This year will be the year of Higgs particle.

The discovery of ‘Higgs-like’ boson will be reported with higher statistics in this March.

To confirm it is the Higgs boson, we must check

- relation between the couplings and the particle masses
- W, Z bosons, quarks, leptons
- 3- and 4-body self-interaction of the boson

The Higgs field in the Standard Model provides the masses of all the weak gauge boson and fermions.

\[ \mathcal{L}_{SM} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_Y + \mathcal{L}_H \]

\[ \mathcal{L}_g = -\frac{1}{4} G^a_{\mu\nu}(x) G^{a\mu\nu}(x) - \frac{1}{4} F^a_{\mu\nu}(x) F^{a\mu\nu}(x) - \frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) \]

\[ G^a_{\mu\nu}(x) = \partial_\mu G^a_\nu(x) - \partial_\nu G^a_\mu(x) - g_s f^{a\alpha\beta} G^\alpha_\mu(x) G^\beta_\nu(x) \]

\[ \mathcal{L}_f = \bar{q}_L(x) i \gamma^\mu \left( \partial_\mu - ig_L \gamma^5 G^a_\mu(x) \right) q_L(x) + \cdots \]

\[ \mathcal{L}_Y = \bar{q}_L(x) Y_L u_R(x) \tilde{\Phi}(x) + \bar{q}_L(x) Y_L d_R(x) \Phi(x) + \bar{q}_L(x) Y_L e_R(x) + h.c. \]

Yukawa coupling matrix

Higgs field \( \Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \)

\[ \tilde{\Phi}(x) = i \gamma^2 \Phi^*(x) = \begin{pmatrix} \phi^0(x) \\ -\phi^-(x) \end{pmatrix} \]

No mass scale in the gauge and fermion sector.
The only mass scale arises from the Higgs sector.

\[ L_H = \left| \left( \partial_\mu - ig_2 A_\mu^a(x) - \frac{i}{2} g_1 B_\mu(x) \right) \Phi(x) \right|^2 - V(\Phi) \]

where

\[ m_W^2 = \frac{1}{4} g_2^2 v_0^2, \quad m_Z^2 = \frac{1}{4} (g_2^2 + g_1^2) v_0^2 = \frac{m_W^2 \cos^2 \theta_W}{\cos^2 \theta_W} \]

\[ W_\mu^a(x) = \frac{1}{\sqrt{2}} (A_\mu^a(x) + i A_\mu^a(x)) \]

\[ Z_\mu(x) = A_\mu^0(x) \cos \theta_W - B_\mu(x) \sin \theta_W \]

\[ A_\mu(x) = A_\mu^3(x) \sin \theta_W + B_\mu(x) \cos \theta_W \]

Similarly for the fermions.

Setting \( \Phi(x) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_0 + h(x) \end{array} \right) \) in the unitary gauge

\[ L_Y = \frac{v_0 + h(x)}{\sqrt{2}} \left[ \bar{u}_L Y_u u_R + \bar{d}_L Y_d d_R + \bar{\ell}_L Y_e e_R + \text{h.c.} \right] \]

A bi-unitary transformation by \( f_L \) and \( f_R \)

\[ \left( 1 + \frac{h(x)}{v_0} \right) \left( m_{uA} \bar{u}_A u_A + m_{dA} \bar{d}_A d_A + m_{eA} \bar{e}_A e_A \right) \]

\( A = 1, 2, 3 \) : generation

**Higgs boson**

The couplings of the Higgs boson to the gauge bosons and fermions are proportional to their masses.

The effect of the unitary transformation resides only in the quark charged-current interaction.

**CKM matrix**

--- **masses of the SM particles** except for the higgs boson

<table>
<thead>
<tr>
<th>electric charge</th>
<th>1st gen.</th>
<th>2nd gen.</th>
<th>3rd gen.</th>
</tr>
</thead>
<tbody>
<tr>
<td>quark ( + \frac{2}{3} )</td>
<td>( u )</td>
<td>( c )</td>
<td>( t )</td>
</tr>
<tr>
<td>( + \frac{1}{3} )</td>
<td>( d )</td>
<td>( s )</td>
<td>( b )</td>
</tr>
<tr>
<td>(-1 )</td>
<td>( e )</td>
<td>( \mu )</td>
<td>( \tau )</td>
</tr>
<tr>
<td>charged lepton (-1 )</td>
<td>( 0.51 \text{MeV} )</td>
<td>( 106 \text{MeV} )</td>
<td>( 1.8 \text{GeV} )</td>
</tr>
</tbody>
</table>

**Weak boson**

| \( W^+ \), \( W^- \), \( Z \) |
| 60.4 GeV | 81.2 GeV |

**N.B.**

\[ m_u + m_d + m_e \sim \frac{1}{10} m_{\text{proton}} \]

90% of the nucleon mass comes from QCD dynamics.

--- confirmed by lattice MC calculation
decay branching rate

$$\Gamma(h \to f\bar{f}) = \frac{N_c m_f^2 m_h}{8\pi v_0^2} \left(1 - \frac{4m_f^2}{m_h^2}\right)^{\frac{3}{2}}$$

$$\Gamma(h \to W^+W^-) = \frac{C_W m_w^3}{8\pi v_0} \sqrt{1 - \frac{4m_W^2}{m_h^2}} \left(1 - \frac{4m_W^2}{m_h^2} + \frac{12m_W^4}{m_h^2}\right)$$

$C_W = 2$, $C_Z = 1$

off-shell vector
3- or 4-body decay

one-loop processes

What determines the value of the Higgs VEV \(v_0\)?

4-Fermi effective theory vs \(W\)-exchange interaction

e.g. weak decay \(\mu^- \to e^- + \bar{\nu}_e + \nu_\mu\)

\[\mathcal{L}_F = \frac{G_F}{\sqrt{2}} \left(\gamma_\mu (1 - \gamma_\nu) \bar{\nu}_\nu \gamma_\nu (1 - \gamma_\mu)\mu + \cdots\right)\]

\[\mathcal{L}_{CC} = \frac{g_2}{\sqrt{2}} \left(J_{\mu}^c W^+ W^- + J_{\nu}^c W^\mu W^-\mu\right)\]

\(J_{\mu}^c = \bar{e}_A \gamma_\mu \frac{1 - \gamma_5}{2} e_A + \bar{\nu}_A \gamma_\mu \frac{1 - \gamma_5}{2} \nu_A d_B\)

\[G_F = \frac{g_2^2}{\sqrt{2}} = \frac{1}{8m_W^2} \quad \Rightarrow \quad v_0 = \left(\frac{1}{\sqrt{2} G_F}\right)^{\frac{1}{2}} = 246.26 \text{GeV}\]
Theoretically, it is the location of the minimum of the **scalar potential** of the Standard Model, including quantum corrections at the classical (tree) level

\[ V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \]

\( V(\Phi) \) takes its minimum at \( \langle \Phi \rangle \).

\[ v_0 = \sqrt{\frac{\mu^2}{\lambda}} \]

Even when \( \mu^2 = 0 \), \( \Phi \) can acquire nonzero \( v_0 \).

**Coleman-Weinberg mechanism**

\[ \Delta V(v) = \sum_I c_I \frac{m_I(v)^4}{64\pi^2} \left( \log \frac{m_I(v)^2}{M^2} - \frac{3}{2} \right) \]

\[ V_{\text{eff}}(v) = V(v) + \Delta V(v) \] takes min. at \( v_0 \)

---

A part of the gauge symmetry of the SM is **spontaneously broken** by \( \langle \Phi \rangle \neq 0 \)

A symmetry of the lagrangian is broken by the ground state.

---

**Similarity to magnetism**

Hamiltonian of the **spin** model

\[ H = -\kappa \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j \] invariant under the spatial rotation

\( \vec{s}_i \cdot \vec{s}_j \)  

the lowest E  

\( \vec{M} \)  

\( M = \langle \vec{s}_i \rangle = 0 \)  

\( M \neq 0 \)  

breaks the rotational symmetry
Heating up a magnet looses its magnetism.

One may expect a similar symmetry restoring
Phase Transition to occur in the Higgs sector.

It’s impossible to reach such a high-T on the Earth,
but is though to be realized in the early Universe.

Brief Review of the Big Bang Cosmology

Evolution of the space is described by Einstein equation:

\[ R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}(x)R(x) + \Lambda g_{\mu\nu}(x) = 8\pi G T_{\mu\nu}(x) \]

spatially uniform and isotropic Universe = Friedmann-Robertson-Walker spacetime

\[ ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - R^2_0 a(t)^2 \left[ \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \]

\[ k = \begin{cases} +1 & : \text{closed} \\ 0 & : \text{flat} \\ -1 & : \text{open} \end{cases} \]

\[ T_{\mu\nu}(x) \] uniform and isotropic

\[ \left\{ \begin{array}{l} \rho(t) \quad \text{energy density} \\ P(t) \quad \text{pressure} \end{array} \right. \]

satisfies the conservation law

\[ dE + PdV = 0 \]

\[ \frac{d}{dt} \left( \rho(t)a(t)^3 \right) + P(t) \frac{d}{dt} (a(t)^3) = 0 \]

3 types of Equation Of State

\[ \begin{array}{ll} \text{Matter (nonrelativistic)} & P_m(t) = 0 \\ & \rho_m(t) \propto a(t)^{-3} \\ \text{Radiation (relativistic)} & P_r(t) = \frac{1}{3}\rho_r(t) \\ & \rho_r(t) \propto a(t)^{-4} \\ \text{Vacuum (or Dark Energy)} & P_\Lambda(t) = -\rho_\Lambda(t) \\ & \rho_\Lambda(t) \propto a(t)^{0} \end{array} \]

Which species behaves as radiation depends on temperature.

\[ T \gg m \quad (k_B T \gg mc^2) \]
Friedmann equation = Einstein equation for FRW metric

\[
\left( \frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3} \left( \rho_r(t) + \rho_m(t) + \rho_\Lambda \right) - \frac{k}{R_0^2 a(t)^2}
\]

\[
= \frac{8\pi G}{3} \left( \frac{\rho_r(t_0)}{a(t_0)^4} + \frac{\rho_m(t_0)}{a(t_0)^4} + \rho_\Lambda \right) - \frac{k}{R_0^2 a(t)^2}
\]

Observation: \( k \simeq 0 \), \( \frac{\rho_r(t_0)}{\rho_m(t_0)} = \frac{\Omega_R}{\Omega_M} = \frac{5.0 \times 10^{-5}}{0.27} \simeq 1.9 \times 10^{-4} \)

The early Universe of \( a(t) < 2 \times 10^{-4} \) is dominated by radiation.

- Entropy of radiation \( \propto T^3 a^4 \propto \text{const.} \)
- Temperature of radiation at present \( T_0 = 2.73 \text{K} \)

\[
T = \frac{T_0}{a} \gtrsim 10^4 \text{K} \simeq 1 \text{eV}
\]

At present, only the photons (and maybe the neutrinos) have the equilibrium distribution. [Cosmic Microwave Background]

In the radiation-dominated Universe \( (T \gg 1 \text{eV}) \), species tightly coupled to the plasma can be regarded as in equilibrium, even though the Universe was expanding.

**Criterion for coupling to the plasma**

\[
H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{8\pi G}{3} \rho_r(t)} < \Gamma(t)
\]

Interaction rate determined by the cross section and number density

Then we can define the temperature \( T \) and apply the equilibrium statistical mechanics.

**Interaction rate**

For relativistic species \( m \lesssim T \)

\[
\Gamma^{-1} = \frac{1}{\lambda} \simeq \lambda \quad \text{mean free path}
\]

Total cross section of that species \( \sigma \simeq \frac{\alpha^2}{s} \simeq \frac{\alpha^2}{T^2} \)

For the weak interaction, \( \alpha = \frac{g^2}{4\pi} = \frac{\alpha_{em}}{\sin^2 \theta_W} \)

Number density \( n(T) \approx g_{*n} \frac{\zeta(3)}{\pi^2} T^3 \)

\[
g_{*n} = \sum_{b} g_{b} + \frac{3}{4} \sum_{f} g_{f}
\]

Effective degrees of freedom \( g_{*} \)

\[
\sigma \cdot \lambda = \frac{1}{n(T)}
\]

\[
\tilde{t} = \frac{10}{g_{*n} T^3} \left( \frac{\alpha^2}{T^2} \right)^{-1} = \frac{10}{g_{*n} \alpha^2 T}
\]

\( \tilde{t} \) is the interaction time.
The expansion rate is given by:

$$H(T) = \sqrt{\frac{8\pi G}{3}} \rho_\nu(T) \simeq 1.66\sqrt{g_*} \frac{T^2}{M_{Pl}}, \quad M_{Pl} = 1.22 \times 10^{19}\text{GeV}$$

$$\rho_\nu(T) = g \int \frac{d^3p}{(2\pi)^3} e^{p_\mu p} \frac{1}{p^2 + 1} = g \left\{ \begin{array}{c} \frac{1}{7/8} \frac{\pi^2 T^4}{30} \end{array} \right.$$  

$$g_* = \sum_p g_0 + \frac{7}{8} \sum_p g_F \quad g_* = 106.75$$

when all the SM particles are relativistic

At $T = 100\text{GeV}$

$$H(T) = 1.66\sqrt{106.75} \times \frac{10^4}{1.22 \times 10^{19}} \text{GeV} \simeq 10^{-14}\text{GeV}$$

$$\Gamma(T) = g_* \frac{\alpha(T)^2 T}{10} \simeq 10^3 \alpha(T)^2 \text{GeV} = (1 - 10)\text{GeV}$$

At temperatures of the weak scale, we can safely regard all the SM particles are in thermal equilibrium.

Comments

If the Universe has been in equilibrium throughout its history, there would be no stars, galaxies, structures and creatures including ourselves.

Nonequilibrium events due to $\Gamma(T) < H(T)$

- decoupling of photons ($T = 1\text{eV}$)
- nucleosynthesis ($T = 1\text{MeV}$)
- decoupling of the Dark Matter
- GUTs Baryogenesis/Leptogenesis

We need to treat time-dependent distribution functions.

We apply the equilibrium statistical mechanics to study static features of the phase transition of the EW symmetry breaking.

Quantum Field Theory at finite temperatures

Le Bellac, 'Thermal Field Theory' (2000)

free energy density as a function of the order parameters

$$= \text{effective potential at finite temperatures}$$

$$V_{\text{eff}}(\varphi; T) = -\Gamma[\varphi(x)] = \int d^4x \quad \Gamma[\varphi] = \text{effective action}$$

$$\text{Tr}\left(e^{-H/T}\right) = N(T) \int_{\mathcal{M}_{\Phi}} [d\phi] \exp\left( - \int_0^{1/T} d^4x E_\phi(\phi) \right)$$

$$\left\{ \begin{array}{c} \phi(0, x) = \phi(1/T, x) \quad \text{boson} \\ \psi(0, x) = -\psi(1/T, x) \quad \text{fermion} \end{array} \right.$$  

$$k^0 = i\omega_n = i\pi 2n T \quad \text{mode sum}$$
free energy vs order parameter (Higgs VEV) at finite T

\[ V_{\text{eff}}(\nu; T) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(T) \end{pmatrix} \]

\[ \nu_C = \lim_{T \to T_C} v(T) \neq 0 \]

1st order phase transition

Standard Model

\[ V_{\text{eff}}(\nu; T) = -\frac{1}{2} \mu^2 \nu^2 + \frac{\lambda}{4} \nu^4 + 2B_{\nu^4} \left[ \log \left( \frac{\nu^2}{\nu_0^2} \right) - \frac{3}{2} \right] + \tilde{V}(\nu; T) \]

\[ B = \frac{3}{64 \pi^3 \nu^6} \left( 2m_W^4 + m_B^4 - 4m_0^4 \right) \]

\[ \tilde{V}(\nu; T) = \frac{T^4}{2 \pi^2} \left( 6I_B(\alpha_W) + 3I_B(\alpha_Z) - 6I_F(\alpha_W) \right) \]

\[ a_A = \frac{m_A(\nu)}{T} \]

\[ I_B(\alpha) = \int_0^\infty dx x^2 \log \left( 1 + e^{-x^2 + \nu^2} \right) \]

High-T expansion

\[ a = m/T \ll 1 \]

\[ I_B(\alpha) = -\frac{\pi^2}{45} \frac{\pi^2}{12} \frac{\pi^2}{6} (\alpha^2)^{3/2} = \frac{\pi^4}{16} \frac{\alpha^4}{\pi} \log \frac{\alpha^2}{4\pi} - \frac{\alpha^4}{16} \left( \gamma_E - \frac{3}{4} \right) + O(\alpha^6) \]

\[ I_F(\alpha) = \frac{7\pi^4}{360} \frac{\pi^2}{24} \frac{\pi^2}{4} \frac{\pi^2}{2} = -\frac{\pi^4}{16} \frac{\alpha^4}{\pi} \log \frac{\alpha^2}{\pi} - \frac{\alpha^4}{16} \left( \gamma_E - \frac{3}{4} \right) + O(\alpha^6) \]

\[ +T^4 \alpha^2 \sim T^2 \nu^2 \rightarrow \text{symmetry restoration at high-T} \]

Assuming \( T > m_W, m_Z, m_t \)

\[ V_{\text{eff}}(\nu; T) \simeq D(T^2 - T_0^2) \nu^2 - E T \nu^3 + \frac{\lambda_T}{4} \nu^4 \]

\[ D = \frac{2m_W^2 + m_Z^2 + 2m_0^2}{8\nu_0^2} \]

\[ E = \frac{2m_W^2 + m_Z^2}{4\pi^2} \nu_0^2 \approx 10^{-2} \]

\[ \lambda_T = \lambda - \frac{3}{16\pi^2 \nu_0^2} \left( 2m_W^2 \log \frac{m_W^2}{\alpha_B T^2} + m_Z^2 \log \frac{m_Z^2}{\alpha_B T^2} - 4m_0^2 \log \frac{m_0^2}{\alpha_B T^2} \right) \]

\[ T_0^2 = \frac{\mu^2 - 4B_{\nu^4}}{2D} \]

At \( T_C \), the local min. at \( \nu_C \) degenerates with that at \( \nu = 0 \).

\[ V_{\text{eff}}(\nu_C; T_C) = V_{\text{eff}}(0; T_C) \]

\[ \nu_C = \frac{2ET_C}{\lambda_T} \]

1st order PT
In most cases, the perturbative expansion at high temperatures is not a good approximation.

\( \text{e.g. } \phi^4 \text{ theory} \quad \text{Dolan and Jackiw, Phys. Rev. D9 (1974)} \)

corrections to 2-point function (High-T exp.) \( a = \frac{m}{T} \ll 1 \)

\( \propto \lambda T^2 I_B(m^2/T^2) \sim \lambda T^2 \)

\( \sim \frac{\lambda T^2}{m} \lambda T \log \frac{T}{m} \)

\( \sim (\lambda T^2)^2 \frac{\lambda T}{m^3} \times \frac{m}{T} \sim \lambda T^2 \left( \frac{\lambda T}{m} \right)^2 \)

\( \star \) the leading correction to \( m^2 \sim \lambda T^2 \)

\( \star \) the bubble subdiagram yields the largest corrections

\( \star \) a factor of \( \frac{\lambda T^2}{m^2} \) from a bubble

\( \therefore T \gtrsim \frac{m}{\sqrt{\lambda}} \rightarrow \) loop expansion is invalidated

The leading correction(\( \sim \lambda T^2 \)) to \( m^2 \) can be incorporated by ‘resummation’

\( m^2 \rightarrow m^2 + \Delta_T m^2 = m^2 + \frac{\lambda T^2}{24} \) in the propagator

thermal mass \( \rightarrow \) weakens the PT

A nonperturbative analysis: **Lattice MC calculation**

\( Z(T) = \text{Tr} \left( e^{-H/T} \right) = \int_{\Phi(1/T) = \Phi(0)} [d\Phi \ dU_\mu] \exp \left( -S_E[\Phi, U] \right) \)

\( U_\mu(x) = e^{igA_\mu(x)} \) link variable

**Standard Model (1 Higgs doublet)** [Csikor, hep-lat/9910354]

1st order Phase Transition for \( m_h < 66.5 \pm 1.4 \text{ GeV} \)

\( T_C \approx 90 - 100 \text{ GeV} \)

End point of the Phase Transition at \( m_h = 72.1 \pm 1.4 \text{ GeV} \)

\( m_h = 125 \text{ GeV} \rightarrow \) Cross Over

\( v(T) \) continuously changes from 0 to \( v_0 \) as the Universe cooled down
As expected, we have seen that the broken gauge symmetry was restored at high temperatures.

\[ T = 0 \quad \text{free energy} \quad T > T_G \approx 100 \text{GeV} \]

\[ m_h > 72 \text{GeV} \rightarrow \text{no dramatic event} \]

\[ \text{Recall} \quad H(T) \approx 10^{-14} \text{GeV} \ll \Gamma_{EW}(T) \approx 1 \text{GeV} \]

\text{If the EWPT is 1st order}, some events might occur which affect the present Universe.

\textbf{Why 1st order PT?} \quad \text{e.g. evaporation of water}

PT proceeds through nucleation and growth of bubbles

\[ V_{\text{eff}}(0; T_N) - V_{\text{eff}}(\nu_G; T_N) \]

\[ T_N < T_G \quad \text{nucleation temp.} \]

- non-equilibrium state near expanding bubble walls
- release of the latent heat

\text{If the EWPT is 1st order,} \quad \text{Gravitational wave might be generated}

by bubble collisions and/or by turbulence of the plasma

\[ T[\varphi](x)_{\mu\nu} \rightarrow g_{\mu\nu}(x) + h_{\mu\nu}(x) \]

The energy density of GW may be marginally observed by LISA.

\text{Laser Interferometer Space Antenna} \quad \text{http://lisa.nasa.gov/}

\text{For a review, see M. Maggiore, Phys. Rep. 331 (2000) 283}
the Baryon Asymmetry of the Universe might be generated at PT = scenario of the Electroweak Baryogenesis (EWBG).

Both the baryon and lepton numbers are conserved in the SM at the classic level (=symmetry of the Lagrangian).

However, \((B+L)\) is not conserved at quantum level.

\[
\partial_{\mu} j_{\mu}^{B+L} = \frac{N_f}{16\pi^2} \left[ g_2^2 \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu} \tilde{B}^{\mu\nu} \right] \quad \partial_{\mu} j_{\mu}^{B-L} = 0
\]

\[
B(t_f) - B(t_i) = \frac{N_f}{32\pi^2} \int_{t_i}^{t_f} dt \int d^3 x \left[ g_2^2 \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu} \tilde{B}^{\mu\nu} \right]
\]

\[
= N_f \left[ N_{CS}(t_f) - N_{CS}(t_i) \right]
\]

Chern-Simons number \((A_0 = 0\)-gauge\)

\[
N_{CS}(t) = \frac{1}{32\pi^2} \int d^3 x \epsilon_{ijk} \left[ g_2^2 \text{Tr} \left( F_{\mu\nu} A_\mu - \frac{2}{3} g_2 A_\mu A_\nu \right) - g_1^2 \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} \right]
\]

\(N_{CS} \in \mathbb{Z}\) for classical vacua

\[
\Delta(B + L)\text{-changing process is suppressed}
\]

in the broken phase by \(e^{-\frac{E_{sph}}{T}}\) sphaleron energy

If the interaction of the plasma particles with the bubble wall violates CP-symmetry, some chiral charge is injected in the symmetric phase.

\[
\Delta(B + L) = 0 \quad \text{broken phase}
\]

\[
\Delta(B + L) \neq 0 \quad \text{symmetric phase}
\]

The chiral charge biases the \((B+L)\)-changing process.

Generated \((B+L)\) is frozen in the broken phase.

CP-violating bubble walls are transparent to the vectorlike quantum numbers such as B and L.

Review articles on EWBG

- Rubakov and Shaposhnikov, Phys. Usp. 39 (1996) 461

Even if the EWPT in the SM is of first order, the KM phase is insufficient to generate the BAU.

For the EWPT to be of first order, we must extend the SM.

Which extension makes the EWPT of first order?
bosonic loop correction

\[ V_{\text{eff}}(v; T) \sim - T (m(v))^3/2 \]

\[ \leftrightarrow \quad a^3\text{-term of } I_B(a^2) \]

bosons interacting with the Higgs whose mass behaves as

\[ m(v)^2 \sim g^2 v^2 \quad \text{(for } v \sim 0) \]

e.g. extra scalars in the two-Higgs-double Model (2HDM),

Supersymmetric SM's

\[ m(v)^2 = m_0^2 + g^2 v^2 \quad (m_0^2 \ll g^2 v^2) \]

2HDM with the discrete symmetry  

\[ \rightarrow \quad \text{vast allowed region of parameters} \]


---

**MSSM**

Minimal Supersymmetric Standard Model

order parameters:  

\[ \langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 + iv_3 \end{pmatrix} \]

8–3 = 5 physical Higgs particles  

3 neutral, 1 charged

\[ V_0 = m_d^2 \Phi_d \Phi_d^\dagger + m_u^2 \Phi_u \Phi_u^\dagger - \left( m_{\epsilon ij} \Phi_d^i \Phi_u^j \right) + \text{h.c.} \]

\[ + \frac{g_1^2 + g_2^2}{8} \left( \Phi_d^i \Phi_d^j - \Phi_u^i \Phi_u^j \right)^2 + \frac{g_2^2}{2} \left| \Phi_d^i \Phi_u^j \right|^2 \]

Higgs self-coupling \( \sim g_1^2, g_2^2 \quad \rightarrow \quad \text{small Higgs mass} \)

radiative corr. from top/stop loops \[ \rightarrow m_h \lesssim 135\text{GeV} \]

\[ m_{H^+}, m_A, m_H \rightarrow \infty \quad \rightarrow \quad \text{SM with relatively light Higgs} \]

extra Higgs bosons

\[ m_H > 200\text{GeV} \quad \rightarrow \quad \text{EWPT becomes SM-like} \]

We expect PT unlike the SM when the extra Higgs are light.

the stop mass matrix

\[ M_t = \begin{pmatrix} m_{\tilde{t}^2}^2 + \left( g_1^2 - g_2^2 \right) (v_1^2 - v_2^2) + \frac{g_2^2}{2} v_3^2 & \frac{g_1^2}{2} (v_1 + A_t v_2) \\ \frac{g_2^2}{2} (v_1 + A_t v_2) & m_{\tilde{b}^2}^2 + \frac{g_1^2}{2} (v_1^2 - v_2^2) + \frac{g_2^2}{2} v_3^2 \end{pmatrix} \]

\[ m_{\tilde{t}^2}^2 = 0 \quad \text{smaller eigenvalue} \]

\[ m_{\tilde{t}^2}^2 \sim g_1^2 O(v^2) \]

makes the EWPT of first order

With \( m_h = 125\text{GeV}, \) it's difficult to leave the BAU.

new class of PT's in the model with a gauge singlet

e.g. Next-to-MSSM (NMSSM) = MSSM with a singlet superfield

'Higgs fields' \( \Phi_u(x), \Phi_d(x) : n(x) \)

\[
V_0 = m_1^2 \phi_1^2 + m_2^2 \phi_2^2 + n^* n - \left( \lambda \lambda \epsilon_{ij} n \phi_i^2 + \frac{\kappa}{3} n \phi_i^3 + h.c. \right) \\
+ \frac{g_1^2 + g_2^2}{8} \left( \phi_1^2 - \phi_2^2 \right)^2 + \frac{g_1^2}{2} | \phi_1^2 |^2 \\
+ | \lambda |^2 n^* n (\phi_1^2 \phi_2 + \phi_2^* \phi_1) + | \lambda \epsilon_{ij} n \phi_i^2 + \kappa n^3 |^2
\]

\( \lambda(n) \rightarrow \mu \) in MSSM
[New self-coupling \( \rightarrow \) heavy Higgs]

\( \langle n \rangle \rightarrow \infty \) with \( \lambda(n) \) fixed \( \rightarrow \) reduced to MSSM

New PT is expected for \( \langle n \rangle = O(100 GeV) \).

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**strong 1st order PT**

without a light stop

KF, Tao and Toyoda,

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CP violating complex parameters: \( \lambda, \kappa, \text{Arg}(n) \)

some combinations of them do not affect EDM of \( n \) and \( \mu \),
which are viable for the baryogenesis

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**Summary**

The Higgs mechanism explains the masses of the weak gauge bosons and fermions in the Standard Model, without spoiling renormalizability and unitarity of the theory.

Although existence of the Higgs boson has not yet been established, various observations seem to support it.

success of the electroweak theory

Even if discovery of a CP-even scalar of 125GeV is established, we must do many to check whether it is the SM Higgs boson.

\( \star \) decay branching ratios of various modes
\( \star \) self-coupling of the scalar boson
If the present vacuum is the result of the Higgs mechanism, the state at very high temperatures is different from it and the broken gauge symmetries are restored.

As the Universe cooled down to weak-scale temperature, the gauge symmetry of the EW theory was spontaneously broken by the expectation value of the Higgs fields.

**Electroweak Phase Transition (EWPT)**

Properties of the EWPT depends on the EW model.

- Standard Model with \( m_h = 125 \text{GeV} \) → **Cross Over**
- MSSM
- NMSSM → open possibility of **1st order EWPT**
- 2HDM → extra bosons, another order parameter

So far, the SM has been tested in experiments and found to be consistent with their results.

We, however, know that some extension of the SM is needed to explain the obvious facts,

- Neutrino mass
- Dark Matter
- Baryon Asymmetry of the Universe

Some of the extended models contain **extra scalar fields**, which may lead to the 1st order EWPT.

In particular, extended Higgs sectors predict **charged Higgs bosons** and **extra neutral Higgs bosons**, which combine with each other to make mass eigenstates.

- decay BR's deviate from the SM prediction
- CP violation in the Higgs sector, ... etc.

**mission of ILC**

Thanks for your attention!