Spontaneous Symmetry Breaking and Restoration at High Temperatures

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There has been a long-time belief that

the fundamental theory of Nature be the simplest.

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large class of symmetries

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complexity of the real world

Breaking of the symmetries will reconcile
the simple world with the complex reality.

various kinds of symmetry breaking

explicit breaking   so slightly that some remnant can be found

a quark mass violates the chiral symmetry

KM phase in the CKM quark mixing matrix violates CP

spontaneous breaking

We shall see this in some detail later.

anomaly   breaking by the regularization

mass scale (cut off)

conformal symmetry
chiral symmetry
supersymmetry

Spontaneous Symmetry Breaking (SSB)

A symmetry of the Lagrangian broken by the vacuum (ground state)

While $\mathcal{L}(\hat{\phi})$ is invariant (up to a total deriv.) under

$$\hat{\phi} \mapsto \hat{\phi}' = \hat{U}\hat{\phi}\hat{U}^{-1} = R\,\hat{\phi},$$

representation matrix

the vacuum is not invariant:

$$\hat{U}|0\rangle \neq |0\rangle$$

For a continuous symmetry, $\hat{U} = e^{i\hat{Q}}$, this is equiv. to

$$\hat{Q}|0\rangle \neq 0$$

An example : ferro-magnetism

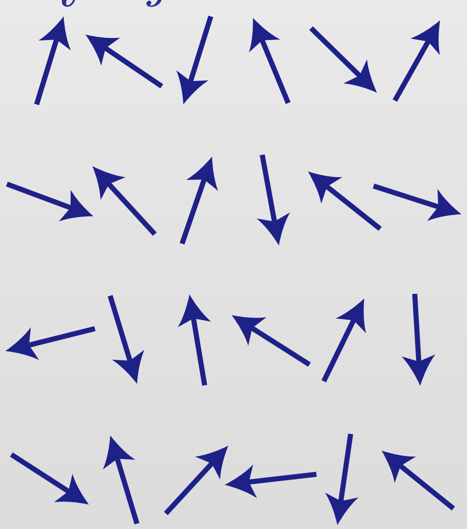
Hamiltonian of the spin model

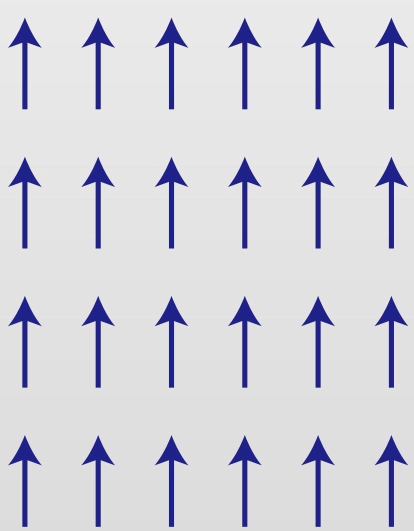
$$H = -\kappa \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$$

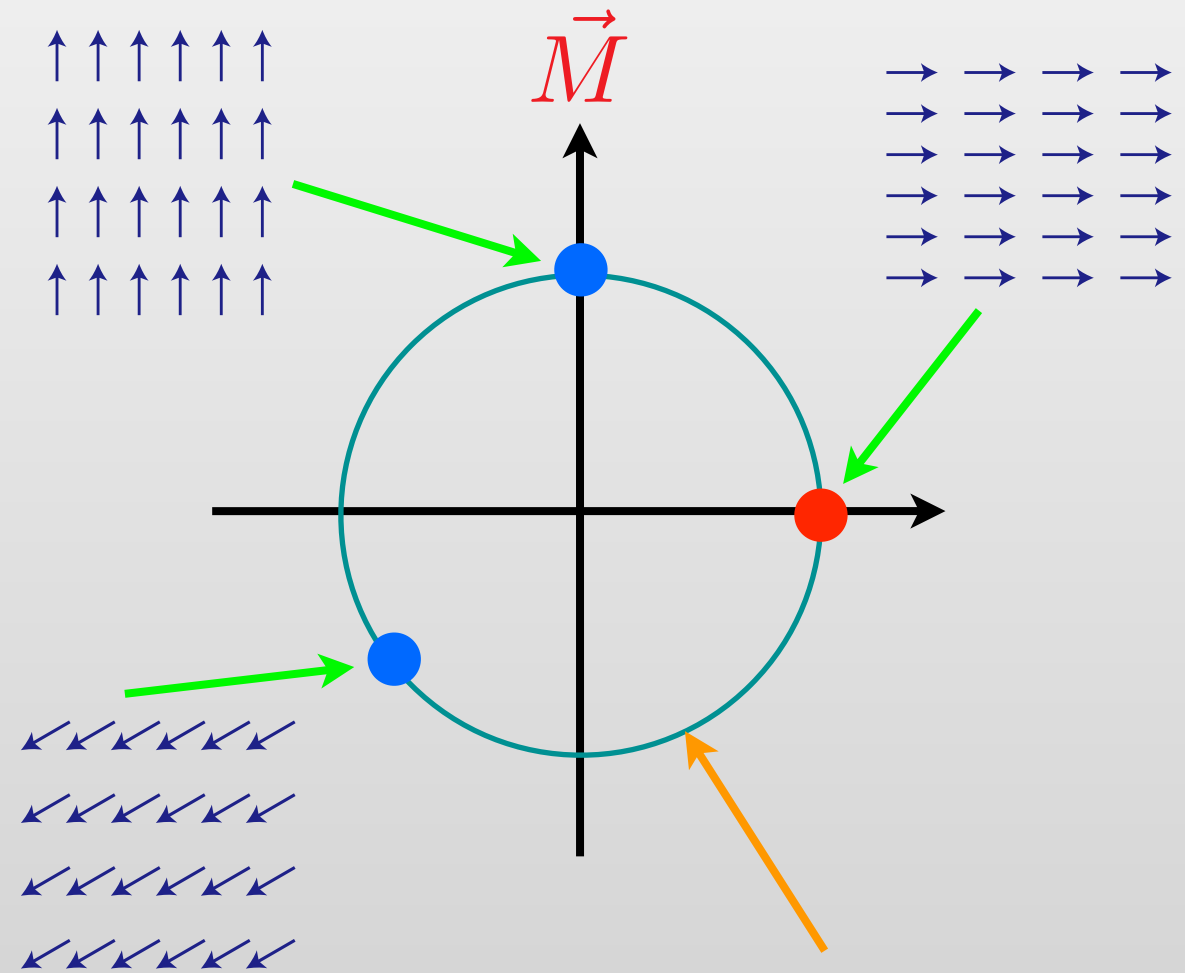
invariant under the spatial rotation

the lowest E

$\vec{s}_i$ $\vec{s}_j$







$M = \langle s_i \rangle = 0$

$M \neq 0$

breaks the rotational symmetry

In QFT, the symmetry is broken by the vacuum,

if an operator which transforms nontrivially has nonzero VEV.

vacuum expectation value

$$\hat{U}\hat{\varphi}_i\hat{U}^{-1} = R_{ij}\hat{\varphi}_j \neq \varphi_i \quad (R_{ij} \neq \delta_{ij})$$

proof)

Let $v_i \equiv \langle 0|\hat{\phi}_i|0\rangle$.

if $\hat{U}^{-1}|0\rangle = |0\rangle \implies v_i \equiv \langle 0|\hat{U}\hat{\varphi}_i\hat{U}^{-1}|0\rangle = R_{ij}\langle 0|\hat{\varphi}_j|0\rangle = R_{ij}v_j$

$(R_{ij} - \delta_{ij})v_j = 0$ holds for ${}^\forall R \neq \delta$ iff $v_i = 0$

対偶 ↓ contraposition

$$v_i \neq 0 \implies \hat{U}|0\rangle \neq |0\rangle$$

As long as the Lorentz symmetry is not broken,
any operator that can acquire nonzero VEV must be a scalar.

e.g.

Higgs field in the SM

$$\mathcal{L} = (D_\mu \Phi)^\dagger D^\mu \Phi - \left[-\mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2\right] + \text{Yukawa int.}$$

invariant under $SU(2)_L \times U(1)_Y$ gauge trf.

$$\Phi_i(x) \mapsto R_{ij}(x)\Phi_j(x)$$
$$\Phi_i^\dagger(x) \mapsto \Phi_j^\dagger(x) R_{ji}^\dagger$$

$v_i \equiv \langle 0|\Phi_i(x)|0\rangle \neq 0$ ⟶ Symmetry is broken

scalar operator

Chiral condensate in the QCD

$$\mathcal{L} = \sum_f \bar{q}_f(x) i\gamma^\mu D_\mu q_f(x)$$

$q_f(x)$: flavor-f, color triplet Dirac spinor

Lorentz symmetry ⟶ $\langle 0 | q_f(x) | 0 \rangle = 0$

$\mathcal{L}$ invariant under $SU(3)_c$ and $U(N_f)_L \times U(N_f)_R$ trf.

$$\simeq SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_A$$

axial anomary

$\bar{q}q = \bar{q}_L q_R + \bar{q}_R q_L$ is *not* invariant under $SU(N_f)_L \times SU(N_f)_R$.

$\langle 0 | \bar{q}(x) q(x) | 0 \rangle \neq 0 \Longrightarrow SU(N_f)_L \times SU(N_f)_R$ is broken to $SU(N_f)_V$

⟶ $\mathrm{dim} SU(N_f) = N_f^2 - 1$ massless NG bosons

For $N_f = 2$, three π mesons

How symmetry is borken depends on the representation of the operator which acquires VEV.

e.g. $SU(2)$ gauge-Higgs system

doublet $\mathcal{L} = (D_\mu \Phi)^\dagger D^\mu \Phi - \left[-\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2\right]$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{fully breaks } SU(2)$$

triplet $\mathcal{L} = D_\mu \boldsymbol{\phi} \cdot D^\mu \boldsymbol{\phi} - \left[-\mu^2 \boldsymbol{\phi} \cdot \boldsymbol{\phi} + \lambda (\boldsymbol{\phi} \cdot \boldsymbol{\phi})^2\right]$

$$\langle \boldsymbol{\phi} \rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \quad \text{breaks } SU(2) \text{ to } O(2) \simeq U(1).$$

$$\begin{pmatrix} O_2 & 0 \\ 0 & 1 \end{pmatrix} \langle \boldsymbol{\phi} \rangle = \langle \boldsymbol{\phi} \rangle$$

Physical consequence depends on
whether the symmetry is global or local.

global sym. Nambu-Goldstone theorem

massless bosons of the same quantum number as the broken generator

local (gauge) sym. Higgs mechanism

massive gauge boson (no massless boson)

other masses, former prohibited by the sym.

fermion mass from the Yukawa coupling

Some kinds of SSB are excluded
--- restrictions from cosmology ---

Consider the Friedmann-Robertson-Walker Universe:

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - R_0^2 a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

scale factor
relative to the present

curvature parameter

radiation dominant era :

$$\rho(t) \propto a(t)^{-4} \qquad a(t) \propto T^{-1} \propto t^{1/2},$$

matter dominant era :

$$\rho(t) \propto a(t)^{-3} \qquad a(t) \propto t^{2/3}$$

stable topological objects

Classical solutions to nonlinear field equations

The conservation of the topological charge guarantees their stability.

Once created in the early Universe, they become to dominate the energy of the Universe to ruin the successful Big Bang Cosmology.

Reviews on the classical solutions in field theories:

S. Coleman, `Classical Lumps and their quantum descendants' in the textbook *Aspects of Symmetry*

Cheng and Li, Chapter 15 of *Gauge theory of elementary particle physics*

domain wall ← SSB of a global discrete symmetry

$\mathbf{Z}_2$ $\quad \mathcal{L}(-\phi) = \mathcal{L}(\phi)$ $\quad \langle\phi\rangle \neq 0$

$\langle\phi\rangle = v$ $\qquad \langle\phi\rangle = -v$

energy per unit area $\sim v^3$

$$\rho_{\rm DW} \sim \frac{v^3 a(t)^2}{a(t)^3} \propto \frac{1}{a(t)}$$

dominates over radiation or matter in the late era

Example of discrete sym.:

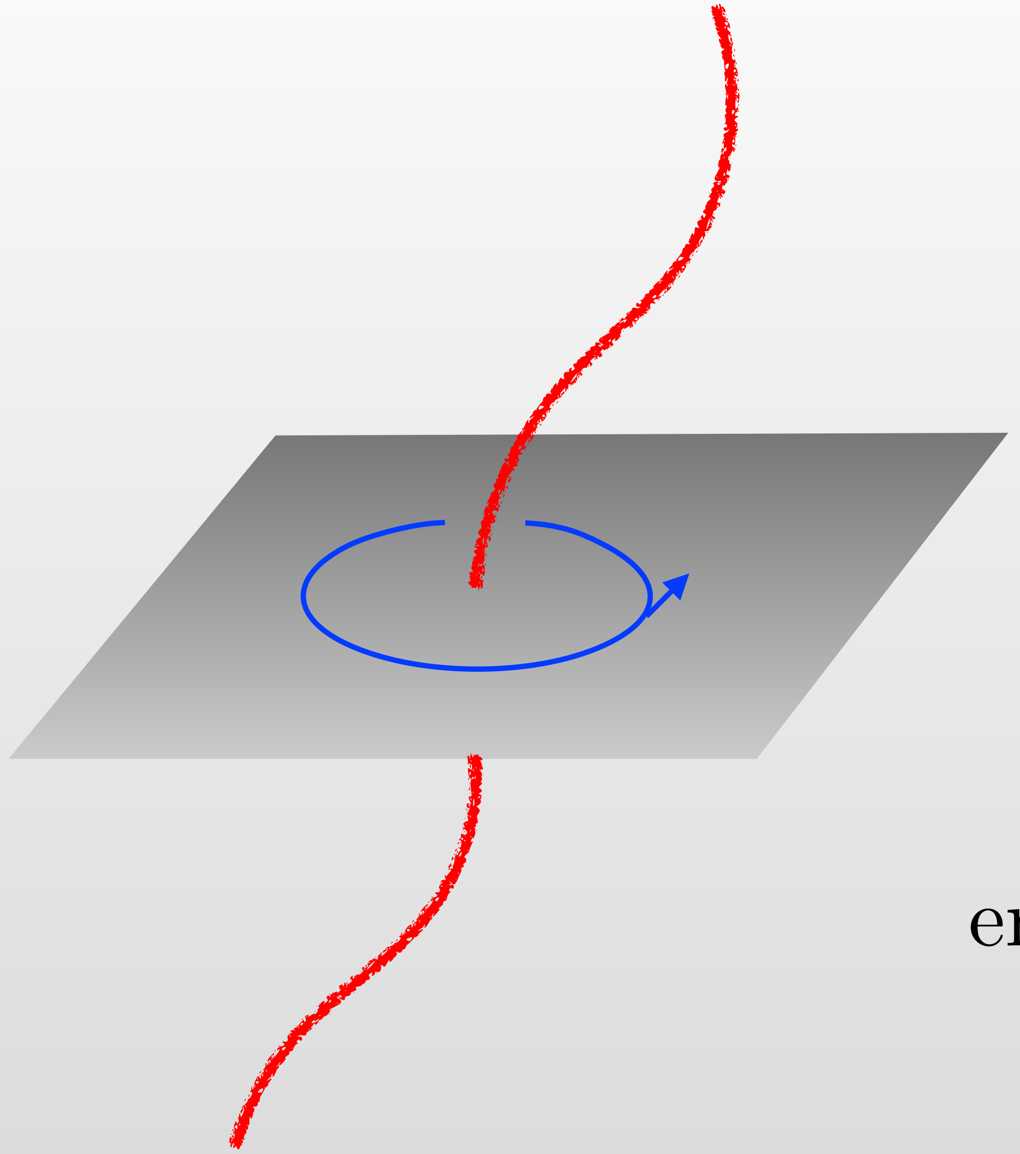
internal $\mathbf{Z}_N$ $\quad V(\phi) \ni \phi^N + \text{h.c.} + |\phi|^4$

$\phi \mapsto e^{2\pi i n/N}\phi \quad (n = 0, 1, \cdots N-1)$

CP symmetry

R-parity in a SUSY model

string or vortex ← SSB of $U(1)$ gauge symmetry



$v = 0$ within the string

$v \neq 0$ outside of the string

phase of $\phi(x)$: $0 \to 2\pi$
around the string

energy density per unit length $\sim v^2$

$$\rho_{\rm string} \sim \frac{v^2 a(t)}{a(t)^3} \propto \frac{1}{a(t)^2}$$

nonabelian monopole ← SSB of $SU(2)$ gauge symmetry to $U(1)$

magnetically charged w.r.t. the $U(1)$

This SSB could occur in some GUTs.

Number of the monopoles > 0.01 x Number of baryons

$$\text{mass scale} \sim v_{\mathrm{GUT}} \gtrsim 10^{15}\mathrm{GeV}$$

Absence of these topological objects is required to a Beyond-the-Standard Model.

How to study a spontaneous symmetry breaking?

Evaluate $\langle 0|\hat{\varphi}|0\rangle$

$\hat{\varphi} = \text{some scalar operator with nontrivial representation}$

Formally, it is expressed, in terms of the path integral, as

$$\langle 0|\hat{\varphi}|0\rangle = \int [d\phi]\,\varphi[\phi] e^{iS[\phi]} \qquad \phi = \{\Phi(x), A_\mu(x), \psi(x), \cdots\}$$

Lattice MC calculation (compact gauge fields)

$\hat{\varphi}$: gauge-invariant operator, otherwise $\langle\hat{\varphi}\rangle = 0$

$\langle \bar{q}(x) q(x)\rangle\,,\ \langle \Phi^\dagger(x)\Phi(x)\rangle\,, \cdots$ can be computed

A perturbative method to study a SSB

effective potential

$$V_{\text{eff}}(v) = \langle 0|\mathcal{H}(\hat{\phi}, \hat{\psi}, \hat{A}_\mu, \cdots)|0\rangle\Big|_{\langle\hat{\varphi}\rangle = v}$$

$$|0\rangle|_{\langle\hat{\varphi}\rangle = v} = \text{the vacuum satisfying } \langle\hat{\varphi}\rangle = v$$

the v minimizing $V_{\text{eff}}(v)$ is the VEV we're looking for

Reviews on the effective potential:

S. Coleman, `Secret Symmetry' in the textbook *Aspects of Symmetry*

e.g. real ϕ^4 theory $\qquad \mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$

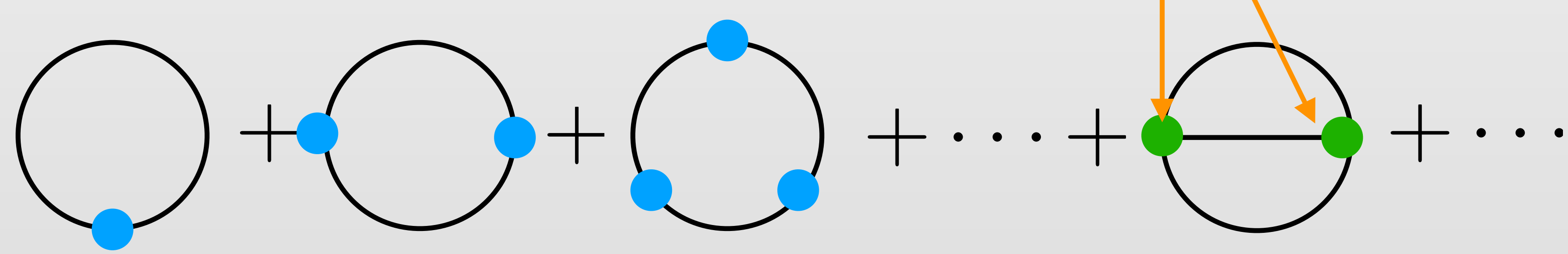
μ^2 could be positive or negative

$$V_{\text{eff}}(v) = V_{\text{tree}}(v) + \text{all vacuum diagrams in the presence of } v$$

$$\mathcal{L} \to \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\left(\mu^2 + \frac{\lambda}{2}v^2\right)\phi^2 - \frac{\lambda}{6}v\phi^3 - \frac{\lambda}{4!}\phi^4$$

mass insertion

$= V_{\text{tree}}(v) +$



2-loop and more

$$= \frac{1}{2}\mu^2v^2 + \frac{\lambda}{4!}v^4 + i\int\frac{d^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\lambda v^2/2}{k^2-\mu^2}\right)^n + \cdots$$

$$= \frac{1}{2}\mu^2v^2 + \frac{\lambda}{4!}v^4 - \frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\log\left(1 - \frac{\lambda v^2/2}{k^2-\mu^2}\right) + \cdots$$

UV divergent

after renormalization,

$$= \frac{1}{2}\mu^2 v^2 + \frac{\lambda}{4!}v^4 + \frac{(\mu^2+\lambda v^2/2)^2}{64\pi^2}\left(\log\frac{\mu^2+\lambda v^2/2}{M^2} - \frac{3}{2}\right) + \cdots$$

M : renormalization scale

the tree level min. at $\begin{cases} v = 0 & (\mu^2 \geq 0) \\ v = \sqrt{6\mu^2/\lambda} & (\mu^2 < 0) \end{cases}$

is modified by the loop corrections.

Even if $\mu^2 \geq 0$, the min. of $V_{\mathrm{eff}}(v)$ is realized at $v \neq 0$ for very small $|\mu^2|$.

That is, the SSB is caused by the radiative corrections.

Coleman-Weinberg mechanism

S. Coleman and E. Weinberg, Phys. Rev. D8 (1973)

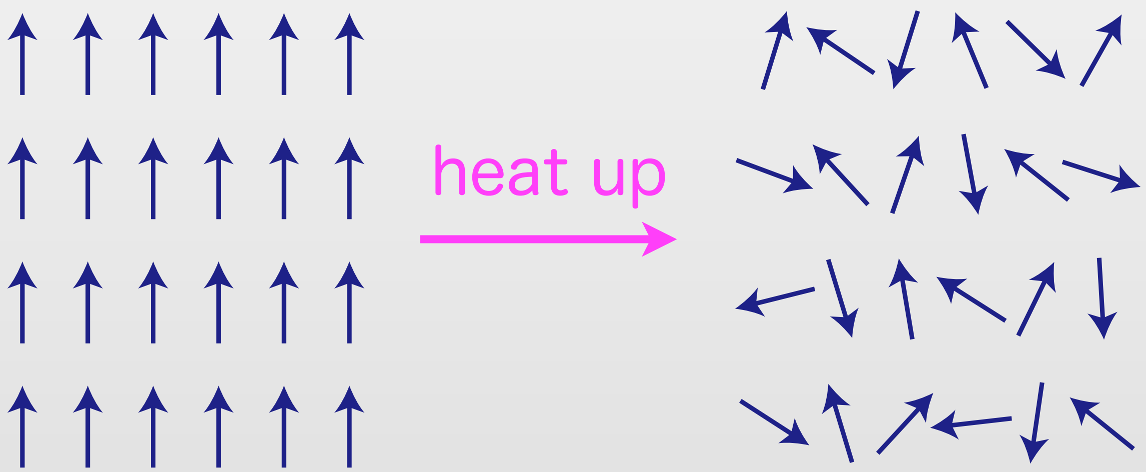
We shall go on to the other topic,

finite-temperature behavior

of spontaneously broken symmetries.

the example of ferro-magnetism

Heating up a magnet looses its magnetism.



$T >$ Curie temperature

Fe: 1043K

Symmetry restoring
phase transition

One may expect a similar symmetry restoring
Phase Transition to in a SSB QFT.

We can apply the equilibrium statistical mechanics to study

static features of the phase transition,

when the Hubble parameter is less than the interaction rates.

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{8\pi G}{3}\rho_r(t)} < \Gamma(t)$$

interaction rate

determined by
the cross section and number density

interaction rate

for relativistic species $\Gamma^{-1} = \bar{t} \simeq \lambda$ mean free path

$m \lesssim T$

total cross section of that species $\sigma \simeq \frac{\alpha^2}{s} \simeq \frac{\alpha^2}{T^2}$

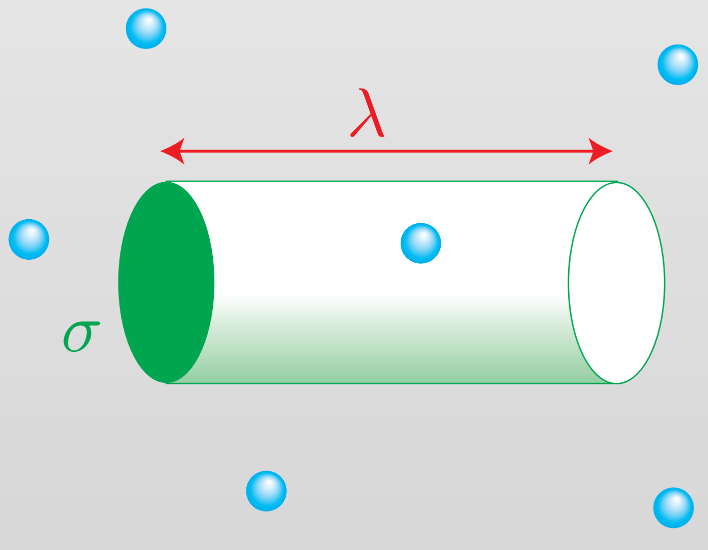
for the weak interaction,

$$\alpha = \frac{g_2^2}{4\pi} = \frac{\alpha_{\rm em}}{\sin^2\theta_W}$$

number density $n(T) \simeq g_{*n} \frac{\zeta(3)}{\pi^2} T^3$

$$g_{*n} = \sum_B g_B + \frac{3}{4}\sum_F g_F$$

effective degrees of freedom



$$\sigma \cdot \lambda = \frac{1}{n(T)}$$

$$\bar{t} = \lambda \simeq \frac{10}{g_{*n} T^3} \left(\frac{\alpha^2}{T^2}\right)^{-1} = \frac{10}{g_{*n}\alpha^2 T}$$

expansion rate $H(T) = \sqrt{\frac{8\pi G}{3}\rho_r(T)} \simeq 1.66\sqrt{g_*}\frac{T^2}{M_{\rm Pl}}$ $\quad M_{\rm Pl} = 1.22\times 10^{19}{\rm GeV}$

$$\rho_r(T) = g\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{|\boldsymbol{p}|}{e^{|\boldsymbol{p}|/T}\mp 1} = g\begin{Bmatrix} 1 \\ 7/8 \end{Bmatrix}\frac{\pi^2}{30}T^4$$

$$g_* \equiv \sum_B g_B + \frac{7}{8}\sum_F g_F \qquad g_* = 106.75$$

when all the SM particles are relativistic

at $T = 100{\rm GeV}$

$$H(T) = 1.66\sqrt{106.75}\times\frac{10^4}{1.22\times 10^{19}}{\rm GeV} \simeq 10^{-14}{\rm GeV}$$

$\wedge\!\!\wedge$

$$\Gamma(T) = g_{*n}\frac{\alpha(T)^2 T}{10} = 10^3\alpha(T)^2{\rm GeV} = (1-10){\rm GeV}$$

↑ EW ↑ QCD

At temperatures of the weak scale, we can safely regard
all the SM particles are in thermal equilibrium at O(100)GeV.

Quantum Field Theory at finite temperatures

Kapusta and Gale, 'Finite-temperature field theory' (2006) 2nd ed.
Le Bellac, 'Thermal Field Theory' (2000)
Landsman and van Weert, Phys. Rep. 145 (1987) 141
Dolan and Jackiw, Phys. Rev. D9 (1974) 3320

zero-temp. QFT

$$\langle \hat{\mathcal{O}} \rangle = \langle 0 | \hat{\mathcal{O}} | 0 \rangle$$

$$V_{\text{eff}}(v) = \langle 0 | \mathcal{H} | 0 \rangle |_{\langle \varphi \rangle = v}$$

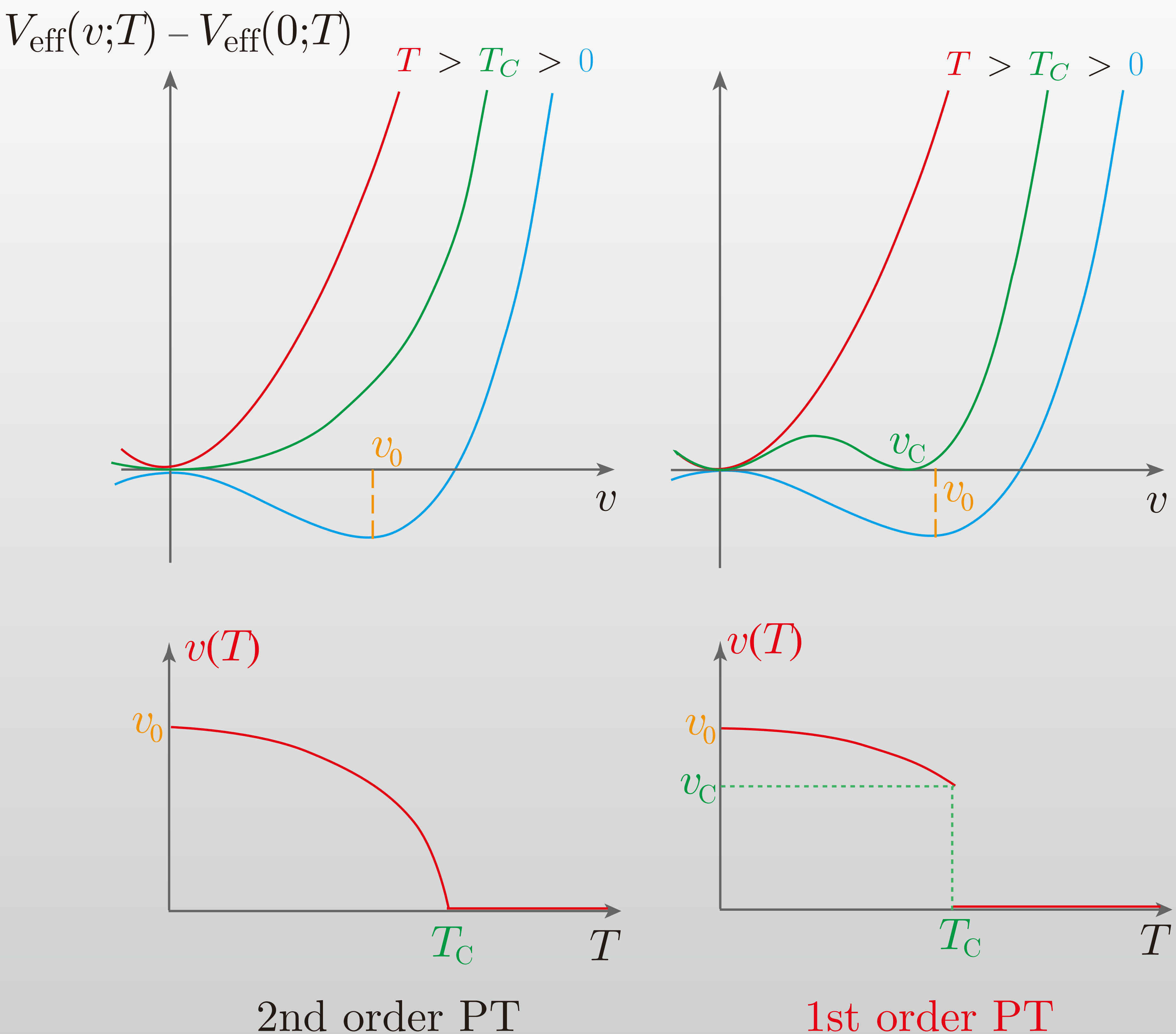
finite-temp. QFT

$$\langle \hat{\mathcal{O}} \rangle = \mathrm{Tr}[\hat{\mathcal{O}} e^{-\hat{H}/T}]/Z(T)$$

$$Z(T) = \mathrm{Tr}[e^{-\hat{H}/T}]$$

$$V_{\text{eff}}(v;T) = (\text{free energy density at } T)_{\langle \varphi \rangle = v}$$

free energy vs order parameter (Higgs VEV) at finite T



order parameter

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(T) \end{pmatrix}$$

$$v_C \equiv \lim_{T \uparrow T_C} v(T) \neq 0$$

↕

1st order phase transition

Path-integral representation of the partition function

$$\mathrm{Tr}(e^{-H/T}) = N(T) \int_{\mathrm{pbc}} [d\phi] \, \exp\left(- \int_0^{1/T} d^4x_E \, \mathcal{L}_E(\phi) \right)$$

euclidean
path integral

$$\begin{cases} \phi(0, \boldsymbol{x}) = \phi(1/T, \boldsymbol{x}) & \text{boson} \quad k^0 = i\omega_n = i\pi 2nT \\ \psi(0, \boldsymbol{x}) = -\psi(1/T, \boldsymbol{x}) & \text{fermion} \quad k^0 = i\tilde{\omega}_n = i\pi(2n+1)T \end{cases}$$

Differences from QFT at T=0

propagators

$$\langle T\phi(x)\phi(0)\rangle \sim \frac{i}{k^2 - m^2} \to \frac{i}{-\omega_n^2 - \boldsymbol{k}^2 - m^2}$$

$$\langle T\psi(x)\bar{\psi}(0)\rangle \sim \frac{i}{\gamma\cdot k - m} \to \frac{i}{i\gamma_0\tilde{\omega}_n - \boldsymbol{\gamma}\cdot\boldsymbol{k} - m}$$

momentum int.

$$\int \frac{d^4k}{(2\pi)^4} \to iT\sum_n \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}$$

Wick's theorem ⟶ Bloch-De Dominicis theorem

Standard Model

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\Phi(x) = e^{i\theta_a(x)\tau_a/2} \begin{pmatrix} 0 \\ (v + h(x))/\sqrt{2} \end{pmatrix}$$

$$V = (-\mu^2 + \lambda v^2) h + \frac{1}{2}(-\mu^2 + 3\lambda v^2) h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

$$\text{tree-level min. at } v_0 = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\text{tree-level mass of the higgs} : m_h^2 = -\mu^2 + 3\lambda v_0^2 = 2\lambda v_0^2$$

For simplicity, we consider the 1-loop corrections from the top quarks, W- and Z-bosons.

Formulas for the 1-loop corrections

1-loop corr. to the effective potential for the real scalar theory

$$-\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\log\left(1-\frac{\lambda v^2/2}{k^2-\mu^2}\right)=-\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\log\big(k^2-m_\phi^2(v)\big)+v\text{-indep. const.}$$

$$m_\phi^2(v)=\mu^2+\frac{\lambda}{2}v^2$$

$$\longrightarrow\frac{T}{2}\sum_n\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\log\left(\omega_n^2+\boldsymbol{k}^2+m_\phi^2(v)\right)\qquad\omega_n=2\pi nT$$

$$=\frac{1}{2}\int\frac{d^4k_E}{(2\pi)^4}\log\big(k_E^2+m_\phi^2(v)\big)-T\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\log\left(1-e^{-\sqrt{\boldsymbol{k}^2+m_\phi^2(v)}/T}\right)$$

euclidean 4-momentum

renormalization

$$=\frac{m_\phi^4(v)}{64\pi^2}\left(\log\frac{m_\phi^2(v)}{M^2}-\frac{3}{2}\right)+\frac{T^4}{2\pi^2}\int_0^\infty dx\,x^2\log\left(1-e^{-\sqrt{x^2+m_\phi^2(v)/T^2}}\right)$$

$$V_{\text{eff}}(v;T) = -\frac{\mu^2}{2}v^2 + \frac{\lambda}{4}v^4 + \sum_{A=t,W,Z} c_A \left[F(m_A^2(v)) + I_A(m_A(v)/T)\right]$$

renormalized 1-loop corr. finite-T corr.

statistics and degrees of freedom $c_t = -4 \cdot 3_{\text{color}} = -12, \qquad c_W = 6, \qquad c_Z = 3$

$$= -\frac{\mu^2}{2}v^2 + \frac{\lambda}{4}v^4 + 2Bv^4\left(\log\frac{v^2}{v_0^2} - \frac{3}{2}\right) + \bar{V}(v;T)$$

where

$$B = \frac{3}{64\pi^2 v_0^4}\left(2m_W^4 + m_Z^4 - 4m_t^4\right) \qquad \text{(coupling)}^4$$

$$\bar{V}(v;T) = \frac{T^4}{2\pi^2}\left(6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)\right) \qquad a_A = \frac{m_A(v)}{T}$$

$$I_{B,F}(a) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right)$$

High-T expansion of $I_{B,F}(a) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right)$

$a = m/T \ll 1$

IR nonanalyticity

$$I_B(a) = -\frac{\pi^4}{45} + \frac{\pi^2}{12} a^2 - \frac{\pi}{6} \left(a^2\right)^{3/2} - \frac{a^4}{16} \log \frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6)$$

$$I_F(a) = \frac{7\pi^4}{360} - \frac{\pi^2}{24} a^2 - \frac{a^4}{16} \log \frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6)$$

$$\bar{V}(v;T) = \frac{T^4}{2\pi^2}\left(6 I_B(a_W) + 3 I_B(a_Z) - 6 I_F(a_t)\right)$$

$+T^4 a^2 \sim +T^2 v^2$ ⟶ symmetry restoration at high-T

Assuming $T > m_W, m_Z, m_t$

$$V_{\rm eff}(v;T) \simeq D(T^2 - T_0^2)v^2 - ETv^3 + \frac{\lambda_T}{4}v^4$$

$$D = \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v_0^2} \qquad E = \frac{2m_W^3 + m_Z^3}{4\pi v_0^3} \sim 10^{-2}$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4}\left(2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right)$$

$$\log\alpha_{F(B)} = 2\log(4)\pi - 2\gamma_E$$

$$T_0^2 = \frac{\mu^2 - 4Bv_0^2}{2D}$$

At T_C, the local min. at v_C degenerates with that at $v = 0$.

$$V_{\rm eff}(v_C;T_C) = V_{\rm eff}(0;T_C)$$

$$\longrightarrow \quad v_C = \frac{2ET_C}{\lambda_T}$$

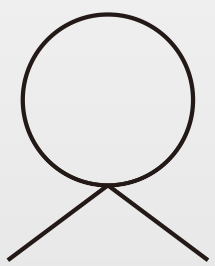
1st order PT

In most cases, the perturbative expansion at high temperatures is *not a good approximation*.

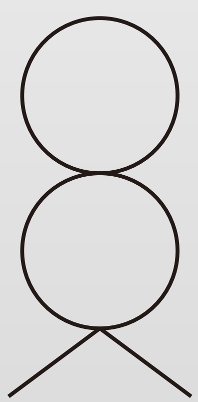
e.g. ϕ^4 theory

Dolan and Jackiw, Phys. Rev. D9 (1974)

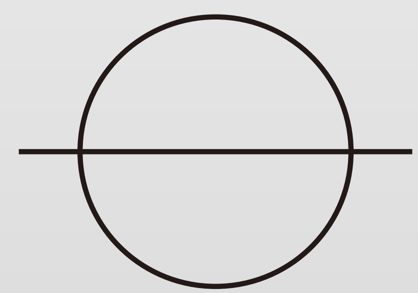
corrections to 2-point function (High-T exp.) $a = \frac{m}{T} \ll 1$



$$\propto \lambda T^2 I'_B(m^2/T^2) \sim \lambda T^2$$

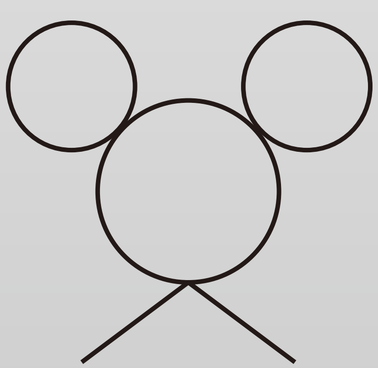


$$\sim \lambda T^2 \frac{\lambda T}{m}$$



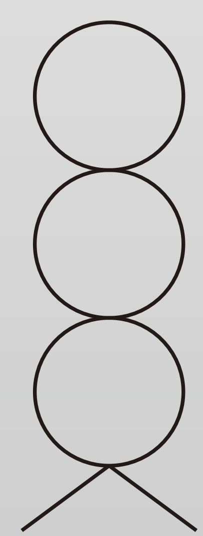
$$\sim \lambda T^2 \lambda \log \frac{T}{m}$$

$$\times \frac{\lambda T^2}{m^2}$$



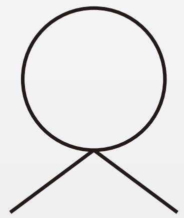
$$\sim (\lambda T^2)^2 \frac{\lambda T}{m^3}$$

$$\times \frac{m}{T}$$



$$\sim \lambda T^2 \left(\frac{\lambda T}{m}\right)^2$$

- the leading correction to $m^2 \sim \lambda T^2$
- the **bubble** subdiagram yields the largest corrections

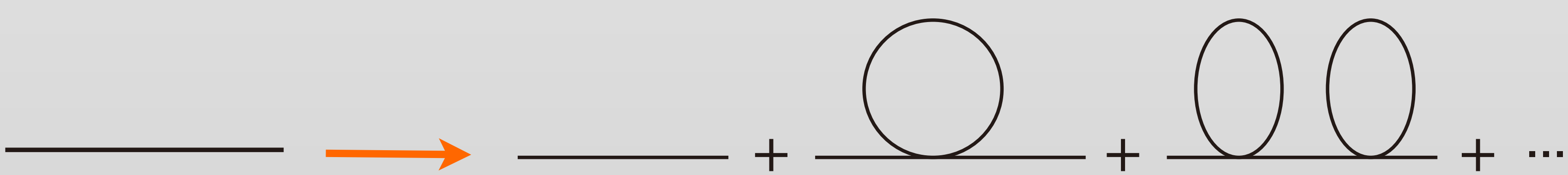


a factor of $\dfrac{\lambda T^2}{m^2}$ from a bubble

$$\therefore T \gtrsim \frac{m}{\sqrt{\lambda}} \longrightarrow$$ loop expansion is invalidated

The leading correction($\sim \lambda T^2$) to m^2 can be incorporated by

‘resummation’



$$m^2 \to m^2 + \Delta_T m^2 = m^2 + \frac{\lambda T^2}{24}$$ in the propagator

thermal mass $\longrightarrow$ weakens the PT

A nonperturbative analysis: Lattice MC calculation

$$Z(T) = \mathrm{Tr}\left(e^{-H/T}\right) = \int_{\phi(1/T)=\phi(0)} [d\Phi \, dU_\mu] \exp\left(-S_E[\Phi, U]\right)$$

$$U_\mu(x) = e^{igA_\mu(x)}$$ link variable

Standard Model (1 Higgs doublet)

[Csikor, hep-lat/9910354]

1st order Phase Transition for $m_h < 66.5 \pm 1.4\,\mathrm{GeV}$

$T_C \simeq 90 - 100\,\mathrm{GeV}$

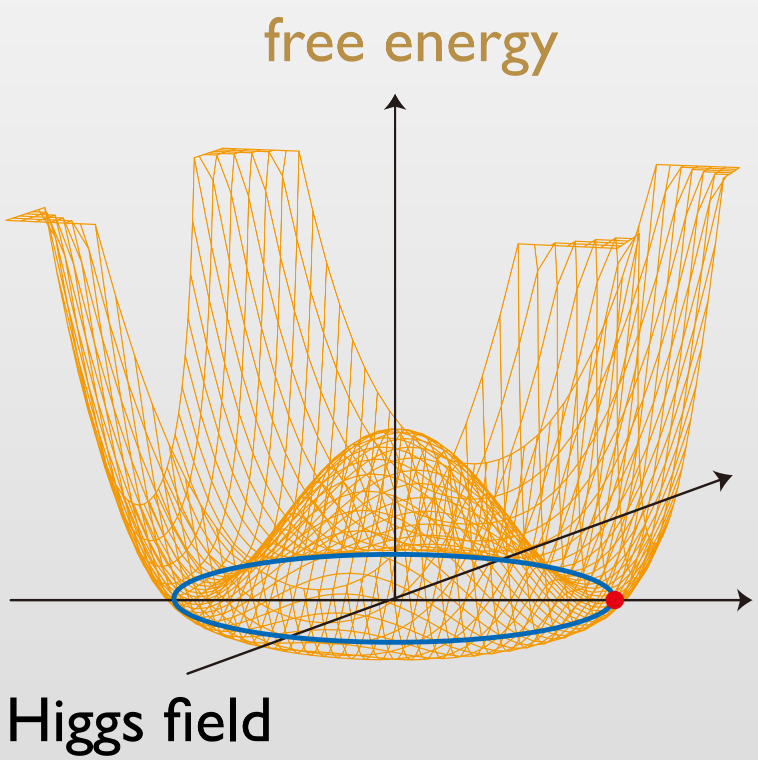
End point of the Phase Transition at $m_h = 72.1 \pm 1.4\,\mathrm{GeV}$

$m_h = 125\mathrm{GeV}$ ➡ Cross Over

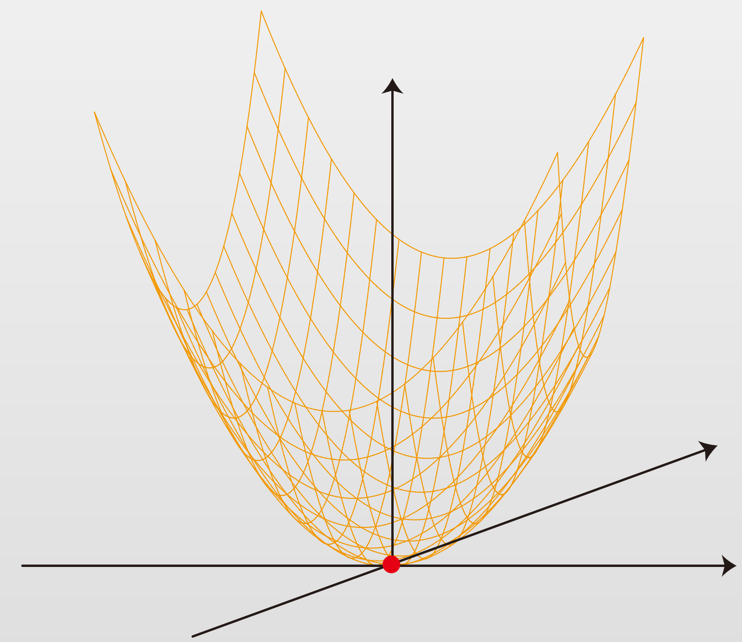
$v(T)$ continuously changes from 0 to v_0
as the Universe cooled down

As expected, we have seen that the broken gauge symmetry was restored at high temperatures.

$T = 0$



$T > T_C \simeq 100\mathrm{GeV}$



EWPT

(Electroweak PT)

← T_C

$m_h > 72\mathrm{GeV}$ → no dramatic event

may be for $m_h > 67\mathrm{GeV}$

Recall

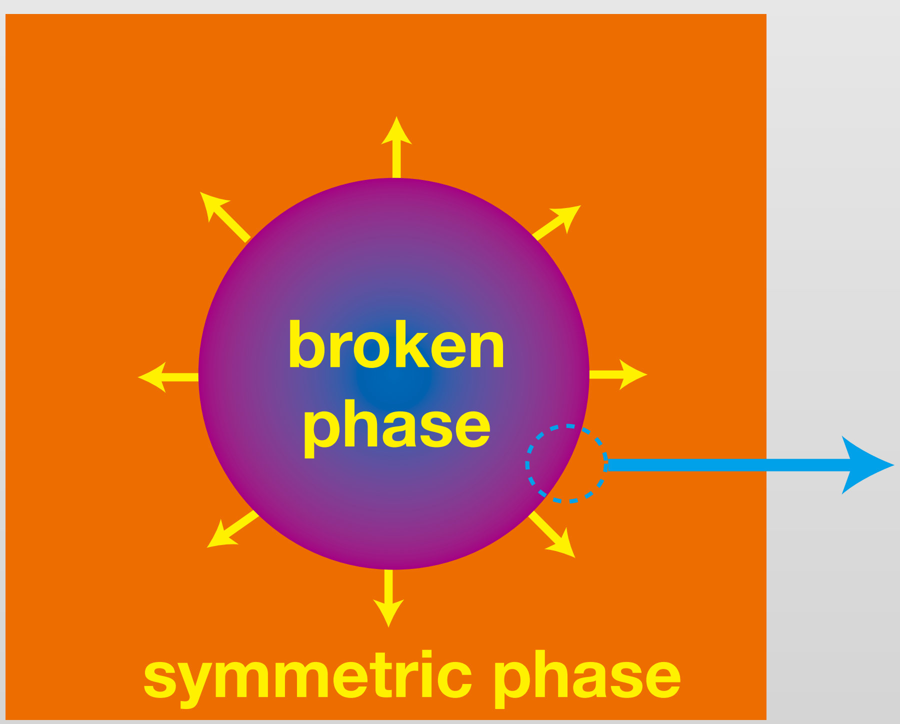
$H(T) \simeq 10^{-14}\mathrm{GeV} \ll \Gamma_{EW}(T) \simeq 1\mathrm{GeV}$

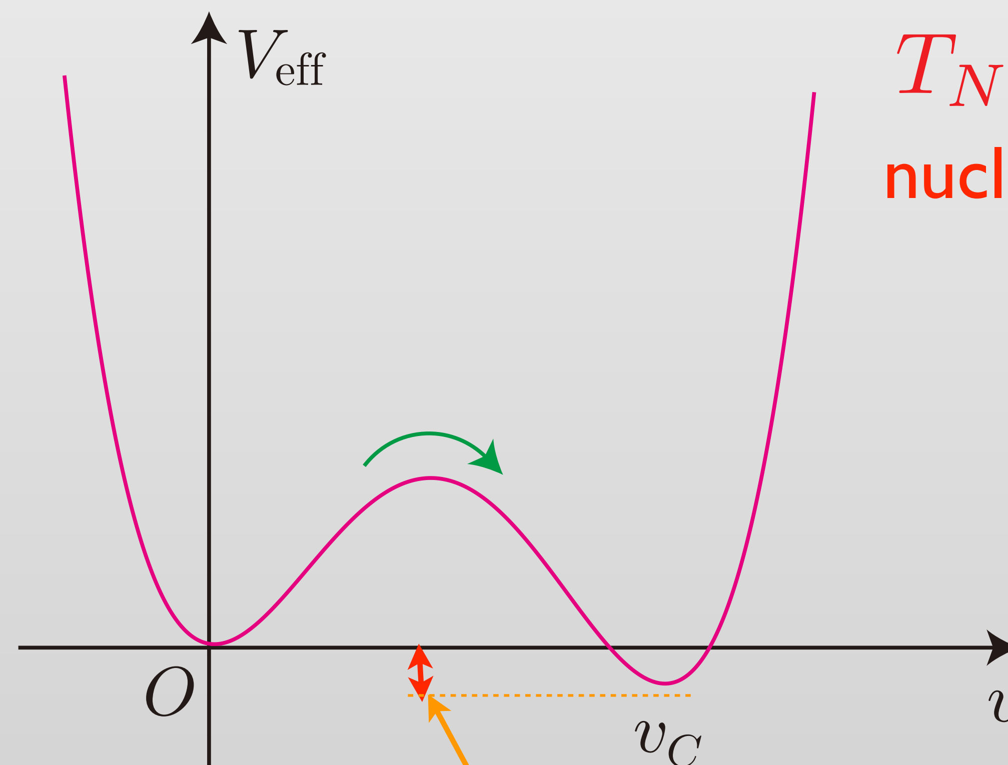
Some extension of the SM predicts the first order EWPT.

If it is of strong first order, some cosmological events could occur.

$$\text{large } \frac{v_C}{T_C}, \quad \text{and/or lare latent heat}$$

The expansion of the bubble of the broken phase within the symmetric phase realizes nonequilibrium state.





$T_N < T_C$

nucleation temp.

$$V_{\text{eff}}(0; T_N) - V_{\text{eff}}(v_C; T_N)$$

||

latent heat

1. Generation of Gravitational Wave

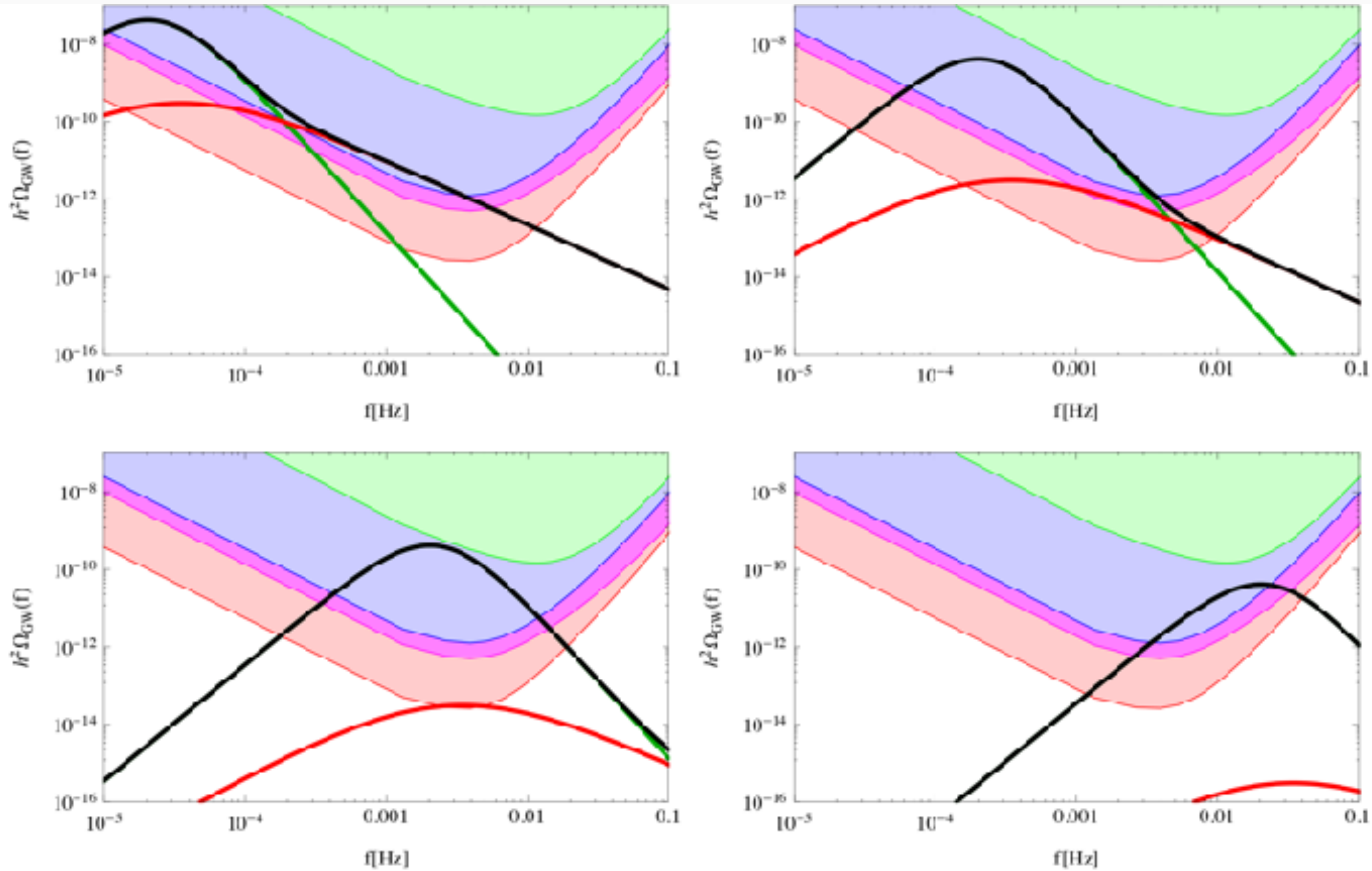


Figure 2. Example of GW spectra in Case 1, for fixed $T_* = 100\,\text{GeV}$, $\alpha = 0.5$, $v_w = 0.95$, and varying β/H_*: from left to right, $\beta/H_* = 1$ and $\beta/H_* = 10$ (top), $\beta/H_* = 100$ and $\beta/H_* = 1000$ (bottom). The black line denotes the total GW spectrum, the green line the contribution from sound waves, the red line the contribution from MHD turbulence. The shaded areas represent the regions detectable by the C1 (red), C2 (magenta), C3 (blue) and C4 (green) configurations.

eLISA config.

arm length/#links

C1: 5Mkm/6

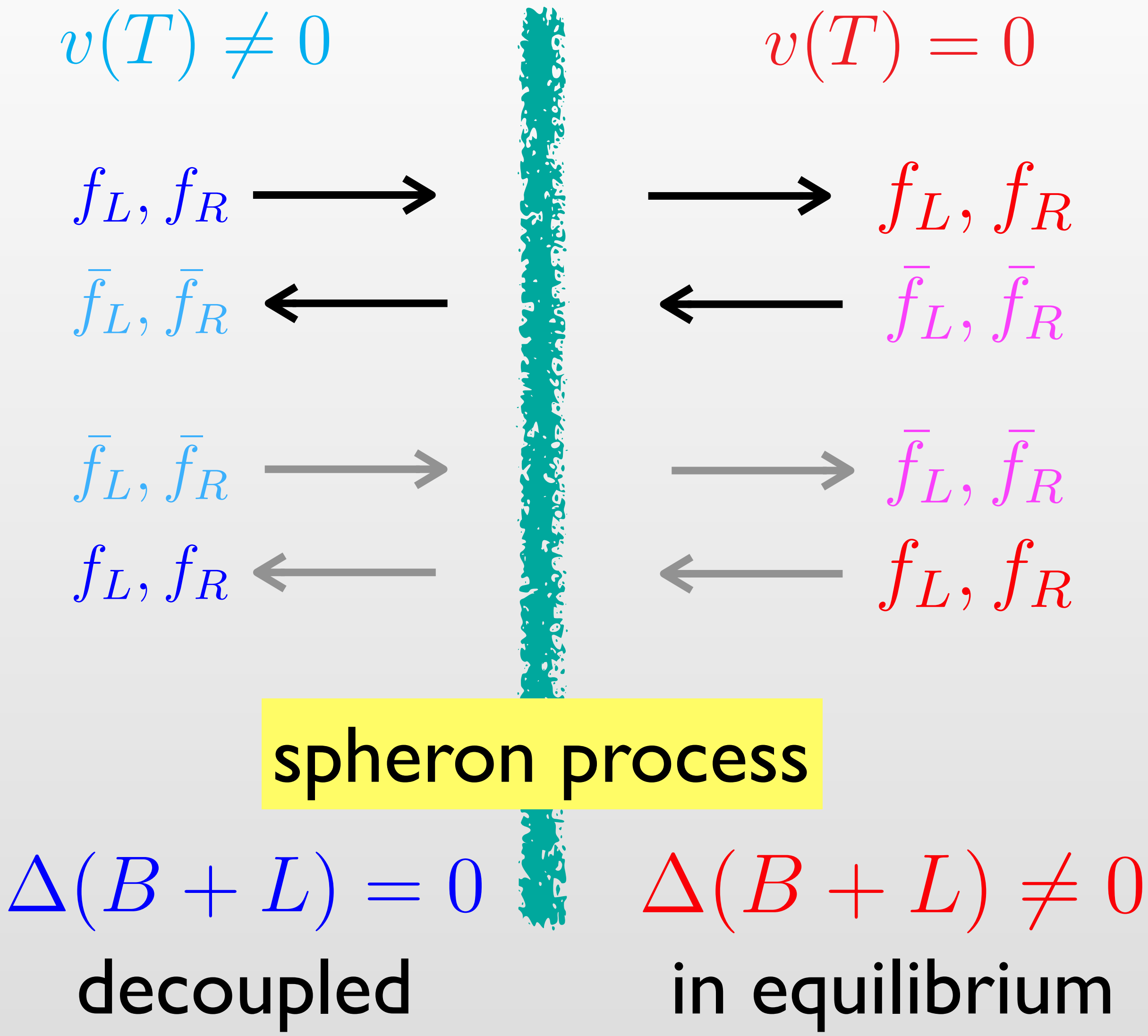
C2: 1Mkm/6

C3: 2Mkm/4

C4: 1Mkm/4

Ref.: C. Caprini, JCAP 04 (2016) 001

2. Electroweak Baryogenesis



CP violation

↓

chiral charge $Q_L - Q_R \neq 0$

accumulated in the sym. phase

↓

bias on the sphaleron process

↓

$(B+L)$ is generated

Reviews : KF, Prog. Theor. Phys. 96 (1996) 475

Rubakov and Shaposhnikov, Phys. Usp. 39 (1996) 461

Riotto and Trodden, Ann. Rev. Nucl. Part. Sci. 49 (1999) 35

Summary

- \`Symmetry' is a guiding principle to build a particle physics mode.
- Except for some symmetries ($SU(3)_c$, $U(1)_{em}$, $U(1)_B$, Lorentz, ...), most of them are broken explicitly (but very slightly) or spontaneously.
- While some of SSB's are prohibited, spontaneously broken symmetries leave their trace — NG bosons, massive vectors, once SSB's occur.
- Spontaneously broken symmetries are restored at high temperatures. The SSB PT could cause a dramatic event such as generation of GW and baryon asymmetry.

Problem radiative symmetry breaking in the massless scalar QED

$$\mathcal{L} = (D_\mu \phi)^* D^\mu \phi - \lambda(\phi^* \phi)^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu = \partial_\mu + ieA_\mu$$

N.B. No mass scale in this $\mathcal{L}$!

1. Set $\phi(x) = \frac{1}{\sqrt{2}}(v + \sigma(x) + i\pi(x))$ to find the effective potential $V_{\text{eff}}(v)$ by taking into account only the 1-loop correction from the gauge boson.

hint: Put $m_A(v)$, which is the gauge-boson mass in the presence of the VEV v, into the 'formula'. Be careful about the degree of freedom of the gauge boson.

2. Determine the ren. scale M by requiring that the minimum of $V_{\text{eff}}(v)$ is realized at v_0. That is, represent M in terms of (v_0, λ, e).

3. Write down the finite-temperature effective potential $V_{\text{eff}}(v; T)$ with the 1-loop correction from A_μ.

4. Study the phase transition of this model by use of $V_{\text{eff}}(v; T)$ obtained in 3. Use the high-T expansion for the function $I_B(a)$. If possible, numerically evaluate $I_B(a)$. Draw $V_{\text{eff}}(v; T)$ as a function of v for several T, and find v_C and T_C if possible.