield

'Higgs fields' $\Phi_u(x),\ \Phi_d(x)\ :\ n(x)$

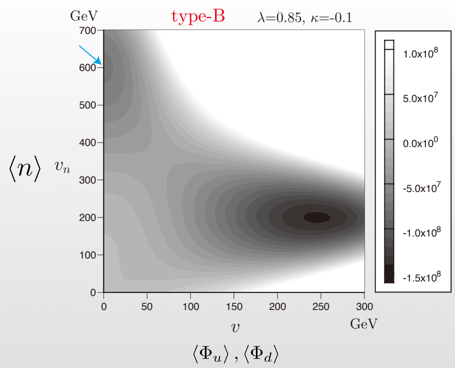
$$\begin{aligned} V_0 &= m_1^2\Phi_d^\dagger\Phi_d + m_2^2\Phi_u^\dagger\Phi_u + m_N^2 n^* n - \left(\lambda A_\lambda \epsilon_{ij} n \Phi_d^i \Phi_u^j + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.}\right) \\ &\quad + \frac{g_2^2 + g_1^2}{8}\left(\Phi_d^\dagger\Phi_d - \Phi_u^\dagger\Phi_u\right)^2 + \frac{g_2^2}{2}\left|\Phi_d^\dagger\Phi_u\right|^2 \\ &\quad + |\lambda|^2 n^* n \left(\Phi_d^\dagger\Phi_d + \Phi_u^\dagger\Phi_u\right) + \left|\lambda\epsilon_{ij}\Phi_d^i\Phi_u^j + \kappa n^2\right|^2 \end{aligned}$$

$\lambda\langle n\rangle \to \mu$ in MSSM

new self-coupling → heavy Higgs

$\langle n\rangle \to \infty$ with $\lambda\langle n\rangle$ fixed → reduced to MSSM

New PT is expected for $\langle n\rangle = O(100\text{GeV})$.



strong 1st order PT without a light stop

KF, Tao and Toyoda, Prog.Theor. Phys. 114 (2005)

CP violating complex parameters: $\lambda,\ \kappa,\ \text{Arg}\,\langle n\rangle$

some combinations of them do not affect EDM of n and μ, which are viable for the baryogenesis

Summary

The Higgs mechanism explains the masses of the weak gauge bosons and fermions in the Standard Model, without spoiling renormalizability and unitarity of the theory.

Although existence of the Higgs boson has not yet been established, various observations seem to support it.

success of the electroweak theory

Even if discovery of a CP-even scalar of 125GeV is established, we must do many to check whether it is the SM Higgs boson.

- decay branching ratios of various modes
- self-coupling of the scalar boson

If the present vacuum is the result of the Higgs mechanism, the state at very high temperatures is different from it and the broken gauge symmetries are restored.

As the Universe cooled down to weak-scale temperature, the gauge symmetry of the EW theory was spontaneously broken by the expectation value of the Higgs fields.

Electroweak Phase Transition (EWPT)

Properties of the EWPT depends on the EW model.

Standard Model with m_h=125GeV → Cross Over

MSSM
NMSSM
2HDM

open possibility of 1st order EWPT

extra bosons, another order parameter

So far, the SM has been tested in experiments and found to be consistent with their results.

We, however, know that some extension of the SM is needed to explain the obvious facts,

- Neutrino mass
- Dark Matter
- Baryon Asymmetry of the Universe

Some of the extended models contain extra scalar fields, which may lead to the 1st order EWPT.

In particular, extended Higgs sectors predict charged Higgs bosons and extra neutral Higgs bosons, which combine with each other to make mass eigenstates.

decay BR's deviate from the SM prediction
CP violation in the Higgs sector, ... etc.

mission of ILC

Thanks for your attention !