Higgs Bosons in the Standard Model and Beyond

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The Standard Model

Successful model of strong and electroweak interactions among elementary particles

- neutral current interaction
- masses of the weak gauge bosons
- CP violation --- Kobayashi-Maskawa phase

etc.

- neutrino mass
- dark matter

$\mathrm{SU}(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory

matter

quarks

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \left(\mathbf{3},\ \mathbf{2},\ \frac{1}{6}\right)$$

$$u_R, \quad c_R, \quad t_R \qquad (\mathbf{3},\ \mathbf{1},\ \frac{2}{3})$$

$$d_R, \quad s_R, \quad b_R \qquad (\mathbf{3},\ \mathbf{1},\ -\frac{1}{3})$$

leptons

$$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \qquad (\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$$

$$e_R, \quad \mu_R, \quad \tau_R \qquad (\mathbf{1},\ \mathbf{1},\ -1)$$

$$Q = I_3 + Y$$

gauge bosons

$SU(3)_c$ G_μ^s $(s = 1 - 8)$ gluon

$SU(2)_L$ A_μ^a $(a = 1 - 3)$ $W_\mu^\pm,\ Z_\mu$

$U(1)_Y$ B_μ A_μ $U(1)_{\rm em}$

spontaneous symmetry breakdown

Higgs mechanism

Masses prohibited by symmetries

gauge bosons — gauge symmetries

quarks and leptons — *chiral* gauge symmetries

in reality,

$$m_W = 80.4\mathrm{GeV} \qquad m_Z = 91.2\mathrm{GeV}$$

$$m_t = 174\mathrm{GeV} \qquad m_b = 4.7\mathrm{GeV} \qquad \cdots$$

as a result of ***Higgs mechanism***

sponteneous symmetry breakdown

A symmetry of ***lagrangian*** is broken by the ***ground state*** (vacuum)

ex. O(2) symmetric model of scalar fields

$$\mathcal{L} = \frac{1}{2}\partial_\mu \sigma(x) \partial^\mu \sigma(x) + \frac{1}{2}\partial_\mu \pi(x) \partial^\mu \pi(x) - V(\sigma^2 + \pi^2)$$

invariant under

$$\begin{pmatrix} \sigma \\ \pi \end{pmatrix} \mapsto \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma \\ \pi \end{pmatrix} \qquad \theta : \text{constant}$$

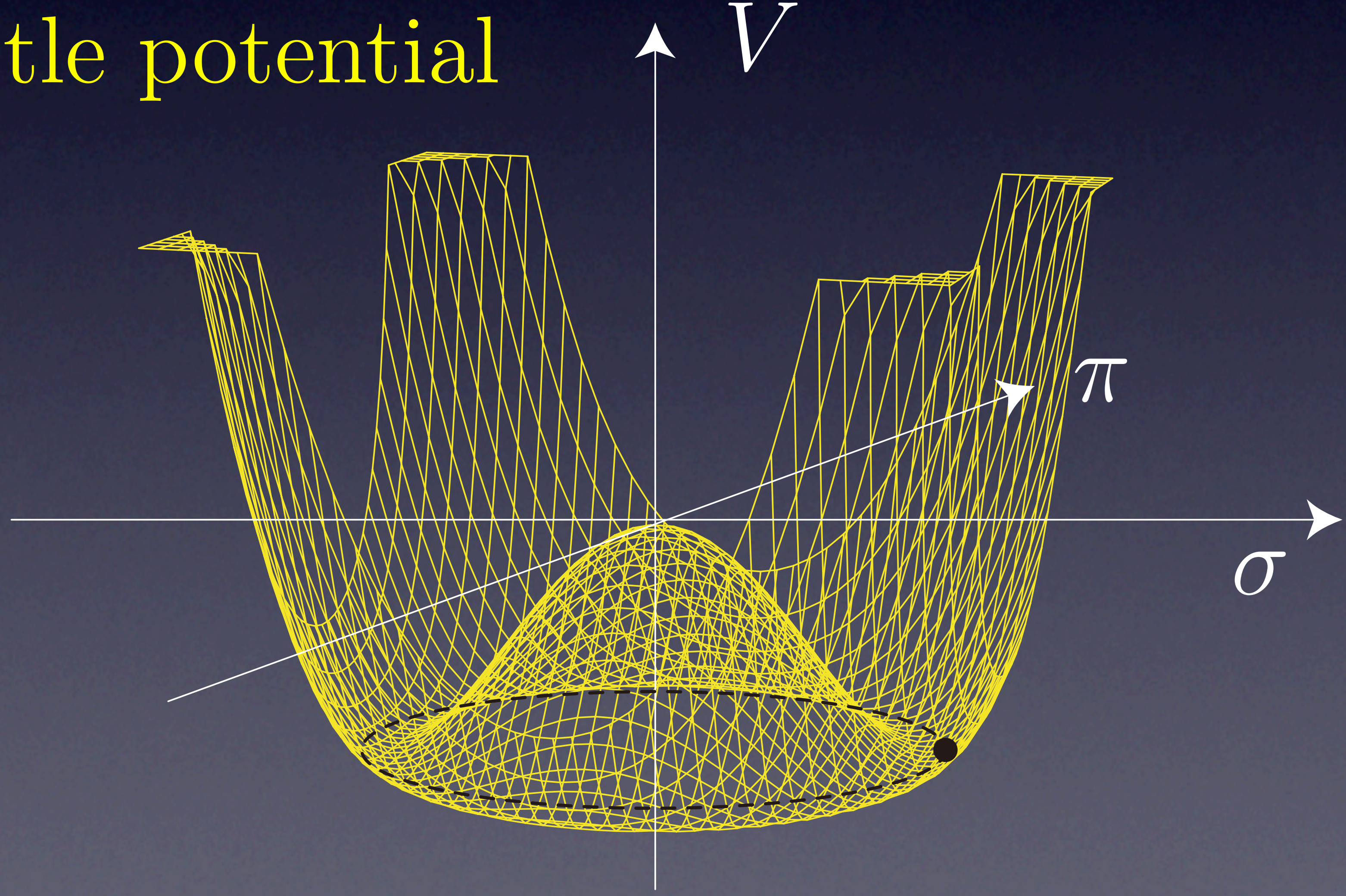
invariant renormalizable potential

$$V = -\frac{1}{2}\mu^2(\sigma^2 + \pi^2) + \frac{\lambda}{4}(\sigma^2 + \pi^2)^2$$

λ must be positive, μ^2 can have any sign.

$\mu^2 < 0 \Rightarrow$ degenerate mass $\sqrt{-\mu^2}$

$\mu^2 > 0 \Rightarrow$ wine bottle potential

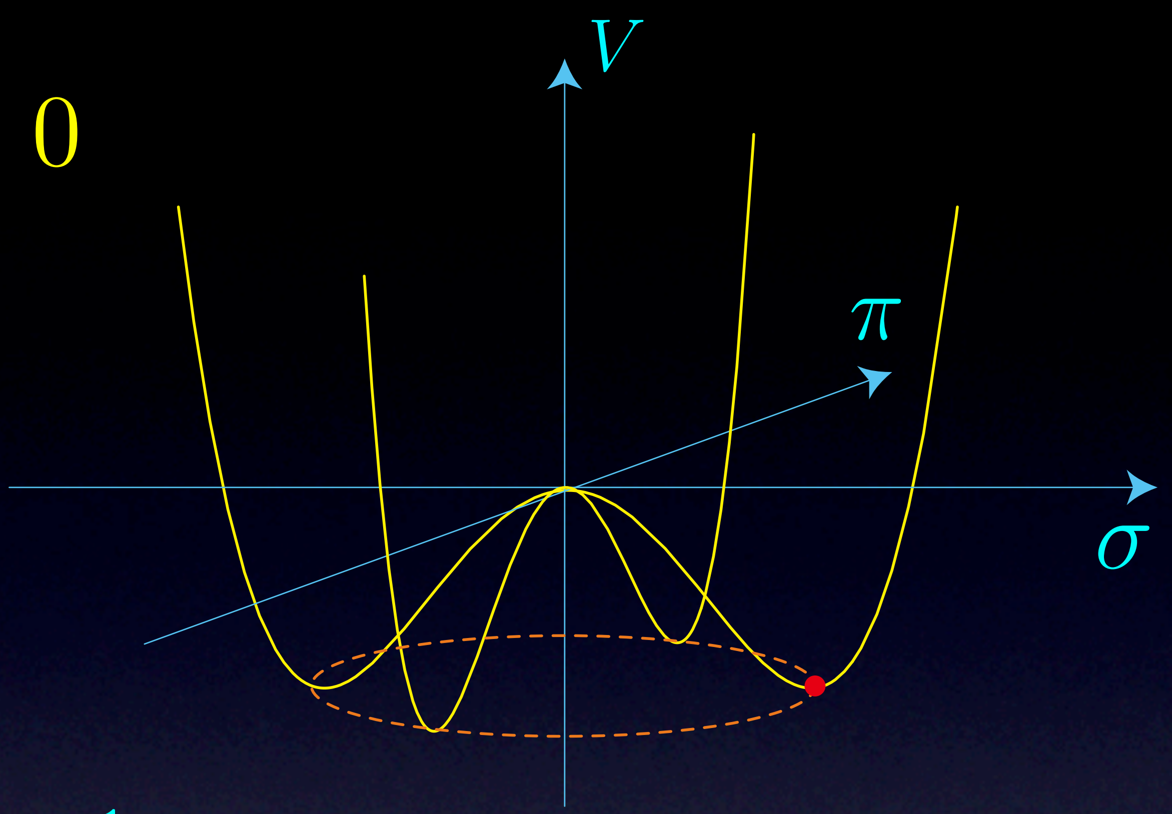


minima at

$$\sigma^2 + \pi^2 = \frac{\mu^2}{\lambda} \equiv v_0^2$$

Upon quantization, one picks up a vacuum

$$\langle 0|\sigma(x)|0\rangle = v_0, \quad \langle 0|\pi(x)|0\rangle = 0$$



fluctuation around the vacuum:

$$\sigma(x) = v_0 + \eta(x)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}\cdot 2\lambda v_0^2\,\eta^2 + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + 0\cdot\pi^2 + \cdots$$

$$m_\eta^2 = \frac{\partial^2 V(v_0, 0)}{\partial\eta^2} = 2\lambda v_0^2$$

$$m_\pi^2 = \frac{\partial^2 V(v_0, 0)}{\partial\pi^2} = 0$$

→ **Nambu-Goldstone boson**

Goldstone's theorem

For each spontaneously broken continuous symmetry, there arises a massless scalar particle, whose quantum number is the same as that of the generator of the broken symmetry.

- Does not rely on perturbation
- Does not require any *elementary scalar fields* in lagrangian
  ex. QCD, Nambu-Jona-Lasinio model

local (gauge) symmetry

$$\frac{1}{\sqrt{2}}(\sigma(x) + i\pi(x)) = \phi(x)$$ complex scalar field

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

invariant under U(1) trf. $\phi(x) \mapsto e^{i\theta}\phi(x) \quad (\theta : \text{const.})$

Require invariance under U(1) gauge trf. $\phi(x) \mapsto e^{i\theta(x)}\phi(x)$

$$\mathcal{L} = (D_\mu \phi)^* D^\mu \phi + \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

$$D_\mu \equiv \partial_\mu + igA_\mu(x)$$

If we require

$$D_\mu \phi(x) \mapsto (\partial_\mu + igA'_\mu(x))e^{i\theta(x)}\phi(x) = e^{i\theta(x)} D_\mu \phi(x)$$

the gauge field must transform as

$$A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) - \frac{1}{g}\partial_\mu \theta(x)$$

Add the gauge-inv. kinetic term of the gauge field to the lagrangian

$$\mathcal{L} = (D_\mu \phi)^* D^\mu \phi + \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

When $\mu^2 > 0$, $\langle 0|\phi(x)|0\rangle = \frac{1}{\sqrt{2}} v_0 e^{i\cdot 0}$

Expressing $\phi(x)$ as $\phi(x) = \frac{1}{\sqrt{2}}(v_0 + \eta(x))e^{i\theta(x)}$

$$(D_\mu\phi)^* D^\mu\phi = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\left(A_\mu + \frac{1}{g}\partial_\mu\theta\right)^2 (v_0 + \eta)^2$$

One can redefine $A_\mu + \frac{1}{g}\partial_\mu\theta$ as A_μ without affecting any other term in $\mathcal{L}$.

The NG mode $\theta(x)$ is *eaten* by the gauege field.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2 v_0^2 A_\mu A^\mu + \frac{1}{2}(\partial_\mu \eta)^2 - \frac{1}{2} 2\lambda v_0^2 \eta^2 + \cdots$$

The NG mode completely disappers.

The gauge field acquires mass.

$$m_A^2 = g^2 v_0^2$$

2 real scalars	***spontaneous sym. breaking*** →	1 real scalars
1 massless vector (2 degrees of freedom)		1 *massive* vector (3 degrees of freedom)

Higgs field in the Standard Model

breaks the gauge symmetry:

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \in (\mathbf{2}, \frac{1}{2}) \text{ of } SU(2)_L \times U(1)_Y$$

$$\mathcal{L} \sim (D_\mu \Phi)^\dagger D^\mu \Phi \quad \text{with} \quad D_\mu \Phi = \left(\partial_\mu + i g_2 A_\mu^a \frac{\tau^a}{2} + \frac{i}{2} g_1 B_\mu \right) \Phi$$

vacuum expectation value (VEV)

$$\langle \Phi(x) \rangle = \langle 0 | \Phi(x) | 0 \rangle = \begin{pmatrix} 0 \\ \frac{v_0}{\sqrt{2}} \end{pmatrix} \Rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$$

gauge boson mass term

$$
\begin{aligned}
(D_\mu \langle\Phi\rangle)^\dagger D^\mu \langle\Phi\rangle &\sim \frac{1}{8}(0\ v_0)\begin{pmatrix} g_2 A^3_\mu + g_1 B_\mu & g_2(A^1_\mu - iA^2_\mu) \\ g_2(A^1_\mu + iA^2_\mu) & -g_2 A^3_\mu + g_1 B_\mu \end{pmatrix}^2 \begin{pmatrix} 0 \\ v_0 \end{pmatrix} \\
&= \frac{1}{8} v_0^2 \left[g_2^2 \left((A^1_\mu)^2 + (A^2_\mu)^2 \right) + (g_2 A^3_\mu + g_1 B_\mu)^2 \right]
\end{aligned}
$$

mass eigenstates

$$
\begin{aligned}
W^\pm_\mu(x) &= \frac{1}{\sqrt{2}} \left(A^1_\mu(x) \mp i A^2_\mu(x) \right), \\
Z_\mu(x) &= A^3_\mu(x) \cos\theta_W - B_\mu(x) \sin\theta_W, \\
A_\mu(x) &= A^3_\mu(x) \sin\theta_W + B_\mu(x) \cos\theta_W,
\end{aligned}
$$

massless photon

Weinberg angle

$$
\cos\theta_W = \frac{g_2}{\sqrt{g_2^2 + g_1^2}}, \qquad \sin\theta_W = \frac{g_1}{\sqrt{g_2^2 + g_1^2}},
$$

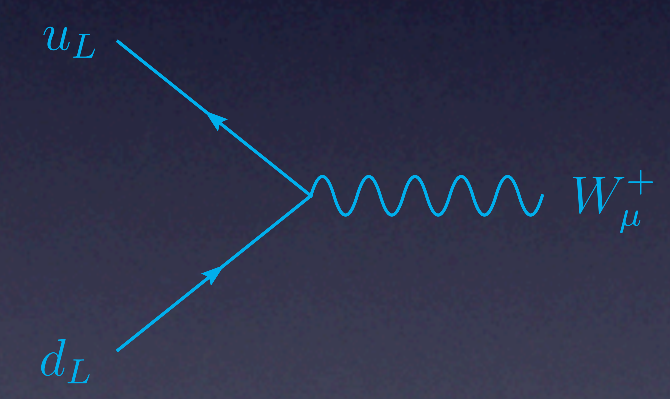
$$(D_\mu\langle\Phi\rangle)^\dagger D^\mu\langle\Phi\rangle \sim m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$$m_W = \frac{1}{2} g_2 v_0 \qquad m_Z = \frac{1}{2}\sqrt{g_2^2 + g_1^2}\, v_0$$

The value of v_0 is determined by the Fermi constant G_F.

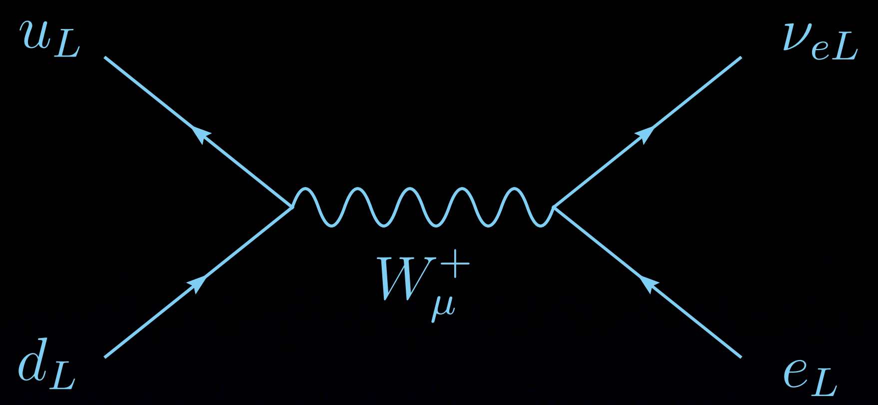
gauge interaction of the fermions

$$\mathcal{L}^{CC} = \frac{g_2}{\sqrt{2}}\left(J_\mu^-(x) W^{+\mu}(x) + J_\mu^+(x) W^{-\mu}(x)\right)$$



charged current of the fermions

$$J_\mu^-(x) = \bar{u}'_{AL}\gamma_\mu d'_{AL} + \bar{\nu}'_{AL}\gamma_\mu e'_{AL}(x) = \left(J_\mu^+(x)\right)^\dagger$$



$$\mathcal{L}_{\rm eff}^{CC} = -\frac{g_2^2}{2m_W^2} J_\mu^+(x) J^{-\mu}(x)$$

Comparing this to definition of the Fermi constant,

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_W^2} = \frac{1}{2v_0^2} \Rightarrow v_0 = \left(\frac{1}{\sqrt{2}\,G_F}\right)^{1/2} \simeq 246.26\text{GeV}$$

Experiments: $\sin^2\theta_W = 0.22, \quad e^2 = 4\pi\alpha = g_2^2 \sin^2\theta_W$

$$m_W \simeq 79.5\text{GeV} \qquad m_Z \simeq 90.0\text{GeV}$$

This predicts $\rho \equiv \dfrac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \qquad (\Delta\rho^{(\rm exp)} < 2.0 \times 10^{-3})$

another important role: generation of fermion masses

$$\mathcal{L} \sim - m\bar{\psi}\psi = -m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

no gauge-invariant mass term in the SM!

Yukawa interaction (gauge eigenstates)

$$\mathcal{L}_Y = f^{(u)}_{AB}\bar{q}'_{AL}u'_{BR}\tilde{\Phi} + f^{(d)}_{AB}\bar{q}'_{AL}d'_{BR}\Phi + f^{(e)}_{AB}\bar{l}'_{AL}e'_{BR}\Phi + \text{h.c.}$$

$$\tilde{\Phi}(x) \equiv i\tau_2\Phi^*(x)$$

$$f^{(u,d,e)}_{AB} \ : \ \text{complex matrix with } A = 1 - N_f = 3$$

$$f^{(u)} \longrightarrow \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \text{ on mass eigenstates}$$

Parameterize the Higgs field as $\Phi(x) = U(\theta(x)) \begin{pmatrix} 0 \\ (v_0 + \varphi(x))/\sqrt{2} \end{pmatrix}$

NG mode---absorbed by the gauge bosons

$$\mathcal{L}_Y = -m_A^{(u)}\left(1+\frac{\varphi}{v_0}\right)\bar{u}_A u_A - m_A^{(d)}\left(1+\frac{\varphi}{v_0}\right)\bar{d}_A d_A - m_A^{(e)}\left(1+\frac{\varphi}{v_0}\right)\bar{e}_A e_A$$

where the fermion masses are proportional to the Yukawa couplings.

$$m_A^{(u,d,e)} = \frac{1}{\sqrt{2}} y_A^{(u,d,e)}$$

Now, the charged-current interaction is expressed as

$$\mathcal{L}^{CC} = \frac{g_2}{\sqrt{2}}\left(\bar{u}_A \gamma^\mu V_{AB}^{\mathrm{CKM}} P_L d_B + \bar{\nu}_A \gamma^\mu P_L e_A\right) W_\mu^+ + \mathrm{h.c.}$$

Properties of the SM Higgs boson

☆ Mass

at the tree level, $m_h^2 = 2\lambda v_0^2$ $(v_0 = 246\text{GeV})$

No reason why m_h lies in the weak scale.

The Higgs self-coupling λ is a free parameter.

There are some theoretical constraints on the Higgs mass, as we shall see.

triviality bound

effective self-coupling at scale Q (neglecting other couplings)

$$\lambda(Q) = \frac{\lambda}{1 - \frac{3\lambda}{4\pi^2}\log\frac{Q^2}{v_0^2}}$$

diverges at $Q_{\max} = v_0 e^{2\pi^2/3\lambda}$

If the theory is valid up to cut-off $\Lambda \Rightarrow \Lambda < Q_{\max}$.

upper bound on $\lambda \to m_h^2 < \dfrac{8\pi^2 v_0^2}{3\log\frac{\Lambda^2}{v_0^2}}$

$$\Lambda = m_{\mathrm{Pl}} \simeq 10^{19}\mathrm{GeV} \to m_h < 180\mathrm{GeV}$$

$$\Lambda = 1\mathrm{TeV} \to m_h < 700\mathrm{GeV}$$

the *modest* triviality bound

unitarity bound

Lee, Quigg, Thacker, Phys. Rev. D16 ('77)

potentially dangerous coupling:

Higgs and the *longitudinal* component of the vector bosons

$$\mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

$s >> m_W^2,\ m_h > m_W$:

$$a_0(s) = -\frac{G_F m_h^2}{8\pi\sqrt{2}}\left[2 + \frac{m_h^2}{s-m_h^2} - \frac{m_h^2}{s}\log\left(1+\frac{s}{m_h^2}\right)\right]$$

$$\xrightarrow{s \gg m_h^2} -\frac{G_F m_h^2}{4\pi\sqrt{2}}$$

$$|a_0| < 1 \to m_h < \left(\frac{4\pi\sqrt{2}}{G_F}\right)^{1/2} \simeq 1.2\text{TeV}$$

experimental bounds

LEP2 experiments (95%CL)

$m_h \geq 114.4\text{GeV}$ direct search at $\sqrt{s} = 189 - 209\text{GeV}$

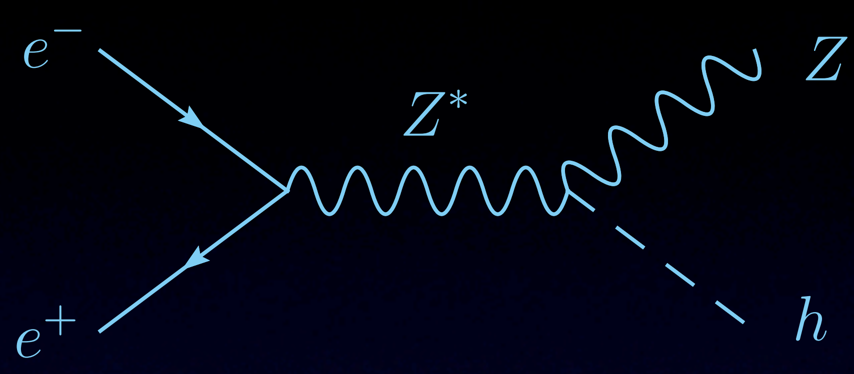
Phys. Lett. B565 ('03) 61

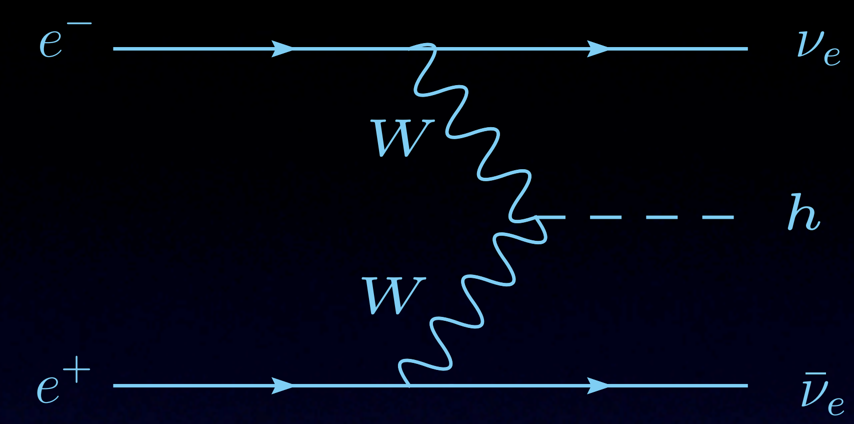
$m_h \leq 185\text{GeV}$ EW precision measurements

http://lepewwg.web.cern.ch/LEPEWWG/

☆ Production

e^+e^- collider



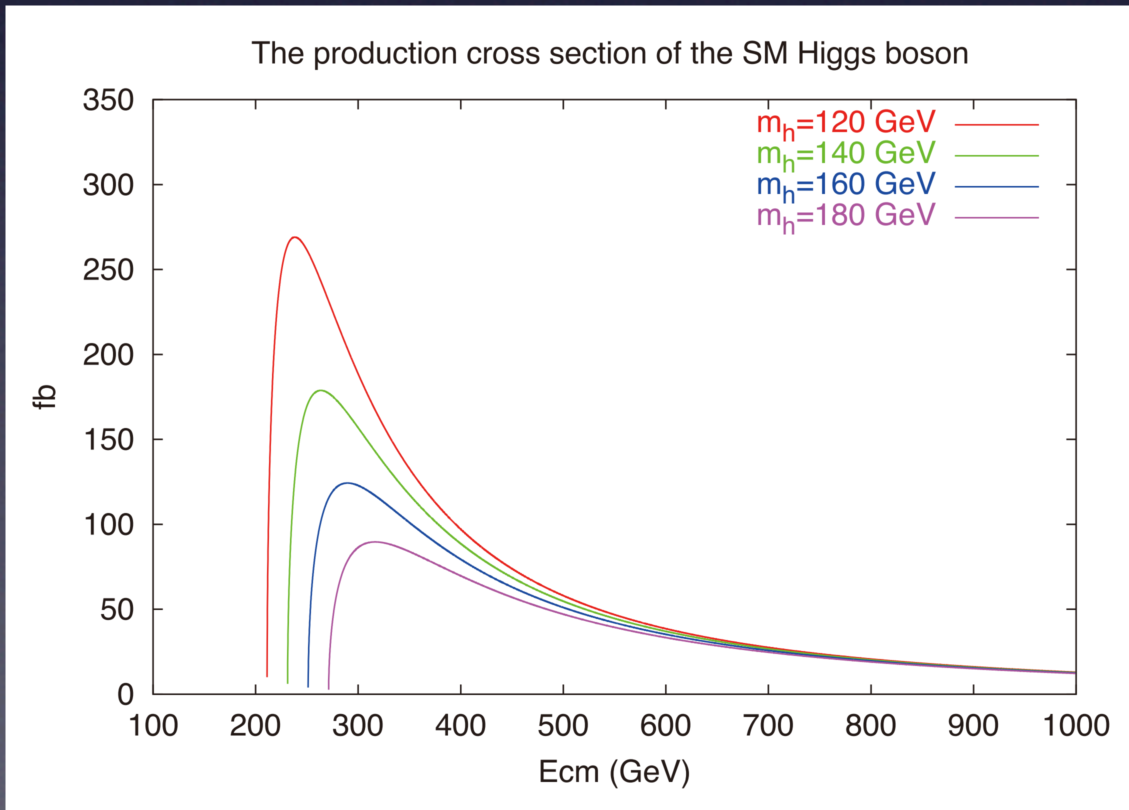


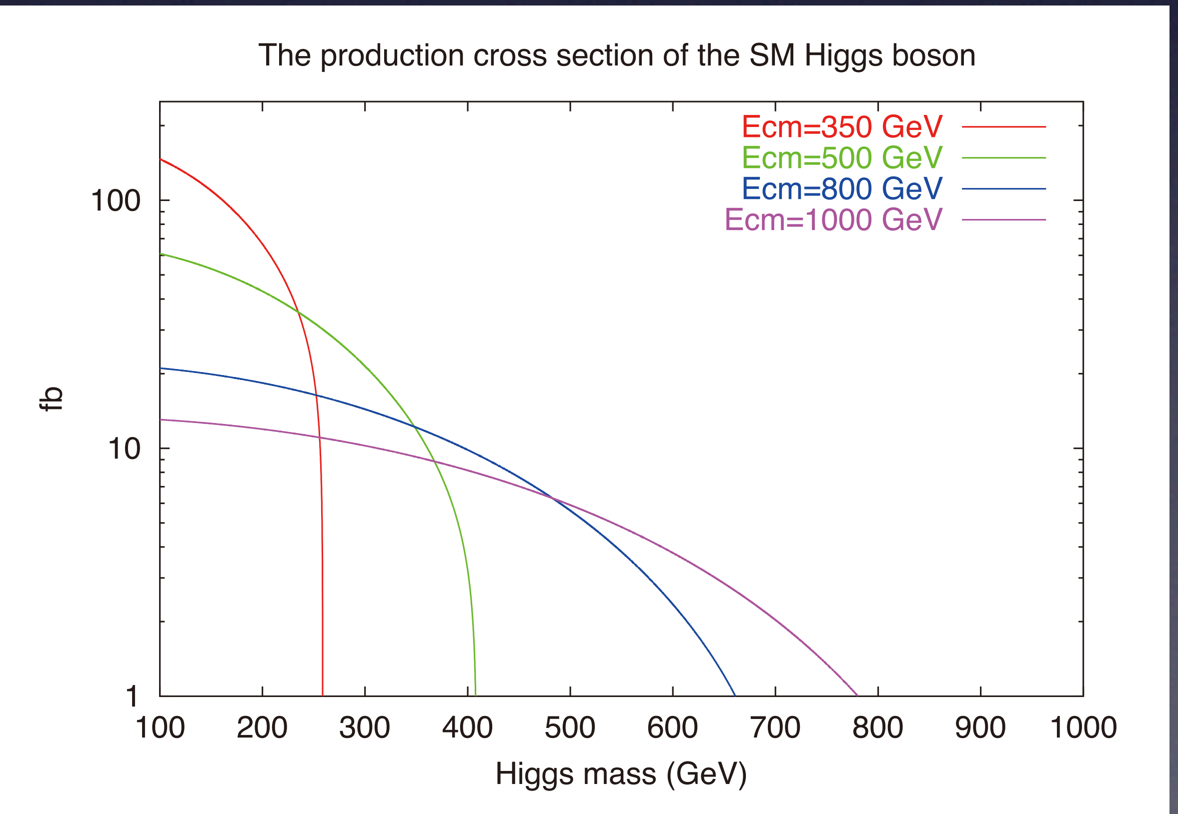
Higgs strahlung

main in LEP2

Vector boson fusion

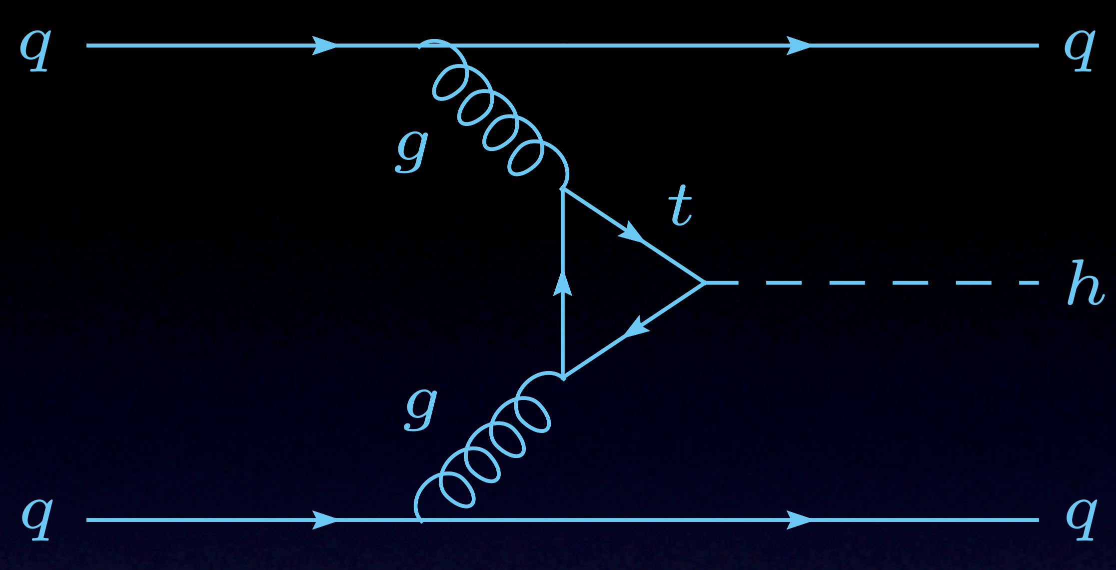
important for $\sqrt{s} > 1\text{TeV}$





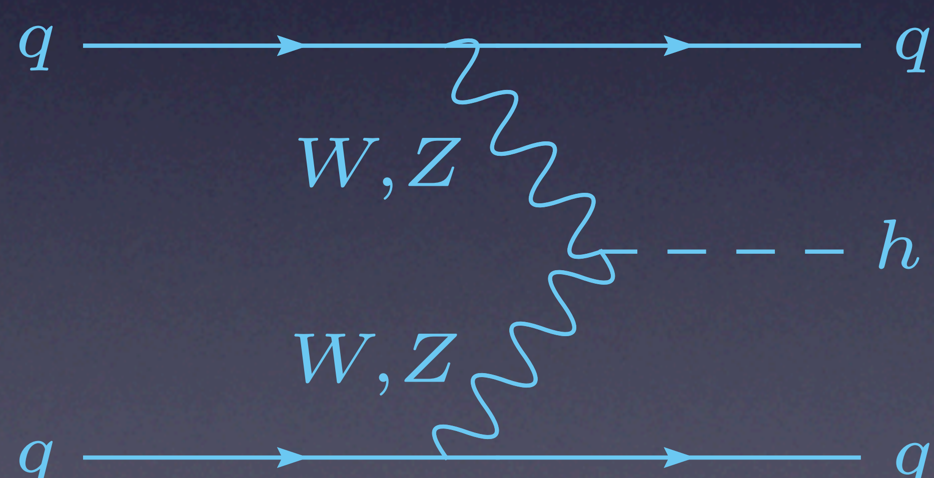
Senaha, D-thesis

LHC



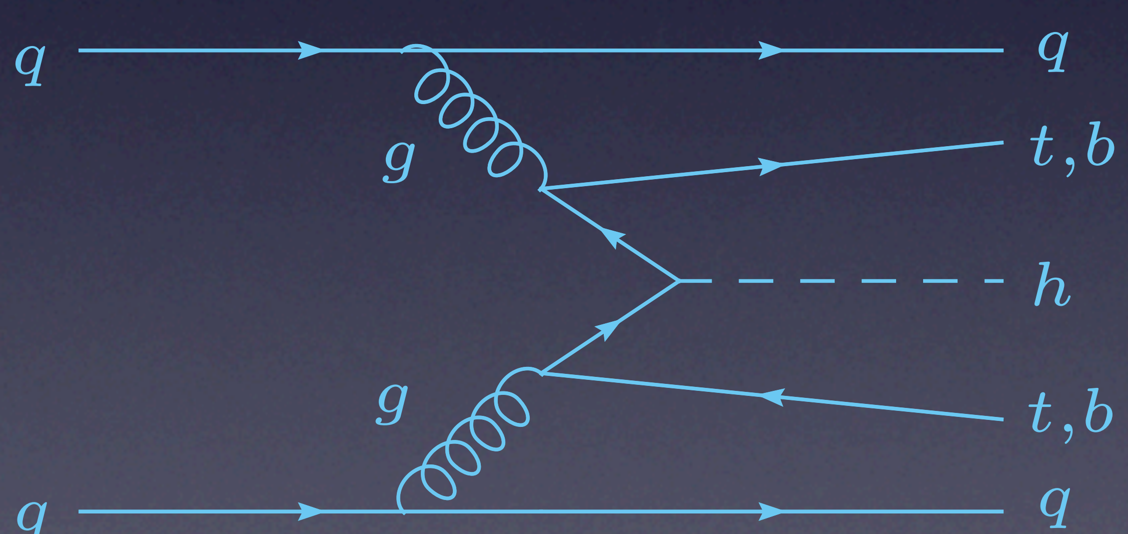
gluon fusion

the largest contribution in LHC

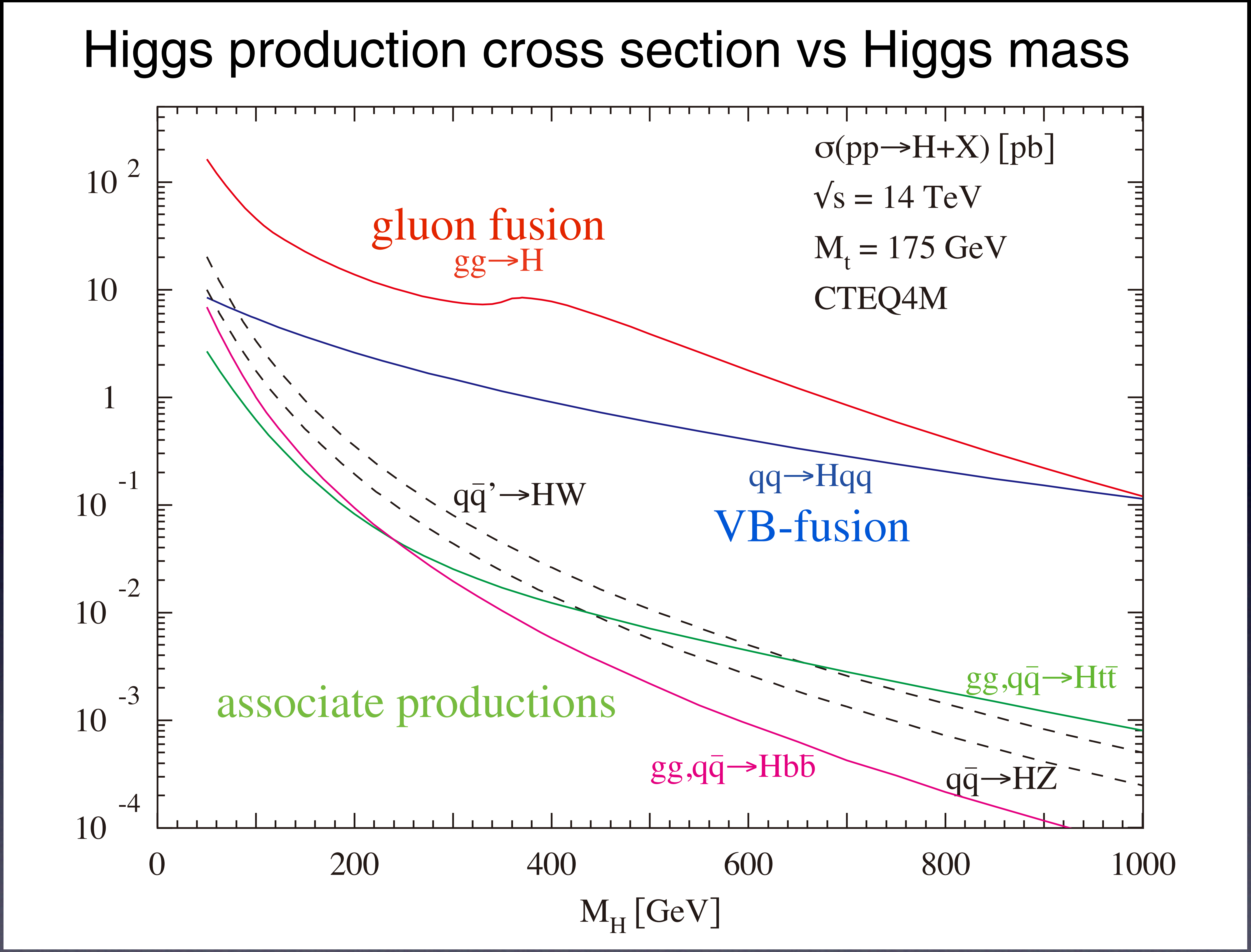


Vector boson fusion

the next largest



associate production

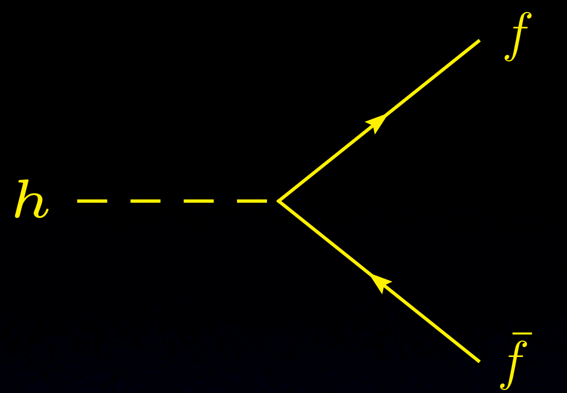


LHC at 14TeV

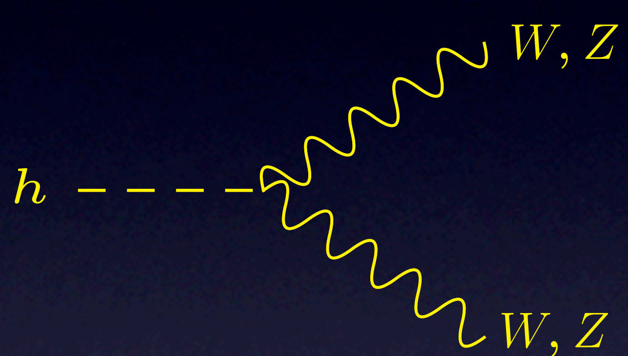
$30\,\mathrm{fb}^{-1}$ in the low luminosity phase

$300\,\mathrm{fb}^{-1}$ in the high luminosity phase

☆ Decay



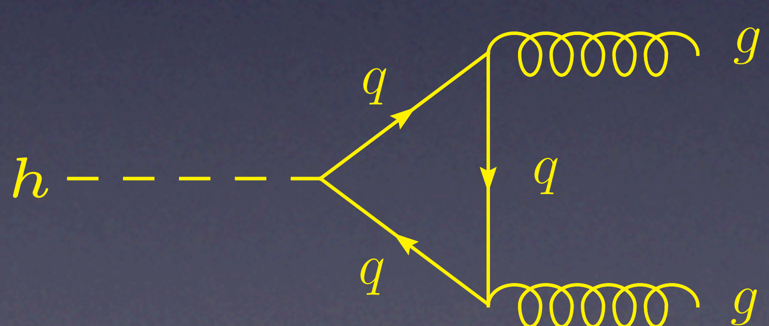
$$\Gamma(h \to f\bar{f}) = \frac{N_c m_f^2 m_h}{8\pi v_0^2}\left(1 - \frac{4m_f^2}{m_h^2}\right)^{3/2}$$

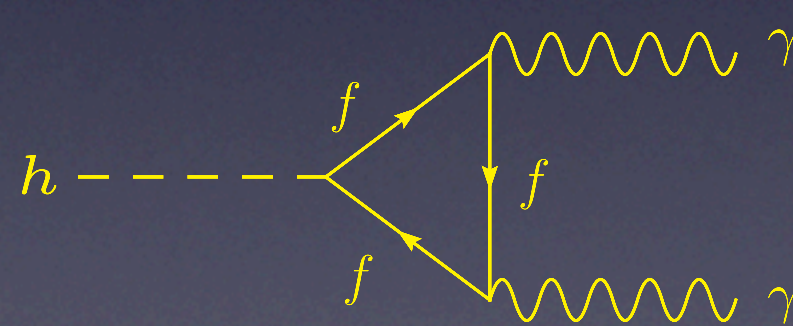


$$\Gamma(h \to VV) = \frac{C_V m_h^3}{8\pi v_0^2}\sqrt{1 - \frac{4m_V^2}{m_h^2}}\left[1 - \frac{4m_V^2}{m_h^2} + \frac{12m_V^4}{m_h^4}\right]$$

$$C_W = 2, \quad C_Z = 1$$

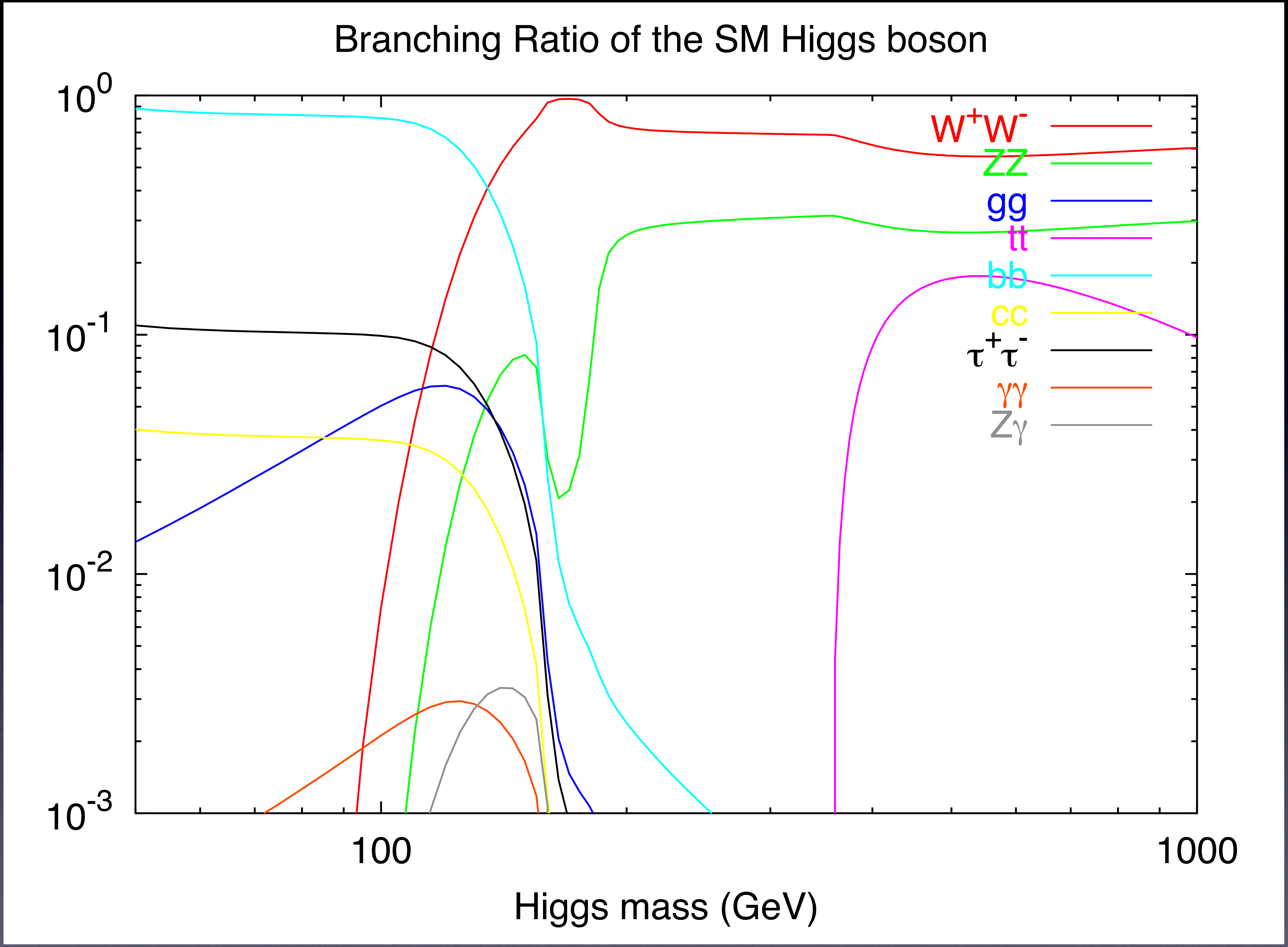
other modes





+ other charged-particle loops

and possible 3- and 4-body decays

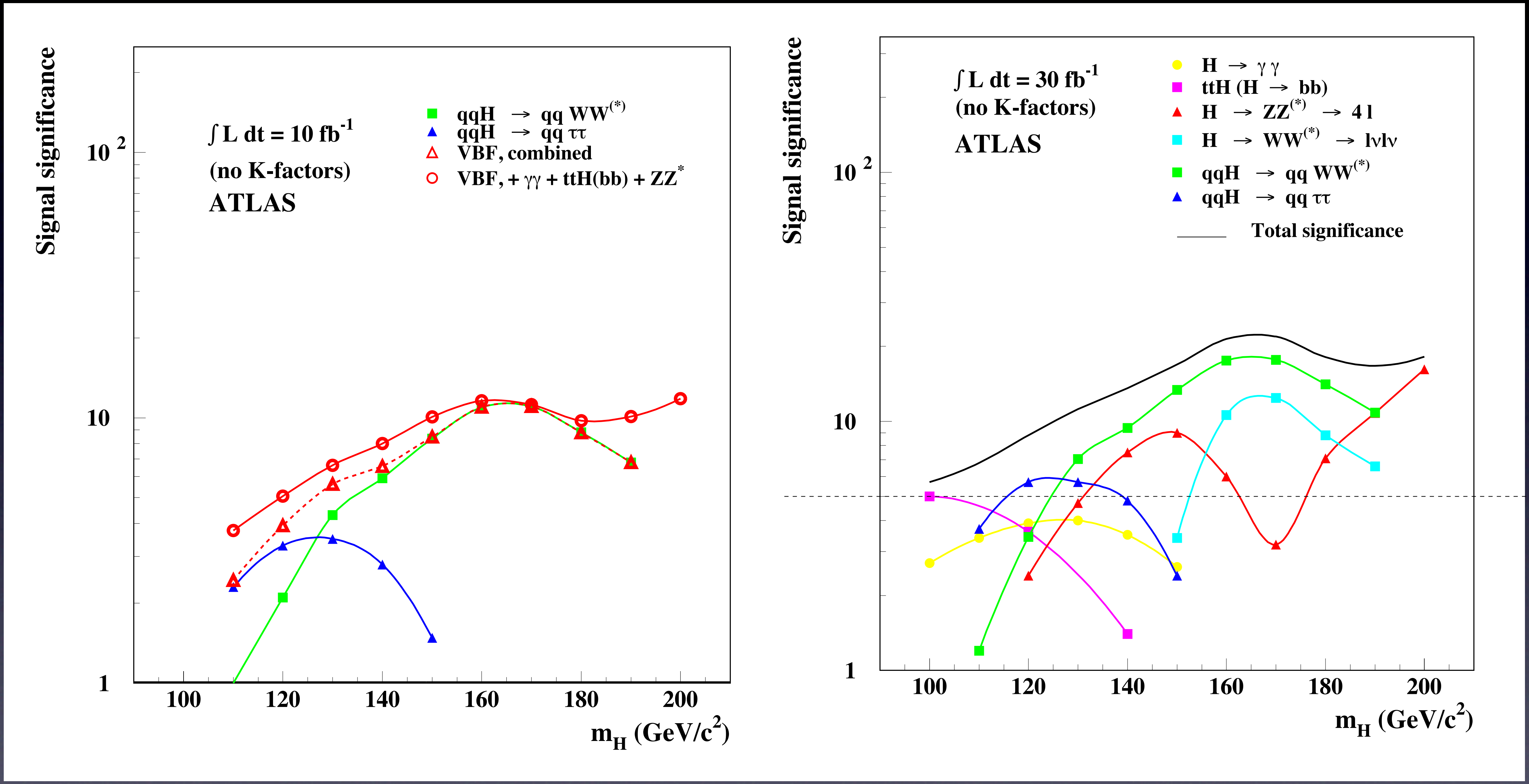


Senaha, D-thesis

Can LHC discover the Higgs boson ?

ATLAS SM Higgs sensitivity

Asai, et al. Eur. Phys. J. C32, s19–s54 (03)



For an integrated luminosity of 30 fb^{-1}, the full mass range can be covered by ATLAS with a significance exceeding 5σ.

Beyond the Standard Model

How to extend the SM:

- gauge symmetry
  ex. $SU(2)_L$ x $SU(2)_R$
- fermion generation
  3 light generations $\longleftarrow \Gamma(Z \to \nu\bar{\nu})$
- Higgs sector
  doublet or singlet $\rho^{\text{tree}} = 1$
- composite (technicolor)
- extra dimension
- ***supersymmetry***

supersymmetry

symmetry between bosons and fermions

$(\phi(x),\ \psi_L(x))$ complex scalar and chiral fermion

chiral multiplet

$(A_\mu(x),\ \chi_L(x))$ massless vector and chiral fermion

vector multiplet

example of SUSY trf. (chiral multiplet, 2-spinor notation)

$$\delta_\zeta \phi(x) = \sqrt{2}\zeta^\alpha \psi_\alpha(x)$$

$$\delta_\zeta \psi(x) = -\sqrt{2}\zeta F(x) + i\sigma^\mu \bar{\zeta} \partial_\mu \phi(x)$$

$$\delta_\zeta F(x) = i\sqrt{2}\bar{\zeta}\bar{\sigma}^\mu \partial_\mu \psi(x)$$

$F(x)$: auxiliary field (scalar)

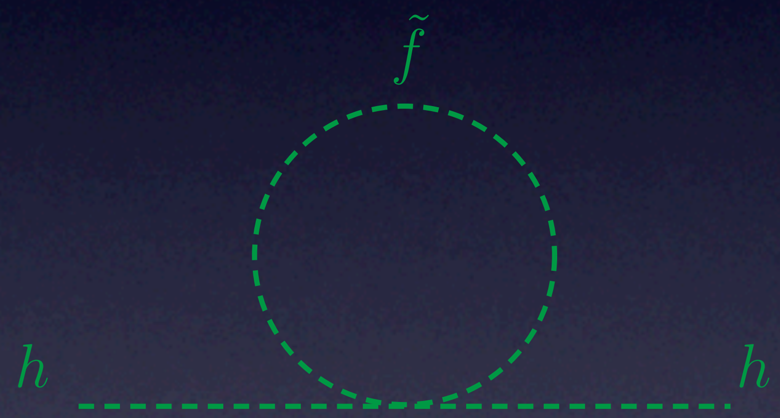
ζ_α: Grassmann spinor parameter

Why supersymmetry?

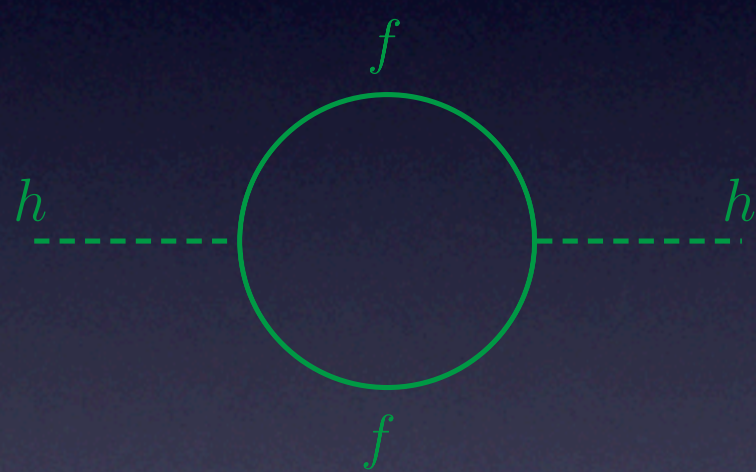
- solution to the hierarchy problem

  difference between the weak scale and cut-off scale ($m_{\rm Pl}$, $m_{\rm GUT}$)

correction to scalar mass expected to be O(m_W)



$\sim \Lambda^2$



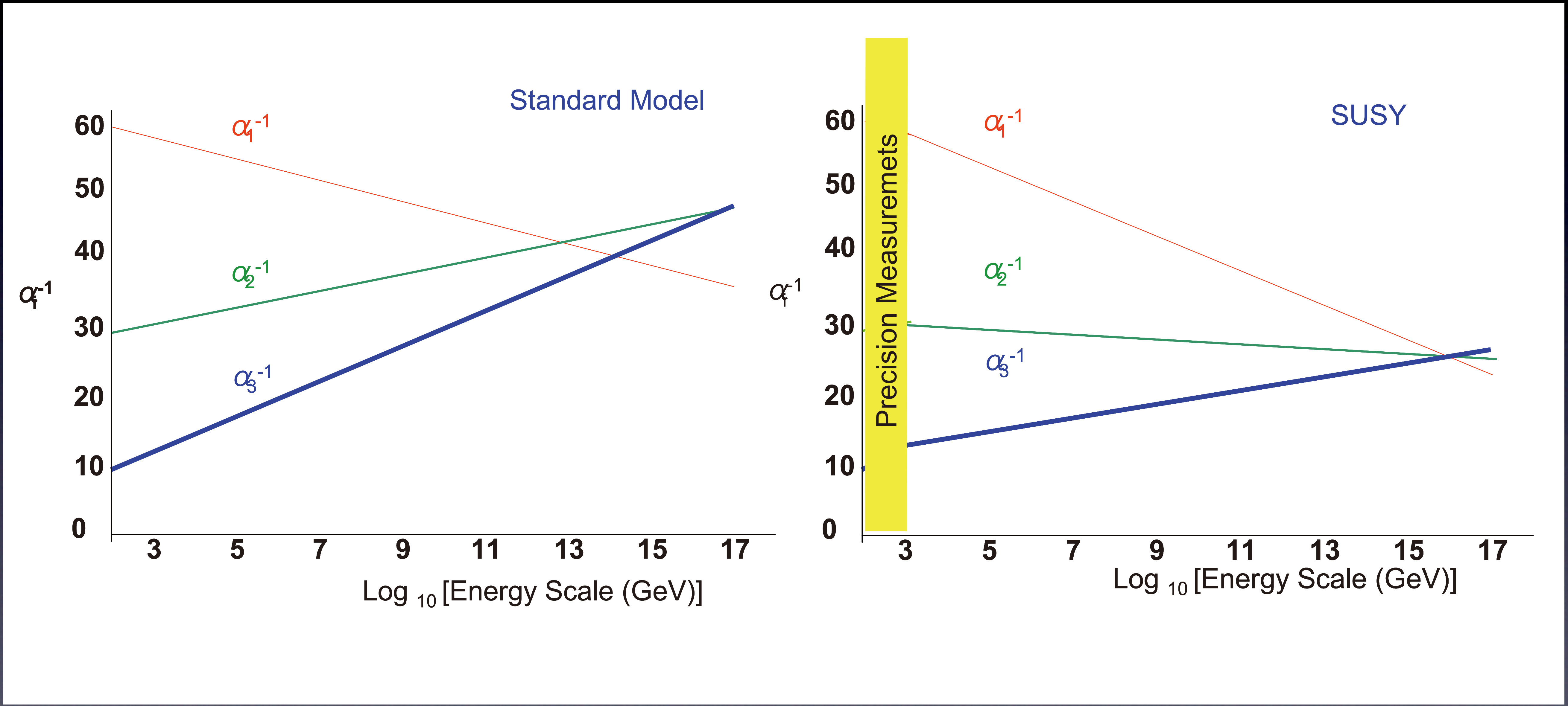
$\sim -\Lambda^2$

SUSY breaking

As long as it is *soft* (by operators of $M^{D<4}$), at most $\log\Lambda$

more likely gauge coupling unification

behavior of the effective couplings depends on particle content



Blair, SLAC Summer Institute 2005

- naturally contains a candidate for Cold Dark Matter

  CDM particle

  stable (τ>age of the Universe), weak-interacting, massive

R-parity

internal symmetry noncommuting with SUSY generator

$$R(\phi_{\rm SM}) = +1 \qquad R(\chi_{\rm SUSY}) = -1$$

Any vertex in R-conserving models contains even number of SUSY particles.

↓

The lightest SUSY particle (LSP) is stable.

If LSP is neutralino or gravitino, CDM may be composed of LSP.

How to construct a Supersymmetric SM?

No pair of SM boson and fermion
with the same quantum numbers

One must introduce a superpartner for each SM particle.

chiral multiplets (sfermion, fermion)

$$Q_A = (\tilde{q}_{AL}, q_{AL}) \quad U_A = (\tilde{u}^c_{AR}, u^c_{AR}) \quad D_A = (\tilde{d}^c_{AR}, d^c_{AR})$$

$$L_A = (\tilde{l}_{AL}, l_{AL}) \quad E_A = (\tilde{e}^c_{AR}, e^c_{AR})$$

vector multiplets (gaugino, gauge boson)

$$(\lambda_3, G_\mu) \quad (\lambda_2, A_\mu) \quad (\lambda_1, B_\mu)$$

How about the Higgs boson?

We need a pair of Higgs doublets.

$$H_u = (\Phi_u, \tilde{\Phi}_u) \quad H_d = (\Phi_d, \tilde{\Phi}_d)$$ (Higgs, Higgsino)

- gauge invariant SUSY Yukawa term

$$\mathcal{W} \sim f^{(u)}_{AB} Q_A U_B H_u + f^{(d)}_{AB} Q_A D_B H_d + f^{(e)}_{AB} L_A E_B H_d$$

different quantum numbers

- gauge anomaly cancellation

All the gauge anomalies in the SM are cancelled by each generation of quarks and leptons

one chiral multiplet → one chiral fermion

Minimal Supersymmetric Standard Model (MSSM)

matter and Higgs = chiral supermultiplet

$$Q_A = (\tilde{q}_{AL}, q_{AL}) \quad U_A = (\tilde{u}^c_{AR}, u^c_{AR}) \quad D_A = (\tilde{d}^c_{AR}, d^c_{AR})$$

$$L_A = (\tilde{l}_{AL}, l_{AL}) \quad E_A = (\tilde{e}^c_{AR}, e^c_{AR})$$

$$H_u = (\Phi_u, \tilde{\Phi}_u) \quad H_d = (\Phi_d, \tilde{\Phi}_d)$$

gauge boson = vector supermultiplet

$$(\lambda_3, G_\mu) \quad (\lambda_2, A_\mu) \quad (\lambda_1, B_\mu)$$

Supersymmetric and gauge-invariant lagrangian

$$\mathcal{L}_{\mathrm{SUSY}} = \mathcal{L}_{\mathrm{matter}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathcal{W}}$$

superpotential

$$\mathcal{W} = \epsilon_{ij} \left(f^{(e)}_{AB} H^i_d L^j_A E_B + f^{(d)}_{AB} H^i_d Q^j_A D_B - f^{(u)}_{AB} H^i_u Q^j_A U_B - \mu H^i_d H^j_u \right)$$

the only mass parameter
in the supersymmetric lagrangian

parameters in $\mathcal{L}_{\text{SUSY}}$

μ; gauge couplings: g_3, g_2, g_1 Yukawa: $f^{(u,d,e)}_{AB}$

- no degeneracy between SM and SUSY particles
- no EW symmetry breaking

⇓

***softly* supersymmetry breaking** terms
which do not affect the cancellation of divergences

$$\mathcal{L}_{\rm soft} = -\tilde{m}_1^2 \Phi_d^\dagger \Phi_d - \tilde{m}_2^2 \Phi_u^\dagger \Phi_u + \epsilon_{ij} \left(\mu B \Phi_d^i \Phi_u^j + \text{h.c.} \right)$$
$$- m_{\tilde{q}AB}^2 \tilde{q}_{AL}^\dagger \tilde{q}_{BL} - m_{\tilde{d}AB}^2 \tilde{d}_{AR}^\dagger \tilde{d}_{BR} - m_{\tilde{u}AB}^2 \tilde{u}_{AR}^\dagger \tilde{u}_{BR}$$
$$- m_{\tilde{l}AB}^2 \tilde{l}_{AL}^\dagger \tilde{l}_{BL} - m_{\tilde{e}AB}^2 \tilde{e}_{AR}^\dagger \tilde{e}_{BR}$$
$$- \epsilon_{ij} \left[\left(f^{(e)} A^{(e)} \right)_{AB} \Phi_d^i \tilde{l}_{AL}^j \tilde{e}_{BR}^* + \left(f^{(d)} A^{(d)} \right)_{AB} \Phi_d^i \tilde{q}_{AL}^j \tilde{d}_{BR}^* \right.$$
$$\left. - \left(f^{(u)} A^{(u)} \right)_{AB} \Phi_u^i \tilde{q}_{AL}^j \tilde{u}_{BR}^* + \text{h.c.} \right]$$
$$- \frac{1}{2} \left(M_3 \lambda_3^s \lambda_3^s + M_2 \lambda_2^a \lambda_2^a + M_1 \lambda_1 \lambda_1 + \text{h.c.} \right)$$

scalar mass term ~ M^2
trilinear scalar int. ~ M^3
gaugino mass term ~ M^3

$\mu B,\ f^{(u,d,e)} A^{(u,d,e)},\ M_3,\ M_2,\ M_1 \in \mathbf{C}$ ⟶ CP violation

Higgs sector in the MSSM

After eliminating the auxiliary fields D, we obtain the Higgs potential,

$$V_0 = m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u - \epsilon_{ij}(\mu B \Phi_d^i \Phi_u^j + \mathrm{h.c.}) + \frac{g_2^2 + g_1^2}{8}(\Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u)^2 + \frac{g_2^2}{2}|\Phi_d^\dagger \Phi_u|^2$$

$$(m_1^2 = \tilde{m}_1^2 + |\mu|^2, \quad m_2^2 = \tilde{m}_2^2 + |\mu|^2)$$

self-coupling ~ (gauge coupling)^2 ⟶ light Higgs boson

degrees of freedom

(2 complex doublets=8 real scalars) – 3NG modes
= 5 physical scalars

$h,\ H;\ A$: neutral scalars

$H^\pm$: charged scalar

vacuum structure (tree-level)

$$\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_4 \\ v_2 + i v_3 \end{pmatrix}$$

$$\langle V_0 \rangle = \frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2^2 (v_2^2 + v_3^2 + v_4^2) - |\mu B| v_1 v_2 + \frac{g_2^2 + g_1^2}{32} (-v_1^2 + v_2^2 + v_3^2 + v_4^2)^2 + \frac{g_2^2}{8} v_1^2 v_4^2$$

minimum: (for $m_1^2 + m_2^2 > 2|\mu B|^2$)

$$v_1 = v_0 \cos \beta_0, \quad v_2 = v_0 \sin \beta_0, \quad v_3 = v_4 = 0$$

with $$v_0^2 = \frac{8}{g_2^2 + g_1^2} \frac{m_2^2 \sin^2 \beta_0 - m_1^2 \cos^2 \beta_0}{\cos(2\beta_0)}, \quad \sin(2\beta_0) = \frac{2|\mu B|}{m_1^2 + m_2^2}$$

One usually gives $v_0 = 246\text{GeV}$ and regards $\tan \beta_0$ as a parameter to express m_1^2 and m_2^2 in terms of $(v_0, \tan \beta_0)$.

gauge boson mass

$$m_W^2 = \frac{g_2^2}{4}(v_1^2 + v_2^2) = \frac{g_2^2}{4} v_0^2, \qquad m_Z^2 = \frac{g_2^2 + g_1^2}{4} v_0^2$$

quark and lepton mass

$$m_t = \frac{y_t}{\sqrt{2}} v_2 = \frac{y_t}{\sqrt{2}} v_0 \sin\beta_0, \qquad m_b = \frac{y_b}{\sqrt{2}} v_1 = \frac{y_b}{\sqrt{2}} v_0 \cos\beta_0$$

For a fixed set of (m_t, m_b),

a larger $\tan\beta_0$ corresponds to a smaller y_t and a larger y_b.

Higgs decay branching ratio to quark pairs

Higgs mass (tree-level)

$$\Phi_d(x) = \begin{pmatrix} \frac{v_0 \cos\beta_0 + h_d + i a_d}{\sqrt{2}} \\ \phi_d^- \end{pmatrix}, \quad \Phi_u(x) = \begin{pmatrix} \phi_u^+ \\ \frac{v_0 \sin\beta_0 + h_u + i a_u}{\sqrt{2}} \end{pmatrix}$$

$$\text{mass}^2 \text{ matrix} = \langle \frac{\partial^2 V_0}{\partial\varphi_i \partial\varphi_j} \rangle \qquad \varphi = \text{fluctuation fields}$$

eigenvalues

$$m_A^2 = \frac{2|\mu B|}{\sin(2\beta_0)}$$

$$m_{h,H}^2 = \frac{1}{2}\left[m_Z^2 + m_A^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2(2\beta_0)}\right]$$

$$m_{H\pm}^2 = m_W^2 + m_A^2$$

$$m_h^2 \leq \min\{m_Z^2,\, m_A^2\}, \qquad m_H^2 \geq \max\{m_Z^2,\, m_A^2\}$$

Excluded by LEP2

large corrections from top and stop loops

Okada, Yamaguchi, Yanagida, Prog. Theor. Phys. 85('91)1

$$m_h^2 \leq m_Z^2 \cos^2(2\beta_0) + \frac{6 m_t^4}{4\pi^2 v_0^2} \log \frac{m_{\tilde{q}}^2 + m_t^2}{m_t^2}$$

The radiative corrections alter the mass^2 matrix of the neutral and charged Higgs bosons.

$$(m_H^2)^{\text{tree}} \sim m_Z^2, \qquad \Delta m_H^2 \sim y_t^4 v_0^2 + \cdots$$

$V_0 \to V_{\text{eff}}$: effective potential

neutral Higgs bosons

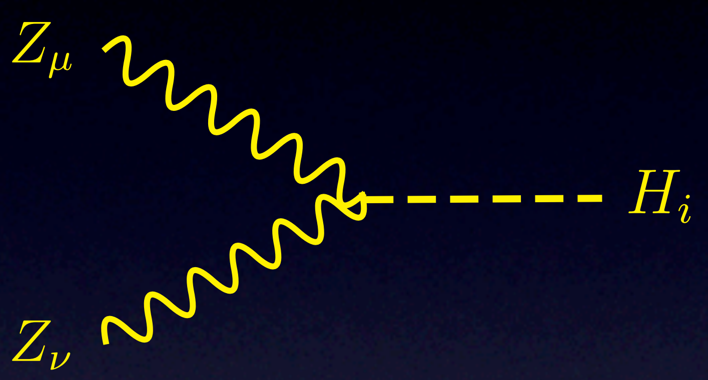
KF, Tao, Toyoda, Prog. Theor. Phys. 109 ('03) 415

$$\mathcal{M}_H^2 = \begin{pmatrix} \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d^2} \rangle & \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial h_u} \rangle & \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial a} \rangle \\ \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial h_u} \rangle & \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_u^2} \rangle & \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_u \partial a} \rangle \\ \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial a} \rangle & \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_u \partial a} \rangle & \langle \frac{\partial^2 V_{\text{eff}}}{\partial a^2} \rangle \end{pmatrix}$$

CP viol. in stop sector $\quad \text{Im}(\mu A_t) \neq 0 \Rightarrow \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_{d,u} \partial a} \rangle \neq 0$

$\longrightarrow$ scalar-pseudoscalar mixing

$$O_H^{-1}\mathcal{M}_H^2 O_H = \begin{pmatrix} m_{H_1}^2 & & \\ & m_{H_2}^2 & \\ & & m_{H_3}^2 \end{pmatrix}$$



$$= i\frac{g_2 m_W}{\cos^2\theta_W} g_{VVH_i} g_{\mu\nu}$$

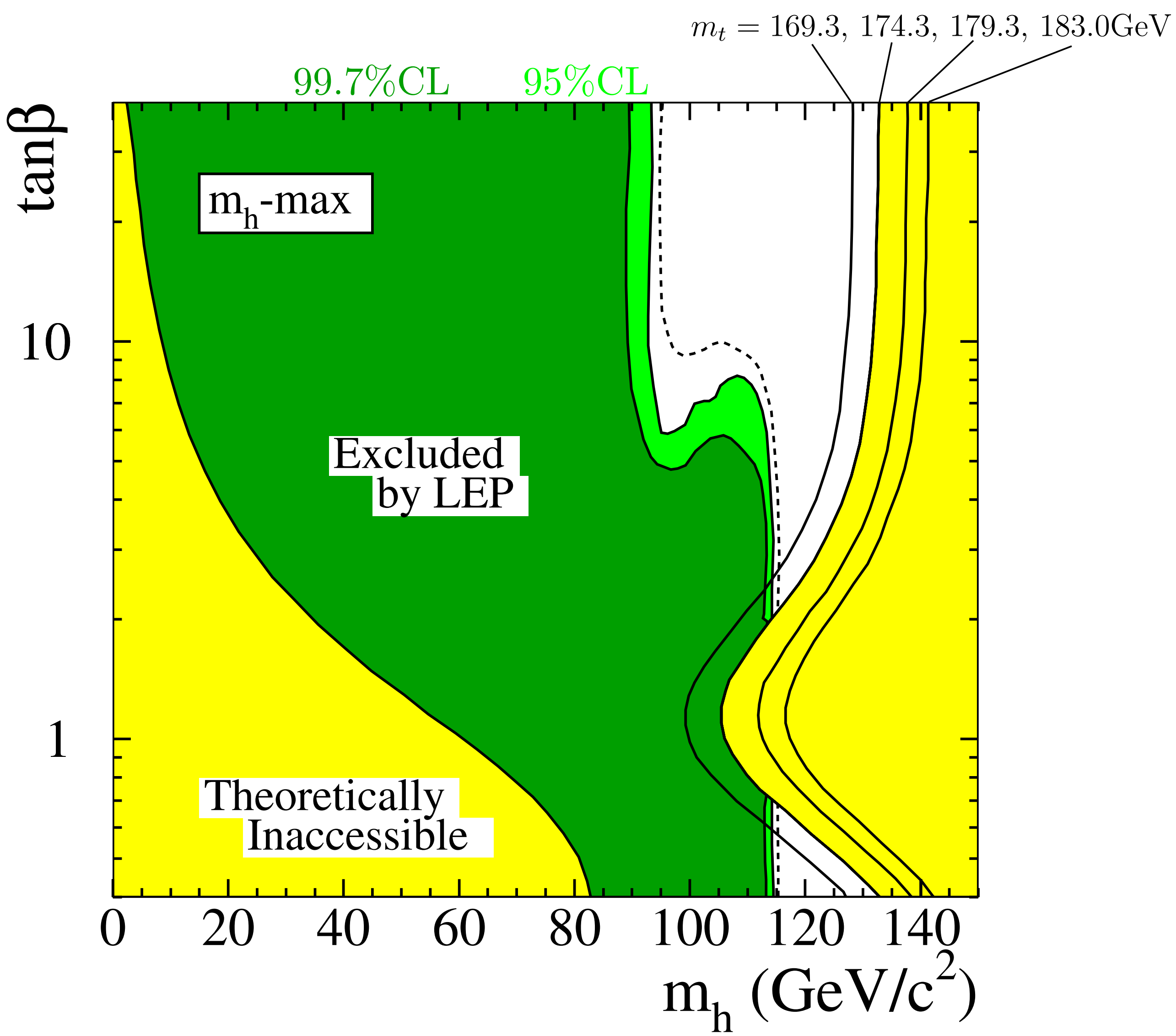
$$g_{VVH_i} = (O_H)_{1i}\cos\beta_0 + (O_H)_{2i}\sin\beta_0$$

Depending on the parameters, a Higgs boson lighter than 114GeV could *not* be produced by the Higgs-strahlung process, so that is *not* excluded by LEP2.

light Higgs scenario

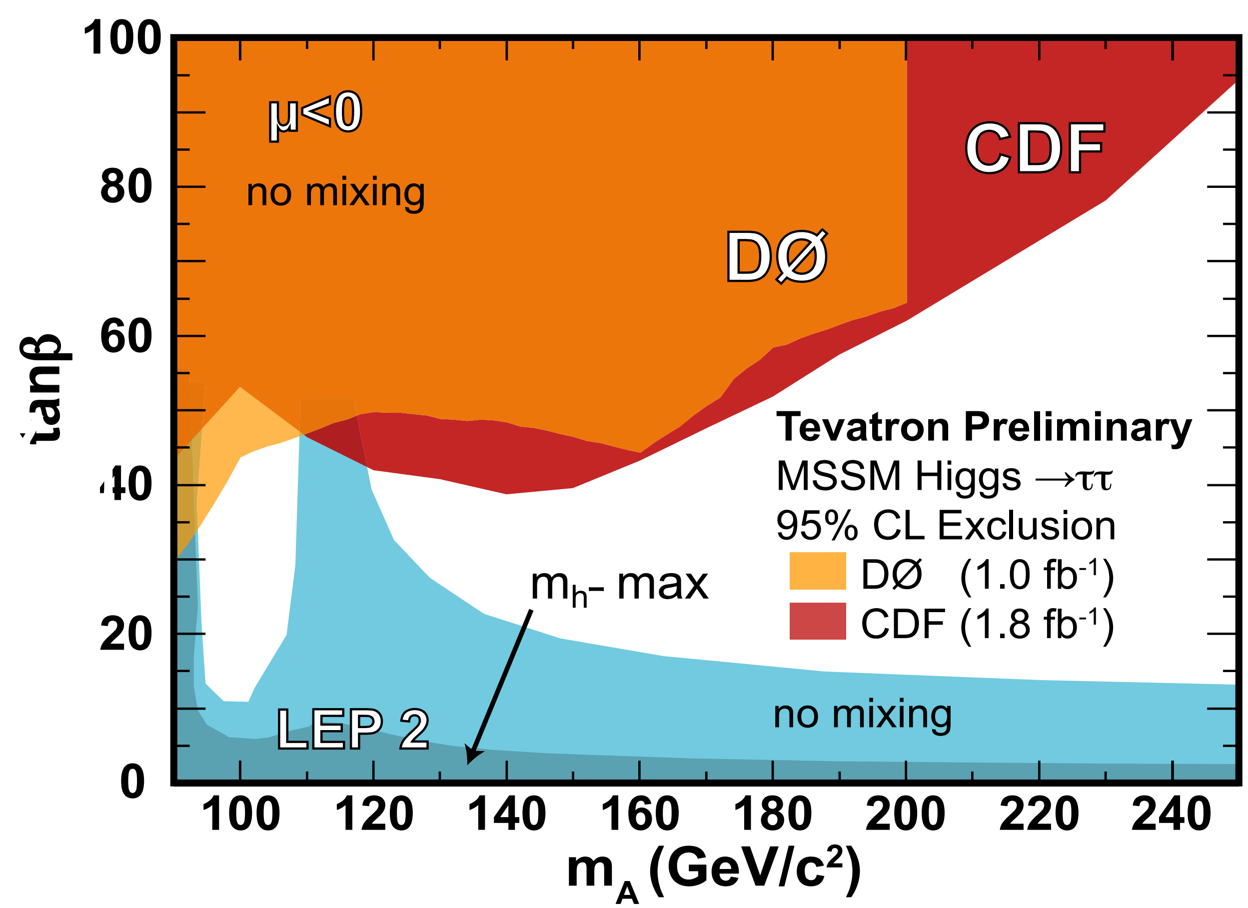
Experimental bounds on Higgs masses

allowed region for the lightest neutral Higgs boson



m_h-max benchmark scenario

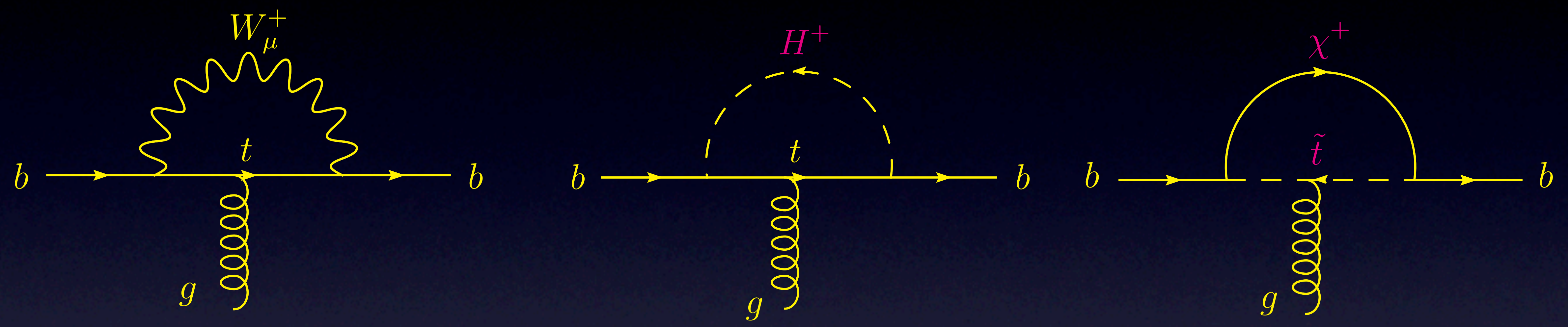
allowed region for the pseudoscalar Higgs boson



Particle Data 2008, "Search for Higgs Bosons" in *Reviews, Tables and Plots*

Phenomenology in the MSSM

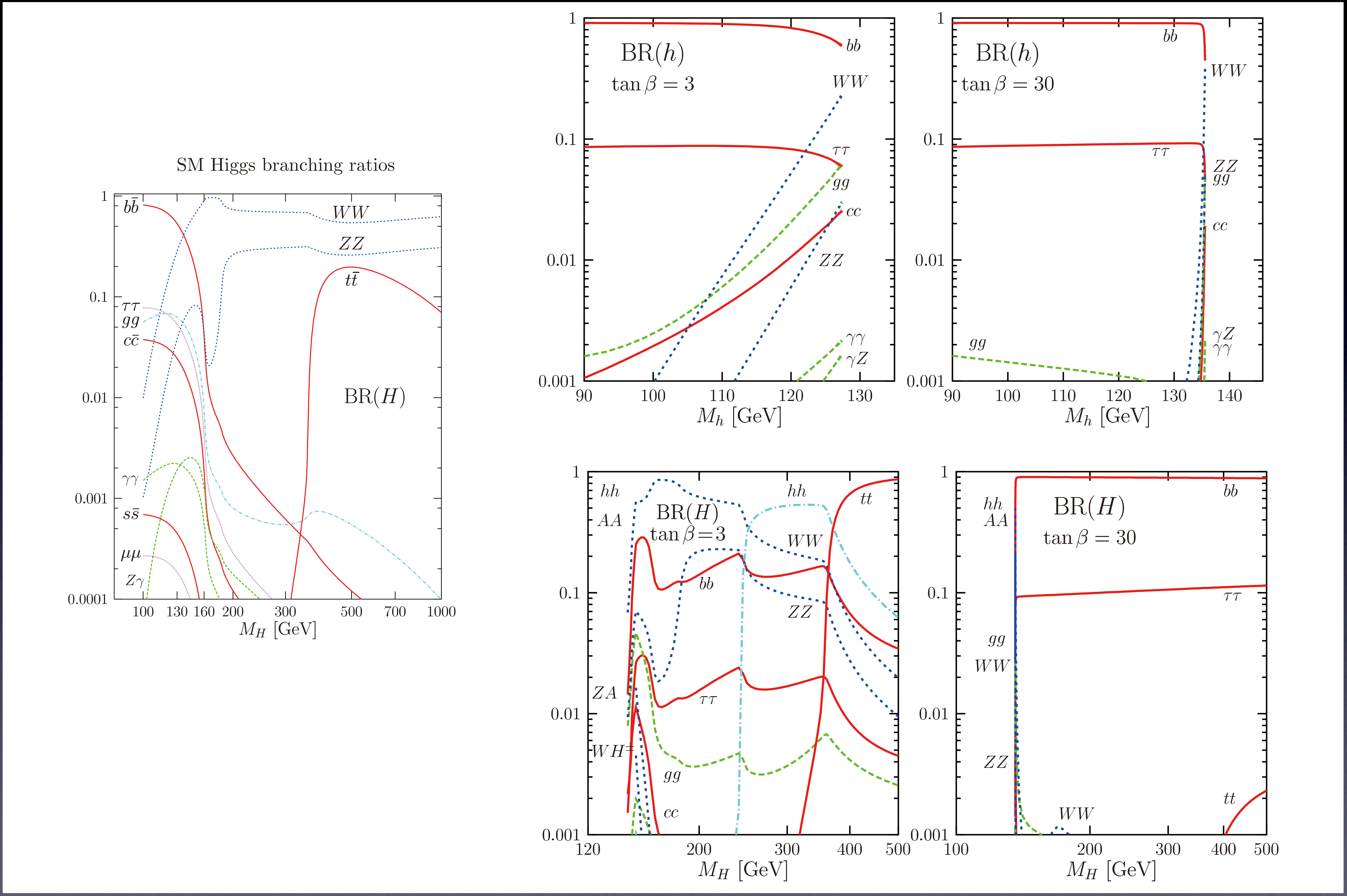
- SUSY corrections to SM amplitudes



- Many parameters
  - spectrum of SUSY particles
  - mixing --- chargino, neutralino, squarks, sleptons
  - new source of CP violation

relative phases among $\mu,\ A_f,\ B, M_3, M_2, M_1$

Branching Ratios of the CP-even Higgs bosons



Djouadi, hep-ph/0503173

Theoretical issues

- **Supersymmetry Breaking**

  soft SUSY-breakng parameters

  scalar mass, A_f, B, gaugino mass

- **μ-problem**

  There is no principle to determine the value of μ-parameter.
  However, it must be in the weak scale for EW symmetry breaking.

  fine-tuning problem *not* resolved in the MSSM?

  This may be resolved by the Next-to-MSSM.

$$\mu H_d H_u \leftarrow \lambda \langle N \rangle H_d H_u$$

Epilogue

A Higgs boson may be discovered within 3–5 years at LHC.

In order to find which Higgs it is, we must know its properties, such as mass, width and decay BR.

A **SUSY particle** may also be discovered.

squark and/or gluino

When a SUSY particle is discovered,

$m_h \geq 135\text{GeV}$ ⟶ MSSM is excluded.

$m_h \leq 135\text{GeV}$ ⟶ We must study properties of SUSY particles.

I hope that
we are present at discovery of new physics and
at opening of next stage of particle physics.